Image Processing:

2. Camera Calibration

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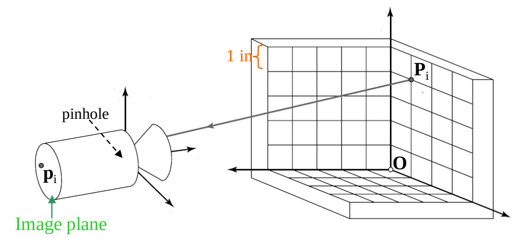
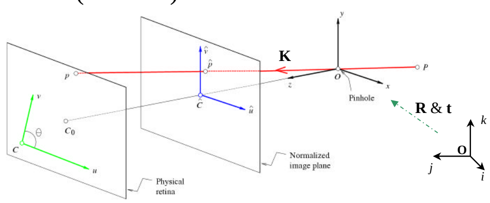
Calibration: 3D to 2D to 3D



$$\mathbf{P}_i = \begin{pmatrix} x_i \\ y_i \\ z_i \end{pmatrix} \Rightarrow \mathbf{p}_i = \begin{pmatrix} x_i \\ y_i \end{pmatrix} \Rightarrow \mathbf{P}_i = \begin{pmatrix} x_i \\ y_i \\ z_i \end{pmatrix}$$

Intrinsic and extrinsic parameters

- To calibrate a camera means we need to find the parameters of **K** (intrinsic) and **R** and **t** (extrinsic).



Perspective projection

$$\begin{cases} u_i = \dfrac{\mathbf{m}_1^T \mathbf{P}_i}{\mathbf{m}_3^T \mathbf{P}_i} \\ v_i = \dfrac{\mathbf{m}_2^T \mathbf{P}_i}{\mathbf{m}_3^T \mathbf{P}_i} \end{cases} \Rightarrow \begin{cases} (\mathbf{m}_1^T - u_i \mathbf{m}_3^T)\mathbf{P}_i = 0 \\ (\mathbf{m}_2^T - v_i \mathbf{m}_3^T)\mathbf{P}_i = 0 \end{cases}$$

where $\mathbf{M} = \begin{pmatrix} \mathbf{m}_1^T \\ \mathbf{m}_2^T \\ \mathbf{m}_3^T \end{pmatrix}$

Linear Least Squares

- Typical method to solve a system of p equations and q unknowns when $p>q$.

$$\begin{cases} u_{11}x_1 + \ldots + u_{1q}x_q = y_1 \\ \vdots \\ u_{p1}x_1 + \ldots + u_{pq}x_q = y_p \end{cases} \Rightarrow \mathbf{Ux} = \mathbf{y}.$$

where $\mathbf{x} = (x_1, \ldots, x_q)^T$, $\mathbf{y} = (y_1, \ldots, y_p)^T$ & $\mathbf{U} = \begin{pmatrix} u_{11} & \cdots & u_{1q} \\ \vdots & \ddots & \vdots \\ u_{p1} & \cdots & u_{pq} \end{pmatrix}$.

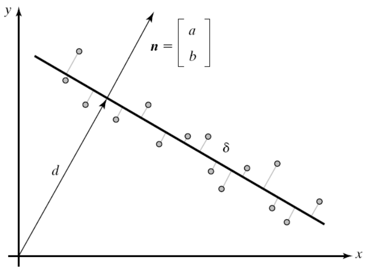
- If $p>q$, there is no solution. Need to define an error function :

$$E = \sum_{i=1}^{n} (u_{i1}x_i + \ldots + u_{iq}x_q - y_i)^2 = |\mathbf{Ux} - \mathbf{y}|^2$$

- To minimize this unction: $\dfrac{\partial E}{\partial x_i} = 0.$
- This yields: $\mathbf{x} = (\mathbf{U}^T\mathbf{U})^{-1}\mathbf{U}^T\mathbf{y}.$

  pseudo-inverse
- In homogeneous systems, we keep thje eigenvector associated to the smallest eigenvalue.

Example: Fitting a line



Estimating the projection matrix

$$\begin{cases}(\mathbf{m}_1^T - u_i\mathbf{m}_3^T)\mathbf{P}_i = 0\\ (\mathbf{m}_2^T - v_i\mathbf{m}_3^T)\mathbf{P}_i = 0\end{cases}$$

known

$$P = \begin{pmatrix} \mathbf{P}_1^T & \mathbf{0}^T & -u_1\mathbf{P}_1^T \\ \mathbf{0}^T & \mathbf{P}_1^T & -v_1\mathbf{P}_1^T \\ \vdots & \vdots & \vdots \\ \mathbf{P}_n^T & \mathbf{0}^T & -u_n\mathbf{P}_n^T \\ \mathbf{0}^T & \mathbf{P}_n^T & -v_n\mathbf{P}_n^T \end{pmatrix}$$

unknown

$$\mathbf{M} = \begin{pmatrix} \mathbf{m}_1^T \\ \mathbf{m}_2^T \\ \mathbf{m}_3^T \end{pmatrix}$$

Least squares solution:

$$\min |P\mathbf{M}|^2$$

Eigenvector **e** associated to smallest eigenvalue λ.

Recovering **K**, **R**, and **t**

$$\rho(\mathbf{Ab}) = \mathbf{K}(\mathbf{Rt}) \Rightarrow \rho\begin{pmatrix} \mathbf{a}_1^T \\ \mathbf{a}_2^T \\ \mathbf{a}_3^T \end{pmatrix} = \begin{pmatrix} \alpha\mathbf{r}_1^T - \alpha\cot\theta\,\mathbf{r}_2^T + u_o\mathbf{r}_3^T \\ \dfrac{\beta}{\sin\theta}\mathbf{r}_2^T + v_o\mathbf{r}_3^T \\ \mathbf{r}_3^T \end{pmatrix}$$

From this equation and

$$\mathbf{r}_i\,\mathbf{r}_j = 0$$
$$\|\mathbf{r}_i^T\| = 1$$

$$\Rightarrow \begin{cases} \rho = \varepsilon/|\mathbf{a}_3|, \text{ where } \varepsilon = \pm 1 \\ \mathbf{r}_3 = \rho\mathbf{a}_3 \\ u_o = \rho^2(\mathbf{a}_1\mathbf{a}_2) \\ v_o = \rho^2(\mathbf{a}_2\mathbf{a}_3) \end{cases}$$

Now, using the fact that $\theta \approx \frac{\pi}{2}$:

$$\begin{cases} \rho^2|\mathbf{a}_1\times\mathbf{a}_2| = \dfrac{|\alpha|}{\sin\theta} \\ \rho^2|\mathbf{a}_2\times\mathbf{a}_3| = \dfrac{|\beta|}{\sin\theta} \end{cases} \Rightarrow \begin{cases} \cos\theta = \dfrac{(\mathbf{a}_1\times\mathbf{a}_3)(\mathbf{a}_2\times\mathbf{a}_3)}{|\mathbf{a}_1\times\mathbf{a}_3||\mathbf{a}_2\times\mathbf{a}_3|} \\ \alpha = \rho^2|\mathbf{a}_1\times\mathbf{a}_3|\sin\theta \\ \beta = \rho^2|\mathbf{a}_2\times\mathbf{a}_3|\sin\theta \end{cases}$$

$$\begin{cases} \mathbf{r}_1 = \dfrac{\rho^2\sin\theta}{\beta}(\mathbf{a}_2\times\mathbf{a}_3) = \dfrac{1}{|\mathbf{a}_2\times\mathbf{a}_3|}(\mathbf{a}_2\times\mathbf{a}_3) \\ \mathbf{r}_2 = \mathbf{r}_1\times\mathbf{r}_3 \end{cases} \Rightarrow \mathbf{R} = \begin{pmatrix} \mathbf{r}_1^T \\ \mathbf{r}_2^T \\ \mathbf{r}_3^T \end{pmatrix}$$

Two solutions $\varepsilon = \pm 1$

The translation is give by: $\mathbf{K}(\mathbf{Rt}) = \rho(\mathbf{Ab}) \Rightarrow \mathbf{t} = \rho\mathbf{K}^{-1}\mathbf{b}$

Degenerate point configuration

We note that $P\mathbf{M} \neq 0$.

The ideal solution (the null space of *P*) is given by: $P\mathbf{l} = 0$.

$$P\mathbf{l} = \begin{pmatrix} \mathbf{P}_1^T & \mathbf{0}^T & -u_1\mathbf{P}_1^T \\ \mathbf{0}^T & \mathbf{P}_1^T & -v_1\mathbf{P}_1^T \\ \vdots & \vdots & \vdots \\ \mathbf{P}_n^T & \mathbf{0}^T & -u_n\mathbf{P}_n^T \\ \mathbf{0}^T & \mathbf{P}_n^T & -v_n\mathbf{P}_n^T \end{pmatrix}\begin{pmatrix} \mathbf{l}_1 \\ \mathbf{l}_2 \\ \mathbf{l}_3 \end{pmatrix} = \begin{pmatrix} \mathbf{P}_1^T\mathbf{l}_1 - u_1\mathbf{P}_1^T\mathbf{l}_3 \\ \mathbf{P}_1^T\mathbf{l}_2 - v_1\mathbf{P}_1^T\mathbf{l}_3 \\ \vdots \\ \mathbf{P}_n^T\mathbf{l}_1 - u_n\mathbf{P}_n^T\mathbf{l}_3 \\ \mathbf{P}_n^T\mathbf{l}_2 - v_n\mathbf{P}_n^T\mathbf{l}_3 \end{pmatrix}$$

$$0 = P\mathbf{l} = \begin{pmatrix} \mathbf{P}_1^T\mathbf{l}_1 - u_1\mathbf{P}_1^T\mathbf{l}_3 \\ \mathbf{P}_1^T\mathbf{l}_2 - v_1\mathbf{P}_1^T\mathbf{l}_3 \\ \vdots \\ \mathbf{P}_n^T\mathbf{l}_1 - u_n\mathbf{P}_n^T\mathbf{l}_3 \\ \mathbf{P}_n^T\mathbf{l}_2 - v_n\mathbf{P}_n^T\mathbf{l}_3 \end{pmatrix}$$

$$\begin{cases}(\mathbf{m}_1^T - u_i\mathbf{m}_3^T)\mathbf{P}_i = 0\\ (\mathbf{m}_2^T - v_i\mathbf{m}_3^T)\mathbf{P}_i = 0\end{cases}$$

$$\begin{cases} \mathbf{P}_i^T(\mathbf{l}_1\mathbf{m}_3^T - \mathbf{m}_1\mathbf{l}_3^T)\mathbf{P}_i = 0 \\ \mathbf{P}_i^T(\mathbf{l}_2\mathbf{m}_3^T - \mathbf{m}_2\mathbf{l}_3^T)\mathbf{P}_i = 0 \end{cases}$$

To obtain a solution the points $\mathbf{P}_i$ cannot be coplanar.

A possible solution is:

$\mathbf{l}_1 = \mathbf{m}_1, \mathbf{l}_2 = \mathbf{m}_2, \mathbf{l}_3 = \mathbf{m}_3$.