Problem Set #9 Unsymmetrical faults

9-1 (Grainger and Stevenson, Jr. Chapter 12, Prob. 12.1)

12.1 A 60-Hz turbogenerator is rated 500 MVA, 22 kV. It is Y-connected and solidly grounded and is operating at rated voltage at no load. It is disconnected from the rest of the system. Its reactances are $X_d'' = X_1 = X_2 = 0.15$ and $X_0 = 0.05$ per unit. Find the ratio of the subtransient line current for a single line-to-ground fault to the subtransient line current for a symmetrical three-phase fault.

Solution:

Single line-to-ground fault:

$$I_a^{(1)} = \frac{1}{j0.15 + j0.15 + j0.15} = -j2.857 \text{ per unit}$$

$$I_a = 3I_a^{(1)} = -j8.571 \text{ per unit}$$

Three-phase fault:

$$I_a = \frac{1}{j0.15} = -j6.667 \text{ per unit}$$

The ratio is 8.571/6.667 = 1.286/1.

9-2 (Grainger and Stevenson, Jr. Chapter 12, Prob. 12.3)

12.3 Determine the inductive reactance in ohms to be inserted in the neutral connection of the generator of Prob. 12.1 to limit the subtransient line current for a single line-to-ground fault to that for a three-phase fault.

Solution:

From Prob. 12.1, $I_a = -j6.667$ per unit for a three-phase fault. Let x be the inductive reactance in per unit to be inserted. Then, for a single line-to-ground fault,

$$I_a = 3I_a^{(1)} = \frac{3}{j(0.15 + 0.15 + 0.05 + 3x)}$$

For a three-phase fault, $I_a = 1/j0.15 = -j6.667$ per unit. Equating the values for I_a, we have

$$3 = -j^2(0.35 + 3x)(6.667)$$

$$x = 0.0333 \text{ per unit}$$

$$\text{Base } Z = \frac{(22)^2}{500} = 0.968\ \Omega$$

$$x = 0.0333 \times 0.968 = 0.3226\ \Omega$$

9-3 (Grainger and Stevenson, Jr. Chapter 12, Prob. 12.4)

12.4 With the inductive reactance found in Prob. 12.3 inserted in the neutral of the generator of Prob. 12.1, find the ratios of the subtransient line currents for the following faults to the subtransient line current for a three-phase fault: (*a*) single line-to-ground fault, (*b*) line-to-line fault and (*c*) double line-to-ground fault.

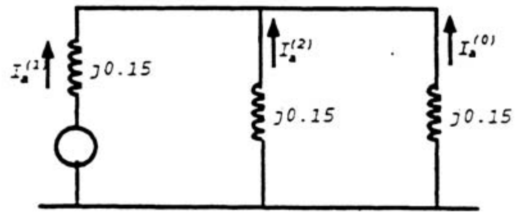
Solution:

(*a*) The ratio equals one since reactance was added to achieve this ratio.

(*b*) The ratio is 0.866 (see Prob. 12.2; this is because the fault current in the line-to-line fault is not affected by the reactance on the neutral).

(*c*) For a double line-to-ground fault,

$$Z_0 = j(0.05 + 3 \times 0.0333) = j0.15$$



$$I_a^{(1)} = \frac{1.0}{j0.15 + \frac{j0.15 \times j0.15}{j0.15 + j0.15}} = \frac{1}{j0.225} = -j4.44 \text{ per unit}$$

$$I_a^{(2)} = I_a^{(0)} = j4.44\left(\frac{0.15}{0.30}\right) = j2.22 \text{ per unit}$$

$$I_f = 3I_a^{(0)} = j6.67 \text{ per unit}$$

$$I_b = 4.44\angle{-90^\circ + 240^\circ} + 2.22\angle{90^\circ + 120^\circ} + 2.22\angle{90^\circ}$$

$$= -3.85 + j2.22 - 1.923 - j1.11 + j2.22 = -5.773 + j3.33 = 6.67\angle{150^\circ} \text{ per unit}$$

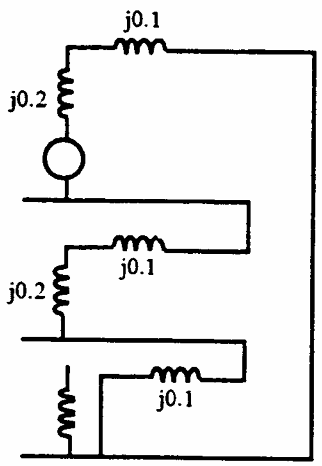
For a three-phase fault,

$$I_a = \frac{1}{j0.15} = -j6.67 \text{ per unit} \qquad \text{Ratio} = \frac{6.67}{6.67} = 1.0$$

9-4 (Grainger and Stevenson, Jr. Chapter 12, Prob. 12.8)

12.8 The reactances of a generator rated 100 MVA, 20 kV, are $X_d'' = X_1 = X_2 = 20\%$ and $X_0 = 5\%$. the generator is connected to a Δ-Y transformer rated 100 MVA, 20Δ-230Y kV, with a reactance of 10%. The neutral of the transformer is solidly grounded. The terminal voltage of the generator is 20 kV when a single line-to-ground fault occurs on the open-circuited, high-voltage side of the transformer. Find the initial symmetrical rms current in all phases of the generator.

Solution:



On the high-voltage side,

$$I_A^{(2)} = I_A^{(0)} = I_A^{(1)} = \frac{1}{j0.3 + j0.3 + j0.1} = -j1.429 = 1.429\angle -90^\circ \text{ per unit}$$

In the generator, $I_a^{(0)} = 0$ and

$$I_a^{(1)} = I_A^{(1)}\angle -30^\circ = 1.429\angle -120^\circ \text{ per unit}$$

$$I_a^{(2)} = I_A^{(2)}\angle 30^\circ = 1.429\angle -60^\circ \text{ per unit}$$

$$\begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} 0 \\ 1.429\angle -120^\circ \\ 1.429\angle -60^\circ \end{bmatrix} = \begin{bmatrix} 2.475\angle -90^\circ \\ 2.475\angle 90^\circ \\ 0 \end{bmatrix} \text{ per unit}$$

Calculating current values we have

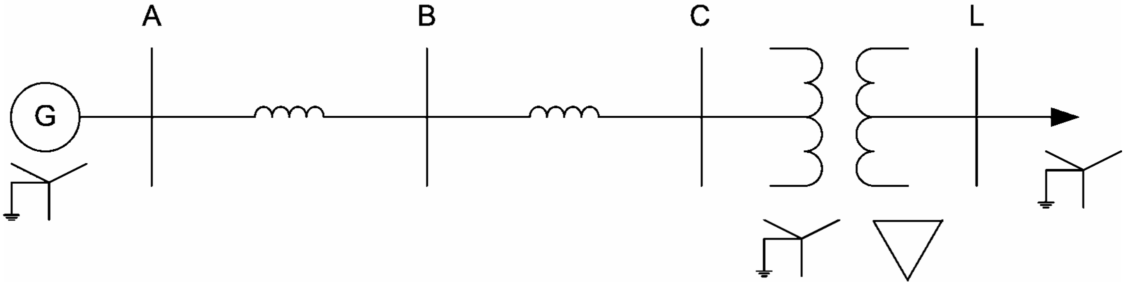
$$\text{Base } I = \frac{100,000}{\sqrt{3} \times 20} = 2887 \text{ A}$$

$$|I_a| = |I_b| = 2.475 \times 2887 = 7145 \text{ A}$$

$$|I_c| = 0 \text{ A}$$

9-5 (Keyhani Lecture)

Consider the power system given below:



Given:

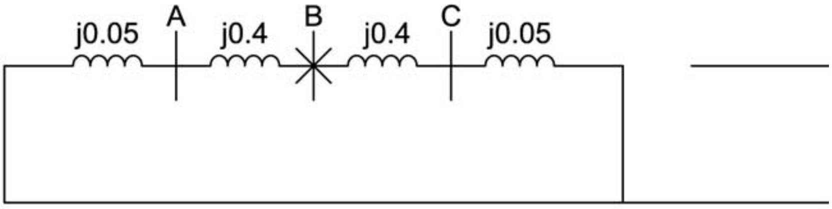
$Z_{G(1)} = Z_{G(2)} = j0.10$ p.u., $Z_{G(0)} = j0.05$ p.u.

$Z_{AB(1)} = Z_{AB(2)} = j0.2$ p.u., $Z_{AB(0)} = j0.4$ p.u.
$Z_{BC(1)} = Z_{BC(2)} = j0.20$ p.u., $Z_{BC(0)} = j0.4$ p.u.
$Z_{T(1)} = Z_{T(2)} = Z_{T(0)} = j0.05$ p.u.
$V_L = 0.9\angle{-4.0°}$ p.u., $S_L = 1+j0.5$ p.u.

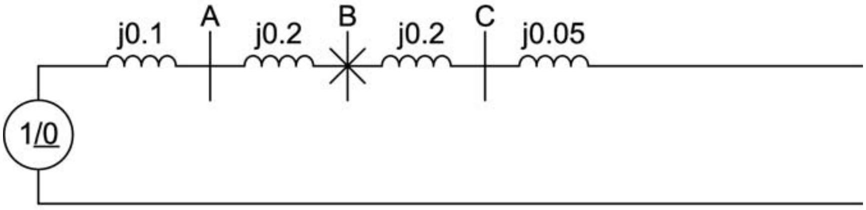
For a single line to ground fault at Bus B, compute the following:

1) The fault currents flowing from Bus A and Bus C to Bus B (faulted bus) when the load is ignored.
2) The same as part (1), but take the load into consideration.
3) The same as part (1), but assume the generator is not grounded.

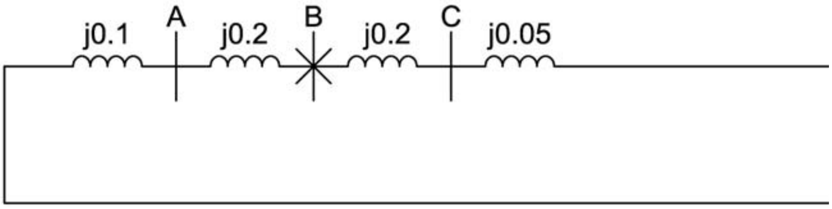
Solution:
1) The sequence network is shown as follows when load is ignored:



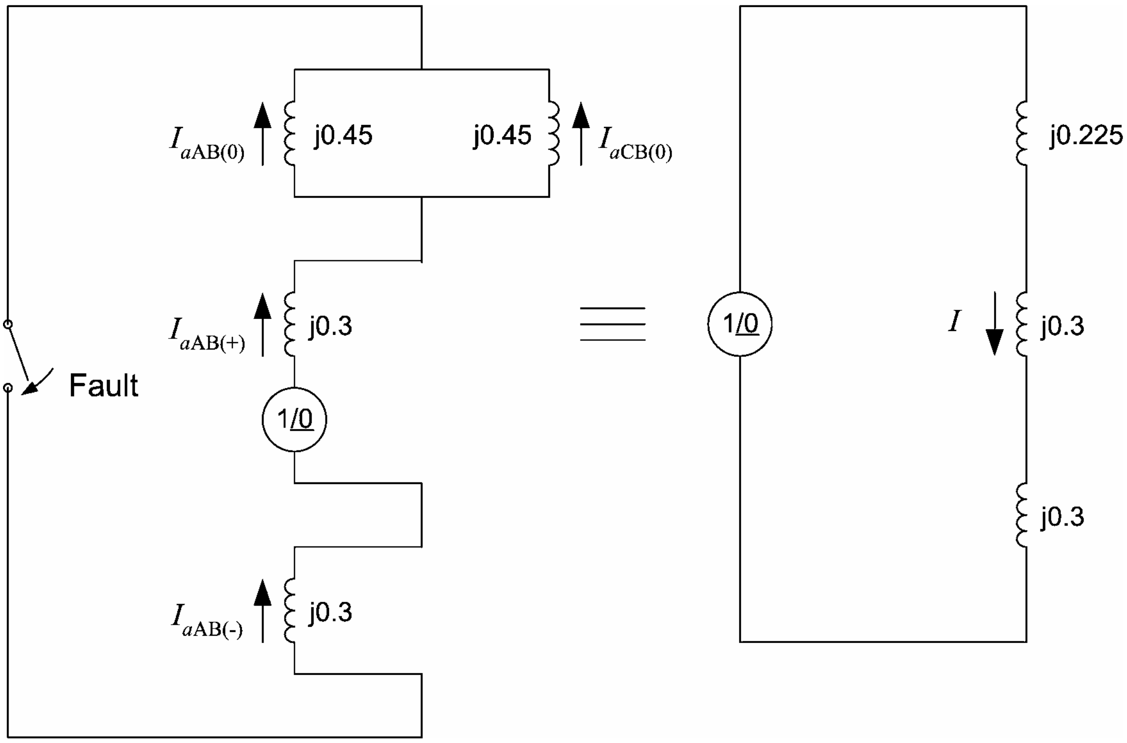
"0" sequence



"+" sequence



"-" sequence



$$I = \frac{V}{Z} = \frac{1\angle 0°}{j0.825} = 1.212\angle -90°$$

$$I_{aAB(0)} = I_{aCB(0)} = \frac{1}{2} I = 0.606\angle -90°$$

$$I_{aAB(+)} = I_{aAB(-)} = I = 1.212\angle -90°$$

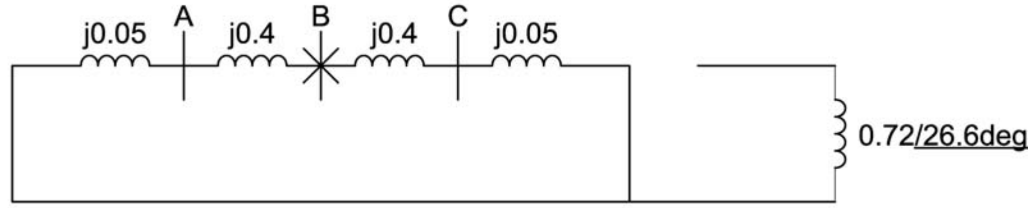
$$I_{aCB(+)} = I_{aCB(-)} = 0$$

$$\begin{bmatrix} I_{aAB} \\ I_{bAB} \\ I_{cAB} \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} 0.606\angle -90° \\ 1.212\angle -90° \\ 1.212\angle -90° \end{bmatrix} = \begin{bmatrix} -j3.03 \\ j0.606 \\ j0.606 \end{bmatrix}$$

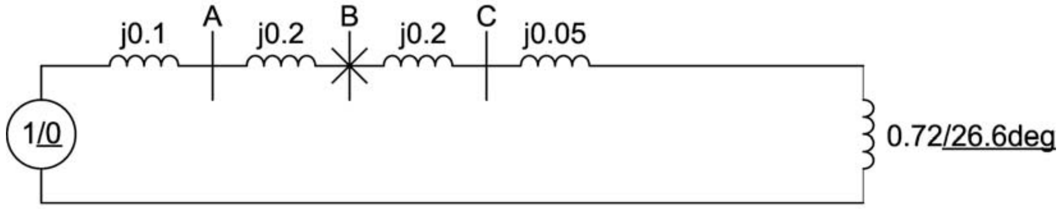
$$\begin{bmatrix} I_{aCB} \\ I_{bCB} \\ I_{cCB} \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} 0.606\angle -90° \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} -j0.606 \\ -j0.606 \\ -j0.606 \end{bmatrix}$$

2) Take the load into consideration

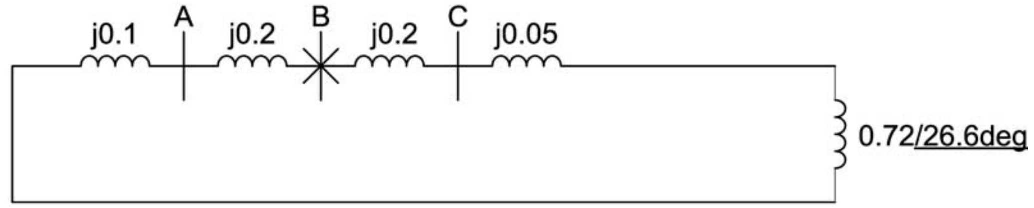
$$Z = \frac{V_L^2}{S_L} = \frac{0.9^2}{1 - j0.5} = 0.65 + j0.32 = 0.72\angle 26.6°$$



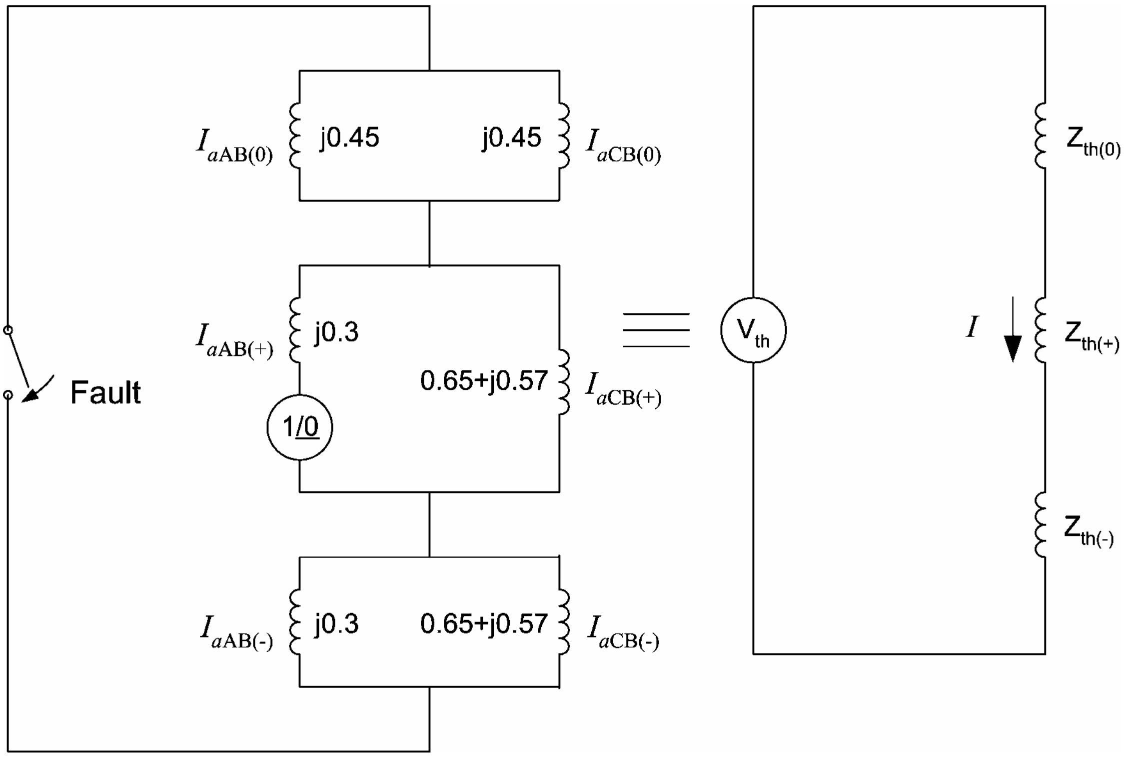
“0” sequence



“+” sequence



“-” sequence



$$Z_{th(+)} = Z_{th(-)} = \frac{j0.3\cdot(0.65+j0.57)}{j0.3+0.65+j0.57} = 0.0496+j0.2336\,,\quad Z_{th(0)} = j0.225$$

V_{th} is Bus B voltage before the fault occurs:

$$V_{th} = V_B = 1\angle 0°\cdot\frac{0.65+j0.57}{j0.3+0.65+j0.57} = 0.779-j0.165 = 0.8\angle -12°$$

$$I = \frac{V_{th}}{Z_{th(0)}+Z_{th(+)}+Z_{th(-)}} = \frac{0.779-j0.165}{0.0992+j0.6922} = 1.1384\angle -93.8°$$

$$I_{aAB(0)} = I_{aCB(0)} = \frac{1}{2}I = 0.5692\angle -93.8°$$

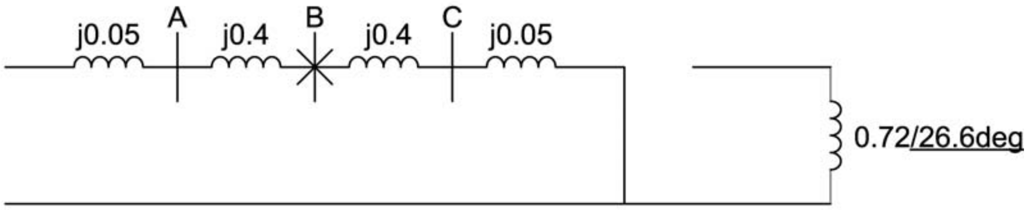
$$I_{aAB(+)} = I_{aAB(-)} = I\cdot\frac{0.65+j0.57}{j0.3+0.65+j0.57} = 0.9062\angle -105.8°$$

$$I_{aCB(+)} = I_{aCB(-)} = I\cdot\frac{j0.3}{j0.3+0.65+j0.57} = 0.3145\angle -57.1°$$

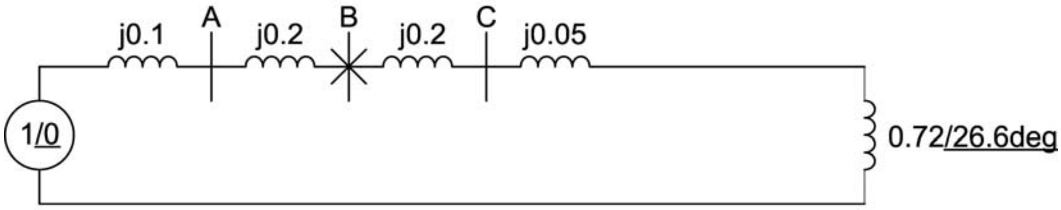
$$\begin{bmatrix} I_{aAB} \\ I_{bAB} \\ I_{cAB} \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} I_{aAB(0)} \\ I_{aAB(+)} \\ I_{aAB(-)} \end{bmatrix} = \begin{bmatrix} -0.5321 - j2.3117 \\ 0.2090 + j0.3040 \\ 0.2090 + j0.3040 \end{bmatrix}$$

$$\begin{bmatrix} I_{aCB} \\ I_{bCB} \\ I_{cCB} \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} I_{aCB(0)} \\ I_{aCB(+)} \\ I_{aCB(-)} \end{bmatrix} = \begin{bmatrix} 0.3039 - j1.0958 \\ -0.2090 - j0.3040 \\ -0.2090 - j0.3040 \end{bmatrix}$$

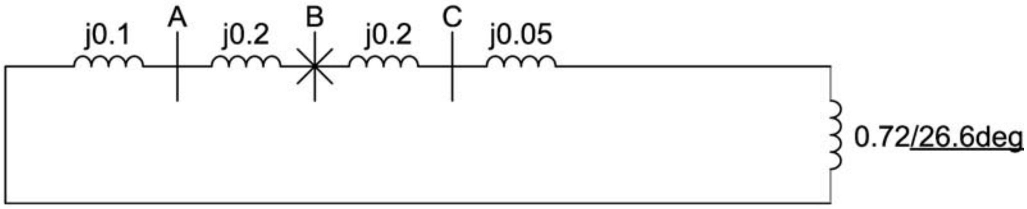
3) Generator is not grounded



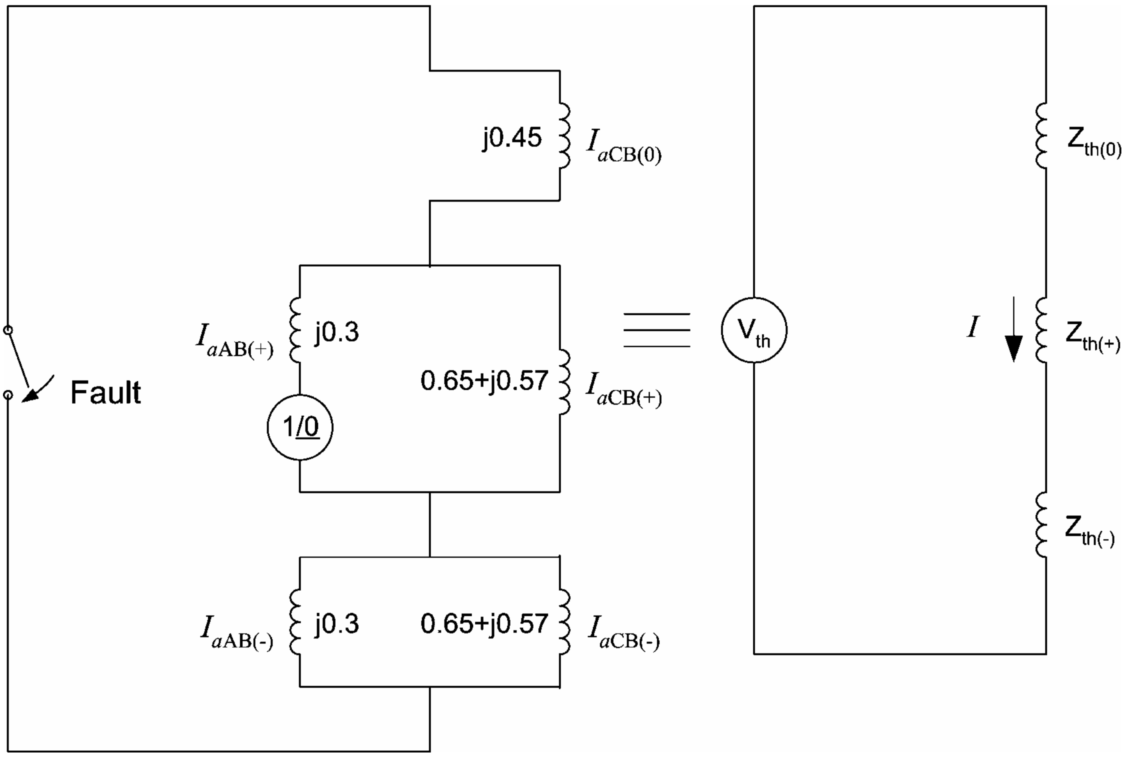
“0” sequence



“+” sequence



“-” sequence



$$Z_{th(+)} = Z_{th(-)} = \frac{j0.3 \cdot (0.65 + j0.57)}{j0.3 + 0.65 + j0.57} = 0.0496 + j0.2336\,, \quad Z_{th(0)} = j0.45$$

V_{th} is Bus B voltage before the fault occurs:

$$V_{th} = V_B = 1\angle 0° \cdot \frac{0.65 + j0.57}{j0.3 + 0.65 + j0.57} = 0.779 - j0.165 = 0.8\angle -12°$$

$$I = \frac{V_{th}}{Z_{th(0)} + Z_{th(+)} + Z_{th(-)}} = \frac{0.779 - j0.165}{0.0992 + j0.9172} = 0.8629\angle -95.8°$$

$$I_{aAB(0)} = 0\,, \qquad I_{aCB(0)} = I = 0.8629\angle -95.8°$$

$$I_{aAB(+)} = I_{aAB(-)} = I \cdot \frac{0.65 + j0.57}{j0.3 + 0.65 + j0.57} = 0.6869\angle -107.8°$$

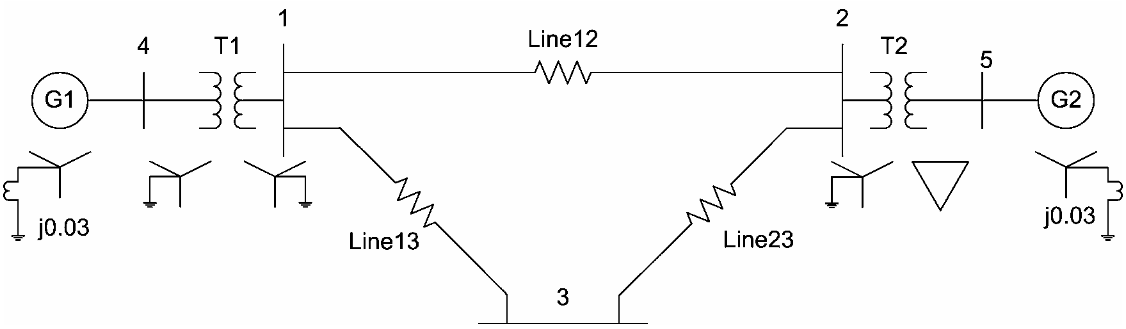
$$I_{aCB(+)} = I_{aCB(-)} = I \cdot \frac{j0.3}{j0.3 + 0.65 + j0.57} = 0.2384\angle -59.05°$$

$$\begin{bmatrix} I_{aAB} \\ I_{bAB} \\ I_{cAB} \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} I_{aAB(0)} \\ I_{aAB(+)} \\ I_{aAB(-)} \end{bmatrix} = \begin{bmatrix} -0.4200 - j1.3080 \\ 0.2100 + j0.6540 \\ 0.2100 + j0.6540 \end{bmatrix}$$

$$\begin{bmatrix} I_{aCB} \\ I_{bCB} \\ I_{cCB} \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} I_{aCB(0)} \\ I_{aCB(+)} \\ I_{aCB(-)} \end{bmatrix} = \begin{bmatrix} 0.1578 - j1.2673 \\ -0.2100 - j0.6540 \\ -0.2100 - j0.6540 \end{bmatrix}$$

9-6 (Keyhani Lecture)
Consider the following system. Derive Thevenin equivalent sequence networks looking at Bus 1.

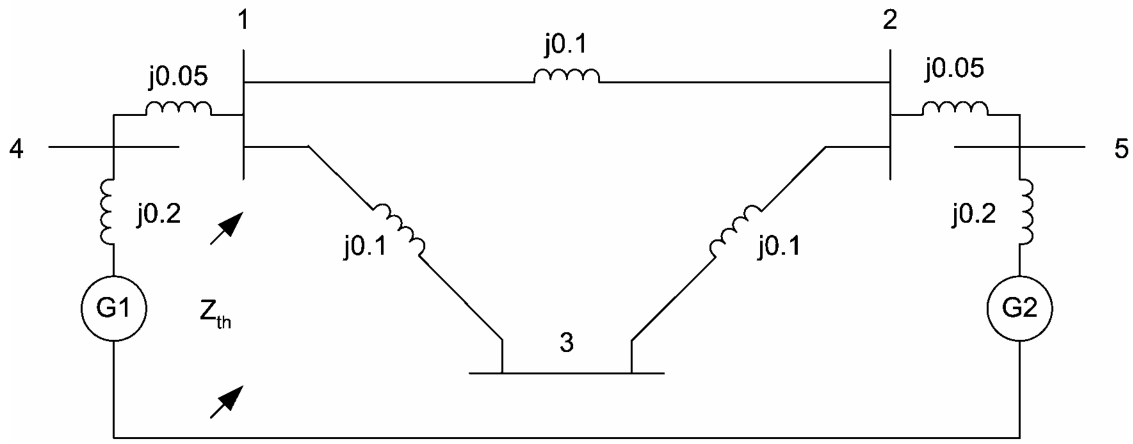
Item	MVA Rating	Voltage Rating	X_1	X_2	X_0
G1	100	25 kV	0.2	0.2	0.05
G2	100	13.8 kV	0.2	0.2	0.05
T1	100	25/230 kV	0.05	0.05	0.05
T2	100	13.8/230 kV	0.05	0.05	0.05
Line12	100	230 kV	0.1	0.1	0.3
Line13	100	230 kV	0.1	0.1	0.3
Line23	100	230 kV	0.1	0.1	0.3

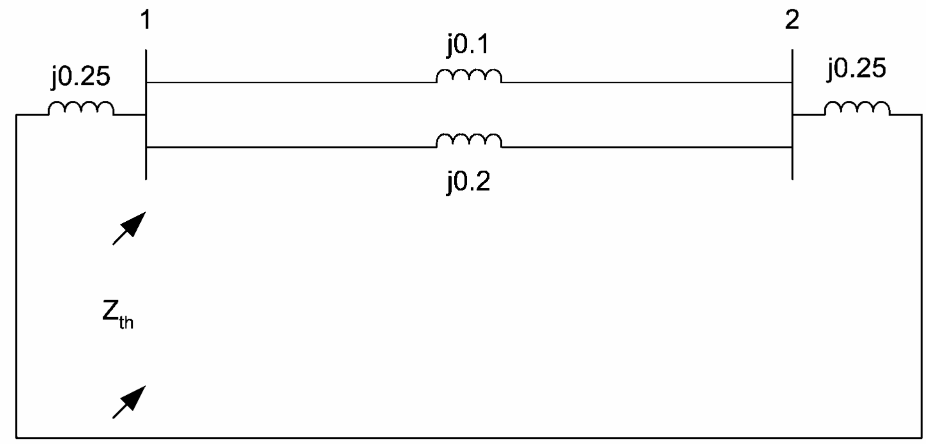


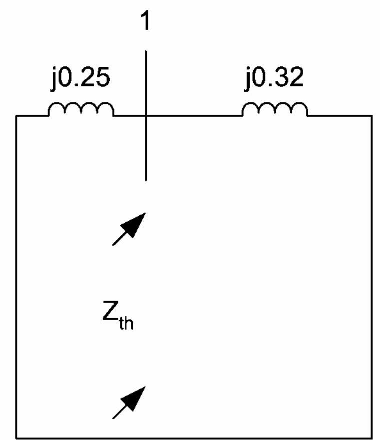
a) Draw the sequence networks.
b) Reduce the networks in (a) to their Thevenin equivalents "looking in" at Bus 1.

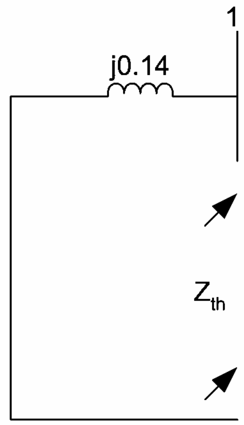
Solution:

a) "+" and "-" sequences:

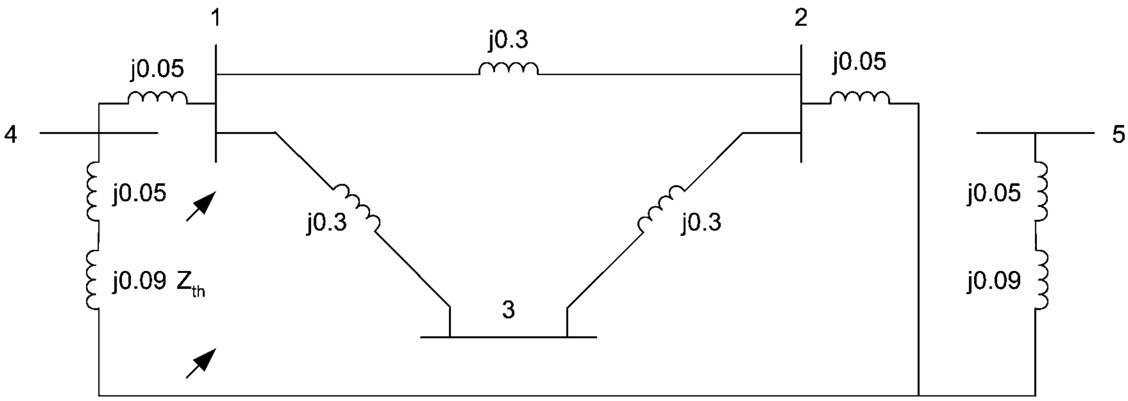

1
2
j0.1
j0.05
j0.05
4
5
j0.2
j0.2
j0.1
j0.1
G1
G2
Z_th
3

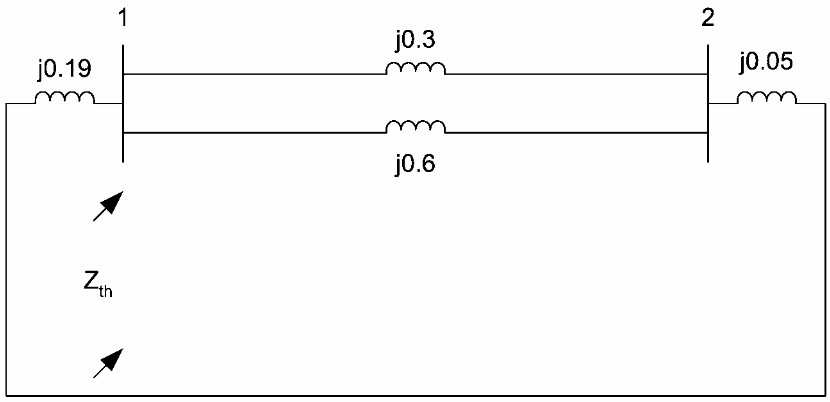

1
2
j0.1
j0.25
j0.25
j0.2
Z_th

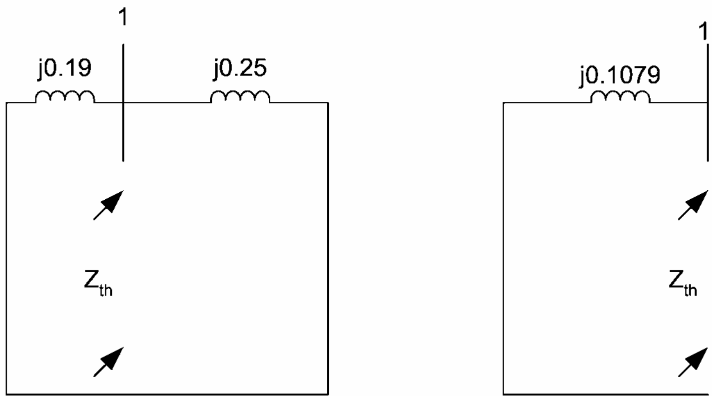

1
j0.25
j0.32
Z_th


1
j0.14
Z_th

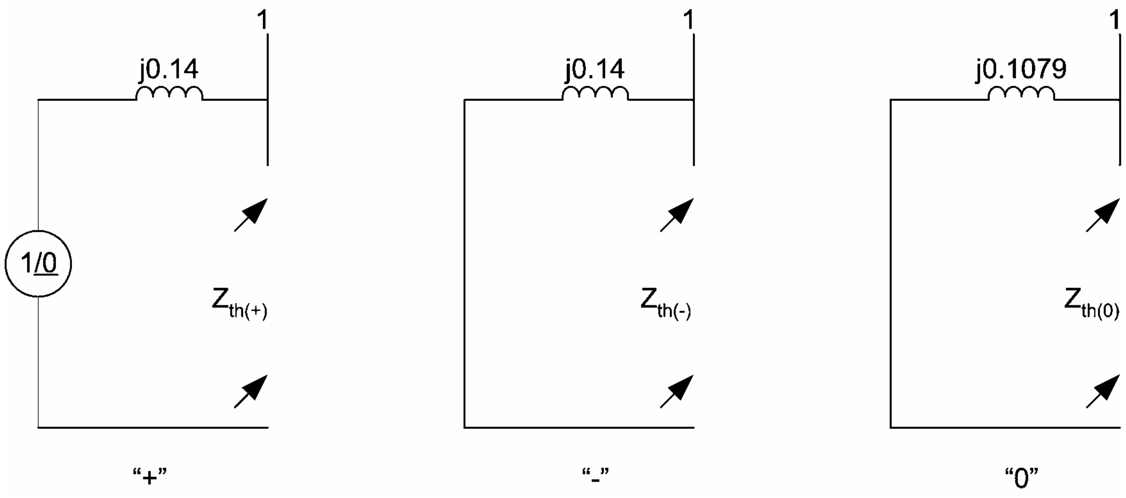
"0" sequence:







Therefore, the Thevenin equivalent sequence networks are:



9-7 (Keyhani Lecture)
For a 3ϕ fault at Bus "1" in Problem 9-6, calculate the fault phase voltages and currents.

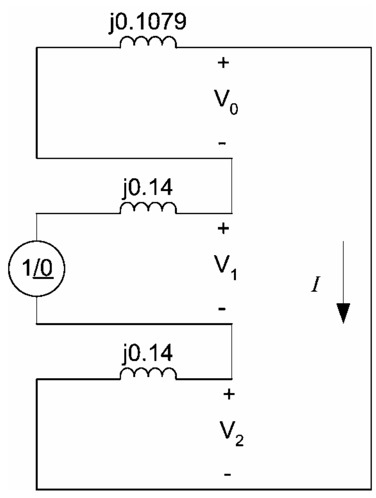
Solution:

$I_1 = \dfrac{1\angle 0°}{j0.14} = -j7.14$, $I_a = -j7.14 = 7.14\angle -90°$, $I_b = 7.14\angle -90° + 240° = 150°$, and
$I_c = 7.14\angle -90° + 120° = 30°$

I_0=I_2=0, V_1=V_2=V_0=0, and V_a=V_b=V_c=0 at Bus 1.

9-8 (Keyhani Lecture)
Repeat Problem 9-7 for a single phase line to ground fault.

Solution:



$$I_0 = I_1 = I_2 = I = \frac{1\angle 0°}{j(0.14+0.14+0.1079)} = \frac{1}{j0.3879} = -j2.578$$

I_a=3I_1=-j7.734

I_b=I_c=0

$$V_1 = 1.0 - j0.14(-j2.578) = 1.0 - 0.36 = 0.64$$

$$V_0 = -(-j2.578 \cdot j0.1079) = 0.279\angle 180°$$

$$V_2 = -(-j2.578 \cdot j0.14) = 0.36\angle 180°$$

$$V_a = V_0 + V_1 + V_2 = 0$$

$$V_b = V_0 + a^2 V_1 + aV_2 = 0.962\angle -116°$$

$$V_b = V_0 + aV_1 + a^2 V_2 = 0.962\angle 116°$$