Problem Set #1 Basic concepts – complex algebra and three-phase balanced circuits

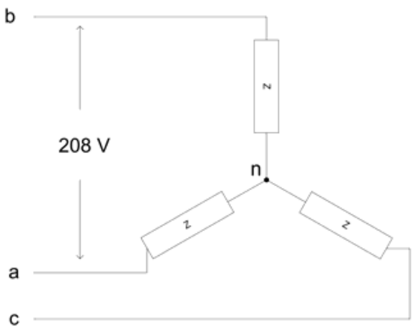
1-1 (Grainger and Stevenson, Jr. Chapter 1, Prob 1.12)
Evaluate the following expressions in polar form:

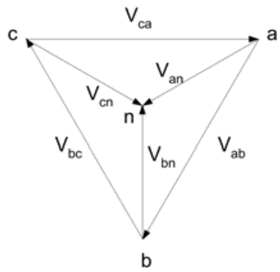
Solution:

a) a-1 = 1.732∠150°
b) 1-a^2+a = 2.00∠60°
c) a^2+a+j = 1.414∠135°
d) ja+a^2 = 1.932∠225°

1-2 (Grainger and Stevenson, Jr. Chapter 1, Prob 1.13)
Three identical impedances of 10∠-15° Ω are Y-connected to balanced three-phase line voltages of 208 V. Specify all the line and phase voltages and the currents as phasors in polar form with V_{ca} as reference for a phase sequence of *abc*.

Solution:





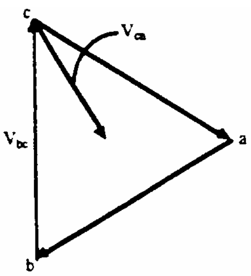
Since V_{ca}=208∠0° V, V_{an}=120∠210° V, V_{bn}=120∠90° V, V_{cn}=120∠-30° V. Therefore,

$$I_a = \frac{V_{an}}{Z} = 12\angle 225° \text{ A},\ I_b = \frac{V_{bn}}{Z} = 12\angle 105° \text{ A},\ I_c = \frac{V_{cn}}{Z} = 12\angle -15° \text{ A}$$

1-3 (Grainger and Stevenson, Jr. Chapter 1, Prob 1.14)
In a balanced three-phase system the Y-connected impedances are 10∠30° Ω. If V_{bc}=416∠90° V, specify I_{cn} in polar form.

Solution:

$$\frac{416}{\sqrt{3}} = 240 \text{ V}$$
$$V_{cn} = 240\angle -60° \text{ V}$$
$$I_{cn} = \frac{240\angle -60°}{10\angle 30°} = 24\angle -90° \text{ A}$$



1-4 (Grainger and Stevenson, Jr. Chapter 1, Prob 1.16)
Determine the current drawn from a three-phase 440-V line by a three-phase 15-hp motor operating at full load, 90% efficiency, and 80% power-factor lagging. Find the values of P and Q drawn from the line.

Solution:

$$|I| = \frac{15 \times 746}{\sqrt{3} \times 440 \times 0.9 \times 0.8} = 20.39 \text{ A}$$
$$P = \sqrt{3} \times 440 \times 20.39 \times 0.8 = 12,431 \text{ W drawn from line}$$
$$Q = \sqrt{3} \times 440 \times 20.39 \times 0.6 = 9,324 \text{ var drawn from line}$$

1-5 (Grainger and Stevenson, Jr. Chapter 1, Prob 1.17)
If the impedance of each of the three lines connecting the motor of Problem 4 (or Prob 1.16) to a bus is 0.3+j1.0 Ω, find the line-to-line voltage at the bus which supplies 440V at the motor.

Solution:

$$I = 20.39\,(0.8 - j0.6) = 16.31 - j12.23 \text{ A}$$

When the reference is voltage to neutral of the motor at the terminal where I is calculated, or $440/\sqrt{3} = 254\angle 0^\circ$ V, the supply bus voltage to neutral is

$$254 + j0 + (0.3 + j1.0)(16.31 - j12.23) = 271.1 + j12.64$$
$$\text{Line-to-line voltage } |V| = \sqrt{3}\,|271.1 + j12.64| = 470 \text{ V}$$

1-6 (Grainger and Stevenson, Jr. Chapter 1, Prob 1.24)
Draw the single-phase equivalent circuit for the motor (an emf in series with inductive reactance labeled Z_m) and its connection to the voltage supply described in Problem 4 and 5 (or Prob. 1.16 and 1.17). Show on the diagram the per-unit values of the line impedance and the voltage at the motor terminals on a base of 20 kVA, 440V. Then using per-unit values, find the supply voltage in per unit and convert the per-unit value of the supply voltage to volts.

Solution:

Per-unit base calculations:

$$\text{Base } Z = \frac{(0.44)^2 \times 1000}{20} = 9.68 \text{ per unit}$$

$$R = \frac{0.3}{9.68} = 0.031 \text{ per unit}$$

$$X = \frac{1.0}{9.68} = 0.1033 \text{ per unit}$$

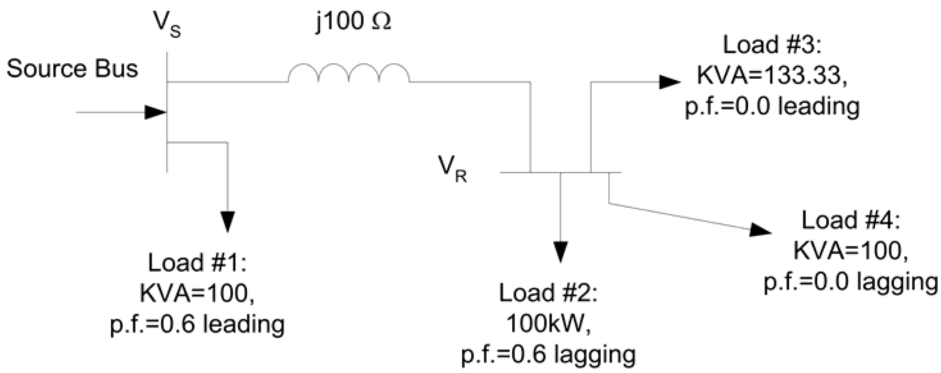
$$\text{Base } I = \frac{20,000}{\sqrt{3} \times 440} = 26.24 \text{ A}$$

$$I = \frac{20.39}{26.24} = 0.777 \text{ per unit}$$

Voltage calculations:

$$\begin{aligned} V &= 1.0 + 0.777\,(0.8 - j0.6)\,(0.031 + j0.1033) \\ &= 1.0 + 0.777 \times 0.1079\angle 36.43^\circ \\ &= 1.0 + 0.0674 + j0.0498 = 1.0686\angle 2.97^\circ \text{ per unit} \\ |V_{LL}| &= 1.0686 \times 440 = 470 \text{ V} \end{aligned}$$

1-7 (Keyhani Lecture 1)
Consider a 3-ϕ distribution feeder as shown below:



Computer the following:

1) The source voltage V_S, if V_R is to be maintained at 4.4 kV (V_R=4.4 kV line value).
2) The source current and power supplied by the source.
3) The total complex power supplied by the source.
4) How much reactive power should be connected to the source bus for obtaining unity power factor at the source bus?

Solution:
Load #1:
p.f.=0.6, leading, cosθ=0.6, sinθ=-0.8,
S_1=100×0.6kW-j100×0.8kVar=60kW-j80kVar;

Load#2:
p.f.=0.6, lagging, cosθ=0.6, sinθ=0.8,
S_2=100kW+j100×0.8/0.6kVar=100kW+j133.33kVar;

Load#3:
p.f.=0.0, leading, cosθ=0.0, sinθ=-1.0,
S_3=-j133.33kVar;

Load#4:
p.f.=0.0, lagging, cosθ=0.0, sinθ=1.0,
S_4=j100kVar;

1) $S_R=S_2+S_3+S_4$=100kW+j100kVar

Assuming $V_{R,L-n}$ has a phase angle of 0°,

$$S_R = 3V_{R,L-n}I_{LR}^*, \qquad I_{LR} = \left(\frac{100 + j100}{3\cdot\frac{4.4}{\sqrt{3}}\angle 0^\circ}\right)^* = 13.14 - j13.14\text{ A}$$

$$V_{s,L-n} = V_{R,L-n} + I_{LR}Z_L = \frac{4400}{\sqrt{3}}\angle 0^\circ + (13.14 - j13.14)\times j100\text{ V} = 4.02\angle 16.35^\circ\text{ kV}$$

$$\left|V_{s,L-L}\right| = \sqrt{3}\cdot 4.02\text{kV} = 6.95\text{kV}$$

2) $$I_{L1} = \left(\frac{60 - j80}{3\cdot 4.02\angle 16.35^\circ}\right)^* = 2.91 + j7.77\text{ A} = 8.29\angle 69.48^\circ\text{ A}$$

$$I_s = I_{L1} + I_{LR} = 16.92\angle -18.5^\circ\text{ A}$$

$$\theta = \angle V_{s,L-n} - \angle I_s = 16.35^\circ - (-18.5^\circ) = 34.85^\circ, \qquad \text{pf} = \cos\theta = 0.82$$

3) $$\begin{aligned} S_s &= 3V_{s,L-n}I_s^* = 3\times 4.02\angle 16.35^\circ \times 16.92\angle 18.5^\circ\text{ kVA} \\ &= 204.06\angle 34.85^\circ\text{ kVA} = 167.46\text{ kW} + j116.61\text{ kVar} \end{aligned}$$

4) S_c=-j116.61 kVar