# ECE 301 Comprehensive Notes

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## 1 Dynamic Programming (Lecture 1)

With dynamic programming, you can figure out how to meet deadlines. This is required study for those unable to make it to class before 8:30.

## 2 Basic Linear Circuit Analysis Review (Lectures 1–2)

### 2.1 Linear Resistive Circuits

A few important facts (you already know).

- If the circuit is linear and ideal, then $V_n = R_n I_n$ (Ohm's Law).
- There are two kinds of sources.
  - Voltage source - always maintains a certain voltage across its terminals.
  - Current source - always maintains a certain current across its terminals.
- Solving the circuit requires $2n$ equations for $n$ nodes.
  - n from Ohm's Law.
  $$\begin{cases} V_1 - R_1 I_1 = 0 \\ \vdots \\ V_n - R_n I_n = 0 \end{cases}$$
  - n from KCL and KVL.
  $$\begin{cases} \sum i = 0 \\ \sum v = 0 \end{cases}$$
- When there are many sources, superposition is applied.

## 2.2 Other Circuit Elements

The capacitor and inductor, obviously.

- Capacitor (Relation: $i = C\,\mathrm{d}v/\mathrm{d}t$)
- Inductor (Relation: $v = L\,\mathrm{d}i/\mathrm{d}t$)

And these things cause there to be differential equations when solving the circuit.

## 2.3 Transients and Response

The inst. ran thru this in a very convoluted manner. Know that *transients* are changes in state (requires some form of memory). They arise from *initial state* and *inputs*.

- Steady-state (or forced, or particular) response
- Zero-input response
- Zero-state response - from input

And here's some other random terms.

- Stable - A system where there may be transients that die out
- Unstable - A system where there are transients that don't die out

At the end of the lecture, he dumped a bunch of random terms on us in rapid fire mode.

- Initial response
- Inputs response
- Total response
- Forced reponse
- Zero-input response
- Zero-state response
- Total transients

I might fix this later. Whatever `Y(V_V)Y` ...

### 2.4 Analysis

Suppose you have a circuit containing a 12 V source, a $R$ $\Omega$ resistor, a $L$ H inductor, and a $C$ F capacitor in series, and that the circuit has been on for a long time. Then clearly for all time $t$, the voltage across the capacitor, $y$, is 12 as the transient term has died.

Now suppose that the KVL equation for this circuit is given by $y''+5y'+6y = 12$, where $y$ is the voltage across the capacitor.

$$y(t) = A$$
$$y'(t) = 0$$
$$y''(t) = 0$$

$$6(A) = 12$$

$$y(t) = A = 2$$

Again, because the transient term has died (the circuit having been on for a long time), $y$ is given by the particular solution to the differential equation alone, which is 2.

Now suppose the equation is changed to $y'' + 5y' + 6y = e^{-t}$.

$$y(t) = Ae^{-t}$$
$$y'(t) = -Ae^{-t}$$
$$y''(t) = Ae^{-t}$$

$$Ae^{-t} - 5Ae^{-t} + 6Ae^{-t} = e^{-t}$$

$$7A = 1$$

$$A = \frac{1}{7}$$

$$y(t) = \frac{1}{7}e^{-t}$$

This time the equation is $y'' + 5y' + 6y = e^{2t}$.

$$y(t) = Ae^{2t}$$
$$y'(t) = 2Ae^{2t}$$
$$y''(t) = 4Ae^{2t}$$

$$4Ae^{2t} + 10Ae^{2t} + 6Ae^{2t} = 6e^{2t}$$

$$20A = 6$$

$$A = \frac{3}{10}$$

$$y(t) = \frac{3}{10}e^{2t}$$

Finally, $y'' + 5y' + 6y = 6\frac{1}{3}\cos t$. There a several ways to solve this equation, such as Method of Undetermined Coefficients, Phasors, and also a strategy that involves converting the cosine to exponentials. The route taken in class was to convert to exponentials. So...

$$y'' + 5y' + 6y = 6\frac{1}{3}\cos t = \frac{1}{2}(e^{-jt} + e^{jt})$$

We now generalize the approach. Suppose $H(s^t + 5s + 6)e^{st} = 6e^{st}$.

$$H(s) = \frac{6}{s^2 + 5s + 6}$$

| Input | S.S. Output |
|---|---|
| $e^{st}$ | $H(s)e^{st}$ |
| $\frac{1}{2}e^{jt}$ | $\frac{1}{2}H(s)e^{jt}$ |
| $\frac{1}{2}e^{-jt}$ | $\frac{1}{2}H(s)e^{-jt}$ |

H(s) is called the *transfer function*.

$$\frac{3}{5(1+j)} = \frac{3}{5\sqrt{2}}e^{-j\frac{\pi}{4}}e^{jt} + \frac{3}{5\sqrt{2}}e^{j\frac{\pi}{4}}e^{-jt}$$

$$\begin{aligned} y_p(t) &= \frac{3}{5\sqrt{2}}(2)\cos\left(t - \frac{\pi}{4}\right) \\ &= \frac{6}{5\sqrt{2}}\left[\frac{\sqrt{2}}{2}\cos t + \frac{\sqrt{2}}{2}\sin t\right] \end{aligned}$$

That concludes notes from "the most important lecture of (my) life." >_>

# 3 Filters (Lecture 3)

There are several types of filters.

- Low-pass filter - passes low-frequency signals and attenuates signals with frequencies higher than the cutoff frequency
- High-pass filter - passes high-frequency signals and attenuates signals with frequencies lower than the cutoff frequency
- Band-pass filter - passes signals with frequencies in a certain range and attenuates other signals with frequencies outside that range (a combination low- and high-pass filter)
- Band-stop (or band-rejection) filter - attenuates signals with signals within a certain range

# 4 Amplification (Lecture 3)

Frankly, the prof's first lecture on amps (lecture 3) was utter crap, but here's some random data. It's probably fits better in a section on complex analysis.

$$H(s) = \frac{6}{s^2 + 5s + 6}$$

| $\omega$ | $\lvert H(j\omega)\rvert$ | $\angle H(j\omega)$ | $H(j\omega)$ |
|---|---|---|---|
| 0 | 1 | 0 | ? |
| 1 | $\frac{6}{\sqrt{50}}$ | $-\frac{\pi}{4}$ | $\frac{6}{5+5j}$ |
| 3 | $\frac{6}{\approx 15}$ | $-\arctan\frac{15}{-3} + \frac{\pi}{4}$ | $\frac{6}{-3+15j}$ |
| $\infty$ | 0 | $-\pi$ | - |

# 5 Transfer Function & Frequency Response (Lecture 4)

Math dump (for some circuit not pictured here yet). Also, wiki.

$$V_L + V_R + V_C = y$$

$$i = C\frac{\mathrm{d}y}{\mathrm{d}t} \quad V_L = L\frac{\mathrm{d}i}{\mathrm{d}t} = LCy''(t)$$

$$V_R = Ri = RCy'(t) + y(t) = x$$

$$y''(t) + \frac{R}{L}y'(t) + \frac{1}{LC}y(t) = \frac{1}{RC}x$$

$$x(t) = e^{st}, \quad y_p(t) = H(s)e^{st}$$

$$H\left(s^2 + \frac{R}{L}s + \frac{1}{RC}\right)e^{st} = \frac{1}{LC}e^{st}$$

$$H(s) = \frac{\frac{1}{LC}}{s^2 + \frac{R}{L}s + \frac{1}{LC}}$$

$s_1$ and $s_2$ are the roots of the transients and occur in some sort of $e^{s_1 t}$ / $e^{s_2 t}$ factors.

To find the frequency response, look at $e^{st}$.

**Poles** The roots of the denominator are called the *poles*, and have the form $s = -\frac{R}{2L} \pm$ some complex part. If the roots are in the left of the complex plane (negative real part), the system is *stable*.

## 5.1 Graphically Computing Gain and Phase

Consider the following transfer function.

$$H(s) = \frac{6}{s^2 + 5s + 6} = \frac{6}{(s+2)(s+3)}$$

Draw $H(j\omega)$ on the complex plane (not pictured yet). No wait, screw that.

$$|H(j\omega)| = \frac{6}{ab}$$

$$\angle H(j\omega) = 0 - (\alpha + \beta)$$

Suppose $\omega = 0$ (DC gain).

$$\text{gain} = \frac{6}{(-2)(-3)} = 1$$

$$\text{phase} = 0 - (0 + 0) = 0$$

Suppose $\omega = 2$.

$$\text{gain} = \frac{6}{\sqrt{8}\sqrt{13}}$$

# 6 Pointless Shit (Lecture ALL)

## 6.1 All Electrical Engineers are Also Mechanical Engineers

Because if you can analyze a linear circuit, a 10000 kg mass is just the same thing.

| RLC Circuit | 10000 kg Mass |
|---|---|
| $v$ | $f$ |
| $p = vi$ | $p = fv$ |
| $R = \frac{v}{i}$ | $B = \frac{f}{v}$ |
| $v = L\frac{\mathrm{d}i}{\mathrm{d}t}$ | $f = M\frac{\mathrm{d}v}{\mathrm{d}t}$ |
| $v = \frac{1}{C}\int i\,\mathrm{d}t$ | $f = Kx$ |


I CAN SEE FOREVER