Sensor Network Localization via Received Signal Strength Measurements with Directional Antennas

Joshua N. Ash and Lee C. Potter
Dept. of Electrical and Computer Engineering, The Ohio State University
2015 Neil Avenue, Columbus, OH 43210
e-mail: {ashj,potter}@ece.osu.edu

Abstract

In wireless sensor networks position awareness is necessary to exploit the communication benefits of directional antennas and for sensors to provide meaningful information about their surroundings. In this paper we evaluate the feasibility and quality of self-localization that can be obtained using received signal strength (RSS) measurements from arrays of directional antennas on each sensor node. We compare the performance of an optimal estimator utilizing all RSS data with a suboptimal one using angles of arrival as an intermediate statistic. Further, we compare the performance bounds of our approach with others based on distance measurements potentially obtained through time of arrival or RSS observations. We demonstrate that sub-meter location accuracy is possible using 802.11 radio frequency communication signals and no assumed model for propagation loss. Results from an outdoor field experiment and our proposed printed circuit board quasi-Yagi directional antenna are also presented.

1 Introduction

High-gain, electronically scanning directional antenna arrays can enhance the performance of wireless networks by decreasing power consumption, reducing interference, and increasing range. Most ad hoc networks use low gain omnidirectional antennas which radiate energy in all directions. In contrast, a directional antenna concentrates energy in a particular direction with a higher gain. The reduction in interference to surrounding nodes increases the spatial reuse factor and therefore increases total network throughput. The directionality of transmissions can also be combined with the routing protocol to reduce network congestion. Further, network connectivity is the crucial ingredient for scalable, sparse, robust networks, and the gain afforded by directional high-efficiency antenna arrays provides a dramatic increase in network connectivity.

To exploit the directional antenna array and achieve many of the benefits listed above, each node needs to know its own location relative to other nodes in the network. In addition, position awareness is typically required of wireless sensors used for monitoring the environment or used for surveillance. Manual assignment of node locations is one possibility, but is often impractical or impossible due to the number of nodes or method of deployment. Equipping each sensor with a GPS receiver is another solution, but is often cost prohibitive and depends on the availability of GPS signals.

Centuries-old solutions to the localization problem are known from navigation and surveying. For ad hoc sensor networks, localization has been addressed by many authors (see, e.g., [1, 2, 3] and references therein). Approaches differ in the sensor wavelength (optical, radio frequency, ultrasound, acoustic), the problem formulation (deterministic versus probabilistic), propagation assumptions, and the computation (distributed versus centralized). Approaches vary in the assumed density of anchor nodes with known location, and some schemes only use soundings from anchors. Received signal strength (RSS) is available from the RF communications typically resident on a node; however, RSS is plagued by the high variability of propagation losses. In many schemes, the RSS is quantized to a single bit to indicate proximity. Ultrasonic and acoustic transducers have been used to measure time difference of arrival, exploiting the relatively slow propagation speeds, and to measure angle of arrival using an array of microphones. Sub-meter accuracy is available from ultrasonic or acoustic measurements, however these modalities require additional hardware and may compromise stealth.

In this paper, we present a low-cost approach to achieve sub-meter accuracy from RF communications signals. We evaluate the quality of different localization estimators that only use received signal strength measurements obtained by directional antennas on the nodes. The location estimates we consider only depend on the relative strengths of signals measured at each antenna and do not depend on distance estimates from an assumed path loss model.

2 The Localization Estimation Problem

Consider N sensor nodes in the plane at two-dimensional locations $[x_n, y_n]$. Each node has N_a selectable antenna beams. Resulting from RF transmissions from node t to node r, the complete set of observable variables is $s_{rt} \in \mathbb{R}^{N_a \times N_a}$ – that is, the received signal strengths from the N_a^2 possible transmit-receive antenna configurations. Each node has a (potentially uninformative) prior distribution on position, $p_n(x_n, y_n)$. In the extreme case, the prior may reflect low-variability node placement information derived from GPS or hand-placement of a sparse subset of nodes, called anchors or beacons.

The *localization service problem* is to estimate the locations, $[x_n, y_n]$, of all nodes and report on the uncertainties of these estimates. The locations may be computed relatively with respect to one another, with unknown translation and rotation, producing a *relative* localization; or, the locations may be required with respect to a fixed, global coordinate system, yielding an *absolute* localization.

The focus in this paper is to establish feasibility and compute performance bounds on the accuracy of localization using the RSS observations from directional antennas. The Cramér-Rao lower bound on location error variance is used to predict performance for efficient estimators and can inform design trade-offs for antennas, communication protocols, and estimation algorithms.

To proceed, we adopt a specific observation model and then compare location estimation using the full observation set with estimation using bearing estimates as simple, intuitive intermediate statistics of the data. Although not sufficient statistics, the bearing angles are found to reduce location precision by only 33%.

We assume the N_a beams are identical and uniformly oriented with spacing $\Delta = 360/N_a$ degrees (Figure 1). A transmission from node t to node r arrives with an angle of arrival (AOA), θ_{rt}, relative to node r's local coordinate system. To model receiver sensitivity, we assume that a packet is received by only a subset of antennas, $\mathcal{V}_{rt}$, for example as determined by θ_{rt}, N_a, and the main beam widths of the antennas. Transmission may be repeated for M packets, yielding M independent measurements of the RF signal strength per antenna. Expressed on a log scale (dBm), the received signal

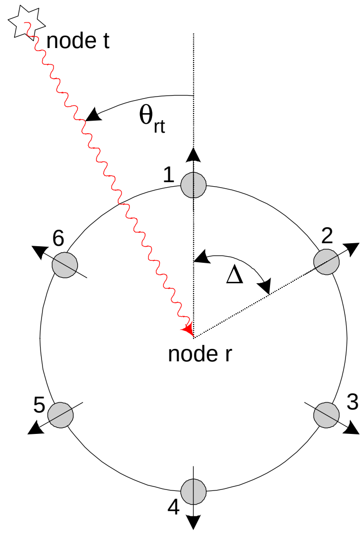

Figure 1: Example six-element directional antenna array at receiving node r. θ_{rt} is the angle of arrival in r's local coordinate system of a signal from transmitter t.

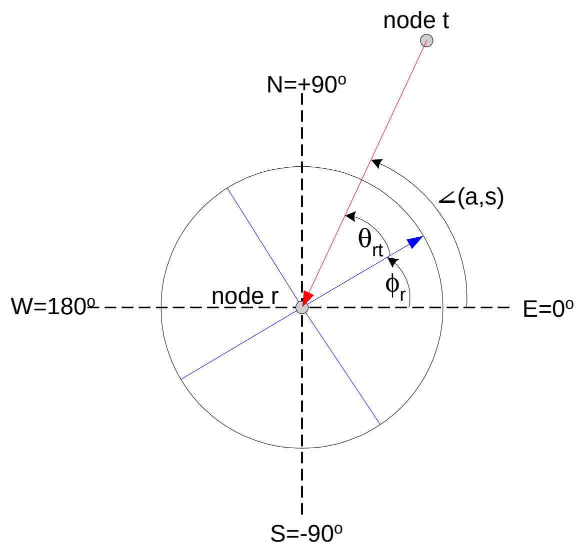

Figure 2: Local coordinate system. Node r's local coordinate system is rotated by an unknown amount, ϕ_r, from the global coordinate reference.

strength for antenna i on node r for transmission from t is

$$s_{irt} = g\big(\angle(r,t) - \phi_r + (i-1)\Delta\big) + \mu_{rt} + S_{irt} \tag{1}$$

where $g(\theta)$ is the gain (dB) of the directional antenna at angle θ, ϕ_r is an unknown orientation of the receiving node (Figure 2), and μ_{rt} is the signal power (dBm) incident upon the antenna array at node r. In general, μ_{rt} depends on the transmit power, the gain of the transmit antenna, and the path loss between t and r, which in turn depends on the propagation environment and the locations of t and r. A commonly used path loss model is the log-distance model [4, 5, 6].

For known transmission power and beam pattern, a probabilistic path loss model could be adopted, directly linking signal strength to distance. However, here we adopt a conservative, but robust, approach and assume that the functional dependence of μ_{rt} on the location parameters is unknown. As such, μ_{rt} is a deterministic but unknown nuisance parameter present in the estimation task. (The added utility of distance dependence is explored in Section 5.)

The random variable S_{irt} in (1) models the variability between measurements. We assume that (on this log scale) S_{irt} is Gaussian distributed $\mathcal{N}(0, \sigma^2_{rss})$ and independent of other measurements. This model is similar to the log-normal shadowing model used by other authors [4, 5]; however, here S_{irt} models the variability of a repeated measurement at the same location, whereas log-normal shadowing is used to model variations in clutter level along different paths of the same length. This Gaussian model, while simple, is consistent with our outdoor field measurements.

For the purposes of deriving performance bounds, we consider a restricted region of an arbitrarily large network, and assume that a packet broadcast from node t can be received by all other nodes. For a given connectivity, this analysis yields a conservative estimate of localization performance for a larger network.

Thus, our estimation task is to estimate the coordinates $\{x_1, \ldots, x_N, y_1, \ldots, y_N\}$ and the unknown placement orientations $\{\phi_1, \ldots, \phi_N\}$. The observation set $\{s_{irt}\}_{i \in \mathcal{V}_{rt}, r \neq t}$ is collected into the measurement vector $\boldsymbol{s}$. For the single measurement case ($M = 1$), this vector contains $N_v(N^2 - N)$ measurements, where N_v is the number of antennas in the set $\mathcal{V}_{rt}$ and, for simplicity, is assumed constant for all (r, t) pairs.

For evaluation of bounds, we choose to consider absolute, rather than relative, localization. Prior information is needed to resolve ambiguities in overall scene translation, rotation, reflection, and scale; we adopt nodes 1, 2 and 3 as anchors with known locations. Selecting anchors allows us to evaluate the utility of the angle-indexed RSS measurements for localization without regard to the quality of prior information. As an estimation performance metric, we report scene RMS error defined as

$$E_{rms} = \sqrt{\frac{1}{N-3}\sum_{n=4}^{N} E[\hat{d}_n^2]}, \tag{2}$$

where $E[\cdot]$ denotes statistical expectation, and $\hat{d}_n$ is the distance between the true location of node n and its estimate. With σ_{xn}^2 and σ_{yn}^2 representing the variance of unbiased estimates of the x and y coordinates of node n, $E[\hat{d}_n^2] = \sigma_{xn}^2 + \sigma_{xn}^2$. As such, the entire scene localization error can be found by adding appropriate elements of the localization covariance matrix. Later, we use the CRB to provide a lower bound on this error, and an average over random node positions is computed by numerical simulation.

3 Localization from RSS: Performance Bounds

In this section we employ the CRB as an indicator of the performance achievable by an efficient estimator for localization using angle-indexed RSS measurements. Adopting the model presented in Section 2, two cases are considered. First, node locations are directly estimated from the raw RSS measurements. In the second approach, the raw measurements are used to estimate arrival angles, $\{\hat{\theta}_{rt}\}_{r\neq t}$, which are then used to estimate locations. Although not sufficient statistics, the arrival angles are intuitive summaries of the data and provide an avenue for simple, distributed computation. The CRB analysis quantifies the relative loss of efficiency when using arrival angles in sub-optimal estimators.

To analyze the performance of the two approaches, we begin by analyzing the variance of AOA estimates.

3.1 AOA estimates, $\hat{\theta}_{rt}$

For the single measurement case ($M = 1$) and a given receiver-transmitter pair (r, t), define the measurement vector $\boldsymbol{s}_{rt}$ to be the stacking of the observation set $\{s_{irt}\}_{i\in\mathcal{V}_{rt}}$. From (1) $\boldsymbol{s}_{rt}$ is a Gaussian random vector distributed $\mathcal{N}(\boldsymbol{\mu}_s, \sigma_{rss}^2\boldsymbol{I})$, where $\boldsymbol{\mu}_s$ is the mean of $\boldsymbol{s}_{rt}$ and is a function of the parameter vector $\boldsymbol{\Theta}_{rt} = [\theta_{rt}, \mu_{rt}]^T$. For the remainder of this section, it is understood we are calculating the arrival from t to r and we drop the rt subscript for simplicity.

The Fisher information matrix (FIM) for the estimation of $\boldsymbol{\Theta}$ is [7]

$$\boldsymbol{I}_\Theta = E[\{\nabla_\Theta \ln f(\boldsymbol{s};\boldsymbol{\Theta})\}\{\nabla_\Theta \ln f(\boldsymbol{s};\boldsymbol{\Theta})\}^T]. \tag{3}$$

Neglecting $\boldsymbol{\Theta}$-independent terms

$$\ln f(\boldsymbol{s};\boldsymbol{\Theta}) = -\frac{1}{2}\left(\boldsymbol{s}-\boldsymbol{\mu}_s\right)^T \boldsymbol{\Sigma}_s^{-1}\left(\boldsymbol{s}-\boldsymbol{\mu}_s\right). \tag{4}$$

Noting $\boldsymbol{\Sigma}_s^{-1} = \frac{1}{\sigma_{rss}^2}\boldsymbol{I}$ and defining the transposed matrix $\boldsymbol{G}_{\Theta s}^T \equiv \nabla_\Theta(\boldsymbol{\mu}_s^T)$, the FIM for estimating $\boldsymbol{\Theta}$ from M independent observations of $\boldsymbol{s}$ is

$$\boldsymbol{I}_\Theta = \frac{M}{\sigma_{rss}^2}\; G_{\Theta s}^T G_{\Theta s}. \tag{5}$$

From the Cramér-Rao inequality, the variance of an unbiased estimate $\hat{\theta}$ is bounded by

$$\mathrm{var}(\hat{\theta}) \geq \left[\boldsymbol{I}_\Theta^{-1}\right]_{1,1}, \tag{6}$$

where $[\cdot]_{1,1}$ denotes the $(1,1)$ element of the bracketed matrix. As a special case when only two antennas, denoted i_1 and i_2, make RSS measurements ($N_v = 2$), the bound reduces to

$$\mathrm{var}(\hat{\theta}) \geq \frac{2\;\sigma_{rss}^2}{M\left(\frac{d\,g(\theta+(i_1-1)\Delta)}{d\theta} - \frac{d\,g(\theta+(i_2-1)\Delta)}{d\theta}\right)^2}. \tag{7}$$

From (7) we see that the performance depends on the true incoming angle and that the error is minimized when the difference between the derivatives is maximized. In general, this is accomplished by making the beamwidths as narrow as possible. As such, there is a trade off between making the beams narrower and decreasing the number of visible antennas.

As an example of AOA performance, we consider $g(\theta)$ from the MD24-12 directional antenna manufactured by Pacific Wireless. Figure 3 illustrates the RMS estimation error in degrees from the CRB (6) and from resulting maximum-likelihood estimates (MLEs) obtained from 2000 Monte Carlo simulations. In this example, $\Delta = 60°$ and the incoming signal was assumed to be seen by only two antennas of the array ($N_v = 2$). The true AOA spanned a 60° arc and started pointing directly at one antenna and ended directly facing the other. $\sigma_{rss} = 12$ dB was selected based on our own experimental tests described in Section 4.

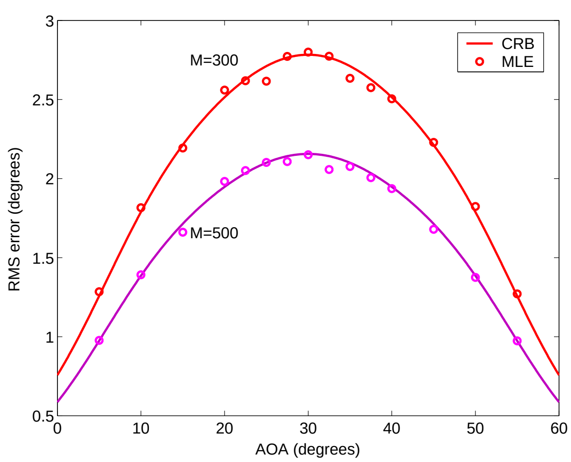

Figure 3: RMS error in estimating angle-of-arrival versus true AOA as predicted from the CRB (-) and from Monte Carlo simulations utilizing a Maximum Likelihood estimator (o). $\Delta = 60°$, $N_v = 2$, $\sigma_{rss} = 12$ dB.

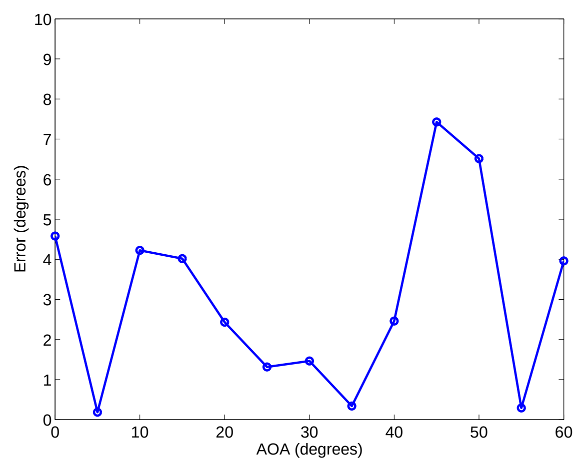

Figure 4: Outdoor field experiment: AOA estimation error versus true AOA. $M = 200$, $N_v = 2$, $\Delta = 60°$, transmission distance = 47 meters.

3.2 Localization estimates

Using the results of the previous section, we now compute the localization performance from intermediate AOA estimates by assuming they are Gaussian with variance given by the bound in (6). This is then compared to the bound based on the entire set of raw RSS measurements each with variance σ^2_{rss}.

Because the observations are assumed Gaussian in each case, the CRBs follow the same form as that in Section 3.1.

Case 1: RSS $\rightarrow (x, y)$'s (direct method)

In this case, the measurement vector $\boldsymbol{s}$ consists of all RSS measurements from all node pairs, $\{s_{irt}\}_{i\in\mathcal{V}_{rt},\forall\ r\neq t}$ and has mean $E[\boldsymbol{s}] = \boldsymbol{\mu}_s$. The unknown parameters $\{x_n, y_n, \phi_n\}_{n=4:N}$ and $\{\mu_{rt}\}_{r\neq t}$ are collected into a parameter vector $\boldsymbol{p}_1$. Defining $\boldsymbol{G}^T_{p_1 s} \equiv \nabla_{p_1}(\boldsymbol{\mu}^T_s)$, the FIM for estimating $\boldsymbol{p}_1$ from M independent observations of $\boldsymbol{s}$ is

$$\boldsymbol{I}_{p_1 s} = \frac{M}{\sigma^2_{rss}}\ \boldsymbol{G}^T_{p_1 s}\boldsymbol{G}_{p_1 s}, \tag{8}$$

and the associated bound on the covariance matrix for estimates $\hat{\boldsymbol{p}}_1$ is

$$\boldsymbol{\Sigma}_{p_1 s} \geq \frac{\sigma^2_{rss}}{M}[\boldsymbol{G}^T_{p_1 s}\boldsymbol{G}_{p_1 s}]^{-1}, \tag{9}$$

where for matrices, $\boldsymbol{A} \geq \boldsymbol{B}$ means that $\boldsymbol{A} - \boldsymbol{B}$ is positive semi-definite.

Case 2: RSS $\rightarrow \hat{\theta}$'s $\rightarrow (x, y)$'s (indirect method)

In this case, the measurement vector $\hat{\boldsymbol{\theta}}$ consists of all $N^2 - N$ AOA estimates $\{\theta_{rt}\}_{r\neq t}$ and has an associated diagonal covariance matrix, $\boldsymbol{\Sigma}_\theta$, whose elements are given by (6). For this case, the unknown parameter vector, $\boldsymbol{p}_2$, only contains positions and orientations $\{x_n, y_n, \phi_n\}_{n=4:N}$. The FIM for estimating $\boldsymbol{p}_2$ from $\hat{\boldsymbol{\theta}}$ is

$$\boldsymbol{I}_{p_2\theta} = \boldsymbol{G}^T_{p_2\theta}\boldsymbol{\Sigma}^{-1}_\theta\boldsymbol{G}_{p_2\theta}, \tag{10}$$

where $\boldsymbol{G}^T_{p_2\theta}$ is analogously defined as $\boldsymbol{G}^T_{p_2\theta} \equiv \nabla_{p_2}(\boldsymbol{\mu}^T_\theta)$, and $\boldsymbol{\mu}_\theta$ is the mean vector of $\hat{\boldsymbol{\theta}}$. The associated covariance of $\boldsymbol{p}_2$ estimates obtained from $\hat{\theta}$'s is

$$\boldsymbol{\Sigma}_{p_2\theta} \geq [\boldsymbol{G}^T_{p_2\theta}\boldsymbol{\Sigma}^{-1}_\theta\boldsymbol{G}_{p_2\theta}]^{-1}. \tag{11}$$

If in (6) $\mathrm{var}(\hat{\theta}) \approx$ constant for all arrival angles, then (11) can be simplified by substituting $\boldsymbol{\Sigma}_\theta = \boldsymbol{I}\ \mathrm{var}(\hat{\theta})$

$$\boldsymbol{\Sigma}_{p_2\theta} \geq \frac{\sigma^2_{rss}}{M}\left[\boldsymbol{G}^T_{\Theta s}\boldsymbol{G}_{\Theta s}\right]^{-1}_{1,1}\left[\boldsymbol{G}^T_{p_2\theta}\boldsymbol{G}_{p_2\theta}\right]^{-1}. \tag{12}$$

For a given network configuration, M, and σ_{rss}, the CRBs $\boldsymbol{\Sigma}_{p_1 s}$ and $\boldsymbol{\Sigma}_{p_2\theta}$ can be computed from (9) and (11) and the resulting RMS localization error determined. In Figure 5 we illustrate the different error rates of the two methods for 15 nodes (including 3 anchors) uniformly distributed in a 100m×100m area. The beam pattern is from the MD24-12 antenna, $N_v = 3$, $\Delta = 60°$, $M = 300$, and the orientations are uniform over $[0°, 360°]$. The plot shows the average and standard error for 1000 random realizations of the 15 node network in the given area. On average, the error using AOA as an intermediate estimate is only 33% greater than using the entire raw RSS data set.

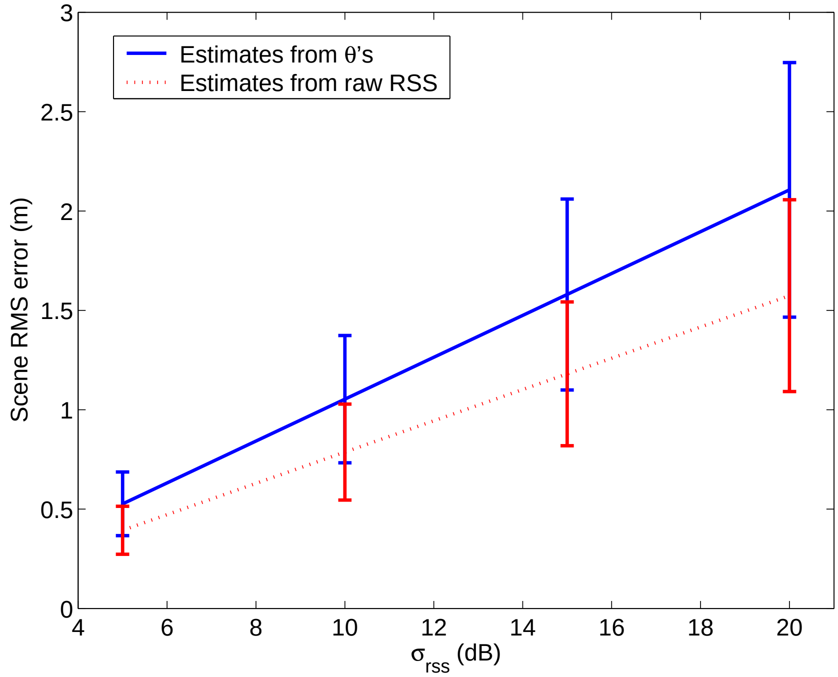

Figure 5: Localization performance using raw RSS data and arrival angles as intermediate estimates. The average error when using angles is 33% greater than when using the complete data set.

4 Bearing from RSS: Outdoor Field Experiment

Using commercial 802.11b transceivers in the 2.4 GHz band, we performed an outdoor field experiment similar to the simulation described above. The transmitter was 47m from the two receiving antennas which were separated by 60°. Four hundred packets were sent by the transmitter and 200 RSS measurements were made by each receiving antenna ($M = 200$). In 5° increments the true AOA of the transmitter was varied from one receiving antenna to the other. The difference between the maximum likelihood estimate and the true AOA is plotted in Figure 4. The larger error near 45° is due to a discrepancy between the gain pattern predicted by the MD24-12 data sheet (which was used in the MLE) and the actual beam patterns of the two-element array. The angle estimates could be improved by using measured beam patterns of the array rather than data sheet values.

From this and other field experiments, we have observed σ_{rss} values ranging from 6 to 12 dB.

5 Comparison to Distance-Based Estimators

Our localization scenario adopted the pessimistic assumption that no distance measurements were available. Other estimators utilize distance information typically obtained from signal time-of-flight measurements or received signal strength and an assumed path loss model. To evaluate the differences between angle-only, distance-only, and hybrid systems we derived the localization CRB for a system utilizing joint angle and distance observations. Equal-error contours are plotted in Figure 6 for the same 15 node network previously considered. The independent variables of the figure are the standard deviation of angle measurements, σ_θ, and the standard deviation of available distance measurements, σ_d.

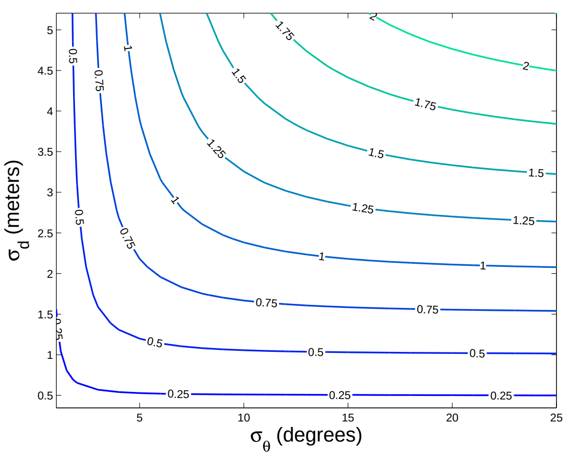

Figure 6: Contours of equal RMS localization error (in meters) as a function of the quality of distance and angle measurements. 15 nodes in a 100m×100m area.

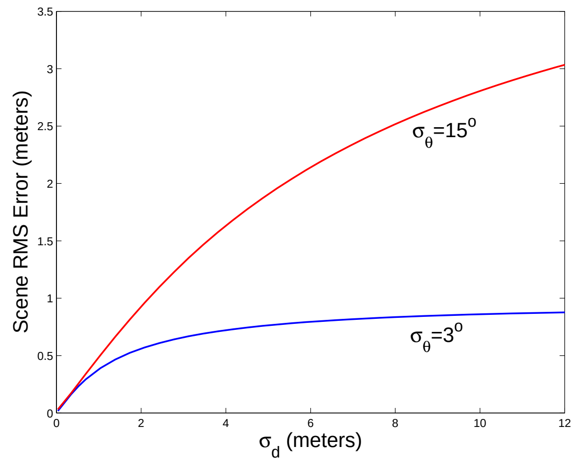

Figure 7: Two vertical slices of Figure 6. Improvements in distance measurements are of little value when the quality of angle estimates is high.

By considering large errors in one type of measurement, the performance of the other measurement modality alone can be inferred from the asymptotic nature of the contours. For example, without angle measurements distance measurements with $\sigma_d = 1.5$m result in RMS localization error of approximately 0.75m.

The figure is also useful in evaluating the utility of one type of measurement in the presence of the other. To illustrate this, two vertical slices of Figure 6, evaluated at $\sigma_\theta = 3°$ and $\sigma_\theta = 15°$, are replotted in Figure 7. For the $\sigma_\theta = 15°$ case, any reduction in σ_d improves the localization estimates. However, for the $\sigma_\theta = 3°$ case, where the angle estimates are less noisy, improving distance estimates has little improvement on overall scene estimation error until σ_d is approximately less than 2m.

In our outdoor field experiment, we had an average AOA error of approximately 3°. Therefore, to improve localization performance, any available distance measurements would need to have $\sigma_d < 2$ m. Further, $\sigma_\theta = 3°$ has an asymptotic RMS error of approximately 0.9 m. For estimates based on distance measurements alone to exceed this performance, $\sigma_d < 1.8$ m is required.

6 Algorithms and Antennas

In the treatment above, network localization is presented as a Bayesian parameter estimation task. The analysis and field tests demonstrated the potential and feasibility of localization using only RF signal strength, as measured by an array of directional antennas. The CRB and the maximum likelihood estimator computed at a central processor provide benchmarks for evaluating decentralized processing schemes with low communication overhead.

6.1 Algorithms

The estimation task is amenable to local processing whereby each node maintains the posterior probability of its location. Such approaches have been employed for localization from distance [8, 9]. For many applications, only relative localization is required, and the problem formulation from Section 2 may be applied without use of anchor nodes.

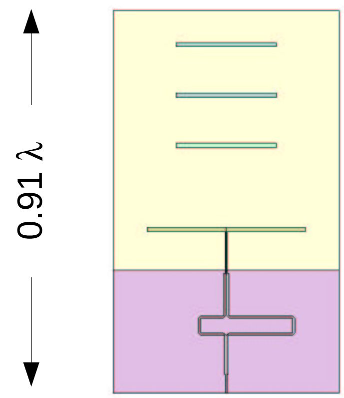

Figure 8: Our uni-planar quasi-Yagi directional antenna designed for printed circuit board construction.

In addition, the basic structure of the decentralized computation is unaltered by use of additional sensing modalities, such as ultrasound [10] or acoustics [11].

Uniqueness of network localization solutions is addressed by the theory of rigid graphs. For distance measurements, a key insight from the theory is that sufficiently high connectivity guarantees, with high probability, a unique solution and computational complexity that scales only linearly with the number of nodes (whereas in general the localization problem is NP-hard [12]). The graph rigidity problem with directions is the dual of the distance case. However, there exists no complete theory for point formations based solely on angles [13]. Yet, with sufficient connectivity and one fixed direction, the direction results may be applied to the abstract graph of links as vertices and angles as edges.

6.2 Antennas

Directional antennas offer many potential performance advantages for sensor networks: e.g., decreased power consumption, reduced interference, increased range, spatial reuse, and geographic routing. Many of these advantages come at the cost of increased complexity in communication protocols. However, the antennas themselves can be fabricated in a low cost, compact package. For example, the uni-planar quasi-Yagi antenna in Figure 8 is our extension of the single-director X-band design of Deal *et al.* [14]. Inexpensively constructed as a printed circuit on a high-dielectric substrate, the antennas may be stacked in a three dimensional array and fed through a single software-controlled switch. The antennas can increase the communication range by more than an order of magnitude, dramatically increasing network connectivity and allowing for sparse networks. A single antenna is less than one wavelength long ($\lambda = 2.25$ inches at 5.25 GHz), and can provide approximately 11 dBi gain at transmitter and receiver, thereby increasing range by a factor of 81 over an isotropic radiator for path loss exponent of 2.5. Additionally, the antennas may be simultaneously fed, producing a nearly isotropic beam pattern for use with existing network protocols that require omni-directional transmission.

7 Conclusion

In this paper we utilized the Cramér-Rao bound to place a benchmark on the quality of localization estimates achievable from received signal strength measurements with directional antennas. The results were favorable indicating that sub-meter RMS localization performance can be achieved for the network sizes considered and for measurement noise levels equivalent to those observed in our own outdoor field experiments. Additionally, we evaluated the performance of sub-optimal estimators based on intermediate estimates of arrival angles. The performance of these estimates was 33% less than the optimal estimates which used the the complete raw data set. The results of an outdoor field experiment were also presented where we obtained an average AOA error of approximately 3° using two MD24-12 antennas placed 47 m from the transmitter. Finally, we briefly considered localization algorithms and then presented our own PCB-based quasi-Yagi directional antenna for use in these applications.

References

[1] *IEEE Signal Processing Magazine, Special Issue on Collaborative Information Processing*, 19(2), March 2002.

[2] R. L. Moses, D. Krishnamurthy, and R. Patterson. A self-localization method for wireless sensor networks. *Eurasip Journal on Applied Signal Processing, Special Issue on Sensor Networks*, 2003:348–358, March 2003.

[3] D. Niculescu and B. Nath. Ad-hoc positioning system using AOA. *Proc. IEEE/INFOCOM 2003*, San Francisco, CA, April 2003.

[4] T.S. Rappaport. *Wireless Communications Principles and Practice.* Prentice Hall, 1996.

[5] N. Patwari, A. Hero III, M. Perkins, N. Correal, and R. O'Dea. Relative location estimation in wireless sensor networks. *IEEE Transactions on Signal Processing*, 51(8):2137– 2148, Aug 2003.

[6] J. Zhao and R. Govindan. Understanding packet delivery performance in dense wireless sensor networks. *Proceedings of SenSys '03*, Los Angeles, CA, 2003.

[7] H. Vincent Poor. *An Introduction to Signal Detection and Estimation.* Springer, New York, 2^{nd} edition, 1994.

[8] A. T. Ihler, J. W. Fisher III, R. L. Moses, and A. S. Willsky. Nonparametric belief propagation for sensor self-calibration of sensor networks. *IEEE Journal on Selected Areas of Communications*, 2004, submitted.

[9] V. Ramadurai and M. L. Sichitiu. Localization in wireless sensor networks: a probabilistic approach. *2003 International conference on Wireless Networks (ICWN03)*, pages 275–281, Las Vegas, NV, June 2003.

[10] N. B. Priyantha, A. Chakraborty, and H. Balakrishnan. The cricket location-support system. *Proc. 6th ACM MOBICOM*, Boston, MA, Aug 2000.

[11] J. N. Ash and R. L. Moses. Acoustic sensor network self-localization: Experimental results. *Proceedings of the 2003 MSS Battlefield Acoustics Symposium*, Laurel, MD, Oct 2003.

[12] J. Saxe. Embeddability of weighted graphs in k-space is strongly np-hard. *17th Allerton Conference in Communications, Control and Computing*, pages 480–489, 1979.

[13] T. Eren *et al.* Sensor and network topologies of formations with direction, bearing and angle information between agents. *Proc. 42nd IEEE conference on Decision and Control*, pages 3064–3069, Maui, Hawaii, Dec 2003.

[14] W. R. Deal, N. Kaneda, J. Sor, Y. Qian, and T. Itoh. A new quasi-yagi antenna for planar active antenna arrays. *IEEE Trans. Microwave Theory and Techniques*, 48(6):910–918, 2000.