

# Analog to Digital Conversion of Ultra-Wideband Signals in Orthogonal Spaces

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**Abstract**— This paper introduces novel techniques to perform analog to digital (A/D) conversion, based on the quantization of the coefficients obtained by the projection of a continuous-time signal over an orthogonal space. The new A/D techniques proposed here are motivated by the sampling of the input signal in domains which may lead to lower levels of signal distortion and significantly less demanding A/D conversion characteristics. As a particular case, we study A/D conversion in the frequency domain where samples of the signal spectrum are taken such that no time-aliasing occurs in the discrete-time version of the signal. We show that the frequency domain analog to digital converter (ADC) overcomes some of the difficulties encountered in conventional time-domain methods for A/D conversion of signals with very large bandwidths, such as ultra-wideband (UWB) signals. The discrete frequency samples are then passed through a vector quantizer with relaxed characteristics, operating over DC levels that change with a speed that is much lower than that required for time-domain A/D conversion. Fundamental figures of merit in A/D conversion and important system trade-offs are discussed for the proposed frequency domain ADC. As an example of this approach, we consider a multi-carrier UWB communications scheme.

## I. INTRODUCTION

High-speed signal processing needed in analog to digital conversion of ultra-wideband signals imposes challenging implementation problems, and sometimes impractical power consumption. Current time-domain A/D conversion encounters technological barriers as time-domain signal features shrink to very fine resolution, on the order of tenths, and sometimes hundredths of nanoseconds. In order to overcome these problems, techniques that aim to relax the speed of the A/D conversion have been proposed. In general, these techniques perform multi-band signal processing [1] in which the spectrum of the signal is channelized into several bands by means of a bank of bandpass filters. A/D conversion thus occurs at a much reduced speed for each one of the resultant bandpass signals. Furthermore, a bank of frequency modulators can be used to shift the signal spectrum so that the center frequency of each sub-band becomes the zero frequency [2], allowing the use of

a bank of identical low-pass filters. Sigma-delta modulation [3] has also been proposed since it enables A/D conversion with low-resolution. The noise penalty associated with the use of few bits in the quantization process is overcome in the sigma-delta scheme by using either signal oversampling or multi-band processing techniques [4, 5]. In particular, when a single bit is used, the implementation is greatly simplified and practical mono-bit digital receivers can be implemented [6]. Since all these techniques are based on time-domain A/D conversion, they suffer from high-speed limitations, making it desirable to channelize the signal spectrum into several sub-bands. The implementation of the bank of bandpass filters needed in the multi-band processing ideas can be potentially troublesome; problems such as spectrum sharing due to the non ideal characteristics of the bandpass filters will affect the overall system performance.

This paper introduces novel approaches to the analog to digital conversion problem that potentially overcome the limitations encountered in the implementation of time-domain A/D converters. These approaches exploit the signal representation in domains other than the classical time domain, which reduces the speed of the comparators that make the quantization of the sampled signal, and potentially improves the distortion versus average bit rate of the A/D conversion. As a particular case, we consider a frequency domain ADC in which samples of the signal spectrum are taken at a rate that guarantees no aliasing in the discrete-time signal domain. In the remainder of the paper, the consequences of carrying out the A/D conversion in new domains are discussed, and fundamental figures of merit in analog to digital conversion are analyzed.

## II. ANALOG TO DIGITAL CONVERSION IN ORTHOGONAL SPACES

The block diagram depicted in Fig. 1 shows the basic orthogonal expansion principle of the proposed A/D conversion. The received signal  $s(t)$  is decomposed every  $T_c$  seconds into  $N$  components which are obtained through the projection over a set of orthogonal bases  $\Phi_i(t) |_{i=0}^{N-1}$ . The coefficients  $a_i |_{i=0}^{N-1}$  are found as

$$a_i = \langle s(t), \Phi_i(t) \rangle_{T_c} = \int_0^{T_c} s(t) \Phi_i^*(t) dt, \quad (1)$$

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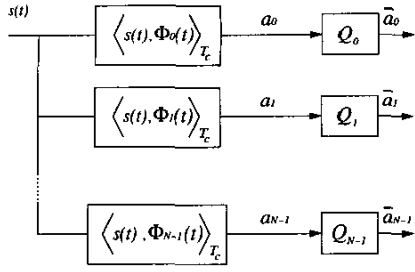


Fig. 1. Block diagram of the analog to digital converter, that expands the received signal using a set of orthogonal basis functions.

where the mean square error criterion is used to approximate the received signal  $s(t)$  in a  $T_c$  second interval as follows

$$\hat{s}(t) = \sum_{l=0}^{N-1} a_l \Phi_l(t). \quad (2)$$

At the end of the conversion time  $T_c$ , the coefficients  $a_l$   $|_{l=0}^{N-1}$  reach a constant value that is fed to a set of quantizers  $Q_l(x) |_{l=0}^{N-1}$  which return the digital words  $\bar{a}_l$   $|_{l=0}^{N-1}$ . These digital values represent the output of this analog to digital converter for the input signal in a  $T_c$  second interval. When only a limited number of coefficients can be used in the A/D signal conversion, some distortion is introduced. This distortion plus the distortion introduced by the quantization process constitute the overall distortion of the proposed A/D conversion, which is analyzed in the following section.

### III. A/D CONVERSION DISTORTION DUE TO LIMITED NUMBER OF COEFFICIENTS AND QUANTIZATION ERROR

The distortion introduced by the A/D converter in orthogonal spaces is generated by the potential limited number of coefficients  $N$ , and by the finite number of bits used in the quantization of the coefficients  $a_l$   $|_{l=0}^{N-1}$ . The first distortion introduces an error  $(e(n) = s(t) - \hat{s}(t))$  in the reconstruction formula (2), where the coefficients  $a_l$   $|_{l=0}^{N-1}$  are calculated as in (1) in order to minimize the MSE distortion. The distortion obtained with  $N$  coefficients can be expressed as

$$D_{\Phi, N} = E_{s, T_c} - \sum_{l=0}^{N-1} \sigma_l^2, \quad (3)$$

where  $E_{s, T_c}$  is the energy of the signal in the conversion interval  $T_c$ ,  $\sigma_l^2$  is the variance of the coefficient  $a_l$ , and the distortion  $D_{\Phi, N}$  is nonnegative by definition. When the distortion reaches the zero value for a number of coefficients  $N^*$ , we have that  $s(t) = \hat{s}(t)$ , where the equality holds in the sense that the approximation error has zero energy.

The distortion introduced by the finite number of bits used in the quantization of the orthogonal domain coefficients will be denoted as  $D_{\Phi, Q}$  which is commonly quantified by the average mean squared error (MSE). Let us define  $\mathcal{A}_l$   $|_{l=0}^{N-1}$  and  $\bar{\mathcal{A}}_l$   $|_{l=0}^{N-1}$  as the random variables associated to the coefficients  $a_l$   $|_{l=0}^{N-1}$  and  $\bar{a}_l$   $|_{l=0}^{N-1}$ , respectively. Thus the MSE of the quantization error is

$$D_{\Phi, Q} = \frac{1}{N} E\{\|\mathcal{A} - \bar{\mathcal{A}}\|^2\} = \frac{1}{N} \sum_{l=0}^{N-1} D_{\Phi_l, Q} \quad (4)$$

where  $D_{\Phi_l, Q} = E\{(\mathcal{A}_l - \bar{\mathcal{A}}_l)^2\} = \epsilon_{\Phi_l}^2 \sigma_{\Phi_l}^2 2^{-2R_l}$ , for a sufficiently high bit rate  $R_l$  [7], where  $\epsilon_{\Phi_l}^2$  is a constant that depends on the probability density function (pdf) of  $\mathcal{A}_l$ , namely  $p(a)$ . Therefore the average distortion introduced by the quantization process is

$$D_{\Phi, Q} = \frac{1}{N} \sum_{l=0}^{N-1} \epsilon_{\Phi_l}^2 \sigma_{\Phi_l}^2 2^{-2R_l} \quad (5)$$

At this point, we would like to find the optimal bit allocation among the  $N$  coefficients, i.e. we want to find the set of rates  $R_l$   $|_{l=0}^{N-1}$  constrained to  $\sum_{l=0}^{N-1} R_l = RN$  such that the distortion in (5) is minimized. This classical optimization problem can be solved using Lagrange multipliers, leading to the following result

$$R_l = R + \frac{1}{2} \log_2 \left( \frac{\epsilon_{\Phi_l}^2 \sigma_{\Phi_l}^2}{\prod_{l=0}^{N-1} \epsilon_{\Phi_l}^2 \sigma_{\Phi_l}^2} \right). \quad (6)$$

The optimum solution assigns more bits to the coefficients with larger variance such that the distortion of all the coefficients is uniform and equal to

$$D_{\Phi_l, Q} = D_{\Phi, Q} = \left( \prod_{l=0}^{N-1} \epsilon_{\Phi_l}^2 \sigma_{\Phi_l}^2 \right)^{1/N} 2^{-2R}. \quad (7)$$

This bit allocation resembles the concept of reverse *water-filling* found in rate distortion theory [8]. Notice that if the variance of one coefficient is sufficiently small, the result rate from Eqn. (6) could be negative, which in practice would mean that the coefficient should be discarded.

It is interesting to compare the performance of the newly defined A/D conversion with the conventional pulse coding modulation (PCM) technique in which each time-domain sample is quantized with the same number of bits  $R$ . The distortion incurred by PCM is  $D_{PCM} = \epsilon_t^2 \sigma_t^2 2^{-2R}$ , where the sub-index  $t$  stands for time,  $\epsilon_t$  depends on the pdf of any sample of the time-domain signal which is assumed stationary, and  $\sigma_t^2$  is the sample's variance. Now, we define a very important figure of merit of the proposed A/D conversion in orthogonal spaces, the orthogonal space A/D conversion gain ( $G_{OSADC}$ ). This figure of merit compares the performance of the proposed A/D method with the performance of a conventional A/D with PCM, which is defined as

$$G_{OSADC} = \frac{D_{PCM}}{D_{\Phi, Q} + D_{\Phi, N}} \quad (8)$$

which is just the ratio between the distortion of PCM and the distortion introduced by both the limited number of coefficients and the quantization error when carrying out the A/D conversion in orthogonal spaces. After substitution, we have

$$G_{OSADC} = \frac{\epsilon_t^2 \sigma_t^2 2^{-2R}}{\left( \prod_{l=0}^{N-1} \epsilon_{\Phi_l}^2 \sigma_{\Phi_l}^2 \right)^{1/N} 2^{-2R} + E_{s, T_c} - \sum_{l=0}^{N-1} \sigma_l^2} \quad (9)$$

It is interesting to analyze the special case in which the number of coefficients reaches the defined value  $N^*$ , which makes zero the distortion pointed out in (3). In that case, (9) can be expressed as:

$$G_{OSADC} = \frac{\epsilon_t^2}{\left( \prod_{l=0}^{N-1} \epsilon_{\Phi_l}^2 \right)^{1/N}} \frac{\frac{1}{N} \sum_{l=0}^{N-1} \sigma_{\Phi_l}^2}{\left( \prod_{l=0}^{N-1} \sigma_{\Phi_l}^2 \right)^{1/N}} \quad (10)$$

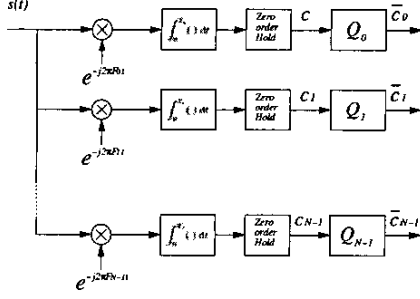


Fig. 2. Block diagram of the frequency domain A/D converter which first takes samples of the signal spectrum and then quantizes the discrete-frequency samples.

where the last equality follows from the fact that the average energy of the coefficients  $a_l \big|_{l=0}^{N-1}$  equals the time sample's variance since the error  $e(n)$  has zero energy. Equation (10) shows the potential gain of the proposed method as the  $G_{OSADC}$  is proportional to the ratio of the arithmetic mean of the orthogonal coefficients variances to the geometric mean of the same variances. Since the arithmetic mean is larger than or equal to the geometric mean, being equal only when all the variances are the same, and in general  $\epsilon_l^2 \leq \left( \prod_{l=0}^{N-1} \epsilon_{\Phi_l}^2 \right)^{1/N}$ , we have that  $G_{OSADC} \geq 1$  or  $D_{PCM} \geq D_{\Phi, Q}$  under the same average bit rate. Notice that a more uneven distribution of the variances leads to a larger gain, which can be advantageous in domains where the variance distribution is known or can be predicted.

The nature of this ADC leads to carrying out digital signal processing (DSP) applications in the same domain used in the A/D conversion itself. The duality between time and some other domains has been extensively studied, and powerful tools are available to carry out the DSP operations. A classical example of this duality is the time-frequency pair, which is hereafter studied in the context of A/D conversion.

#### IV. ANALOG TO DIGITAL CONVERSION IN THE FREQUENCY DOMAIN

The frequency domain emerges as an appealing domain for the analog to digital conversion of signals with very large bandwidths since it relaxes the extremely fine time resolutions needed in time-domain ADCs. Figure 2 shows the block diagram of the frequency domain ADC in which samples of the continuous-time signal spectrum at the frequencies  $F_l \big|_{l=0}^{N-1}$  are taken, leading to the set of frequency coefficients

$$c_l = \int_0^{T_c} s(t) e^{-j2\pi F_l t} dt, \quad l = 0, \dots, N-1. \quad (11)$$

These coefficients are then quantized by a set of quantizers  $Q_l(x) \big|_{l=0}^{N-1}$ , which in turn produce the ADC output digital coefficients  $\bar{c}_l \big|_{l=0}^{N-1}$ . The frequency sample spacing  $\Delta F = F_l - F_{l-1}$  complies with  $\Delta F \leq \frac{1}{T}$  in order to avoid aliasing in the discrete-time domain. Thus, the minimum number of coefficients  $N$  necessary to sample a signal spectrum with bandwidth  $W$ , without introducing time aliasing, is proportional to the time-bandwidth product

$$N = \frac{W}{\Delta F} + 1 \geq WT_c + 1. \quad (12)$$

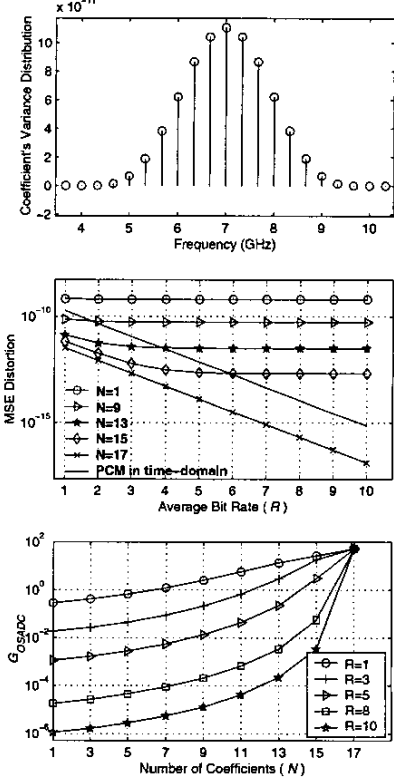


Fig. 3. (a) Raised cosine shaped power spectrum density of the the Gaussian source that is going to be A/D converted. (b) Comparison between the MSE distortion of the time-domain PCM ADC with the MSE distortion of the ADC in frequency domain with optimal bit allocation. (c) The orthogonal space A/D conversion gain ( $G_{OSADC}$ ) against the number of coefficients ( $N$ ) for different average bit rates.

Assuming that the input signal  $s(t)$  is real-valued, the coefficients  $c_l \big|_{l=0}^{N-1}$  are related to the discrete Fourier transform (DFT) coefficients as follows

$$\begin{aligned} \mathbf{S}(k) &= [S(0), S(1), \dots, S(K-1)] \\ &= F_s [c_0, c_1, \dots, c_{N-1}, c_{N-2}^*, \dots, c_1^*], \end{aligned} \quad (13)$$

where it is assumed that samples are taken from 0 Hz,  $*$  denotes the complex conjugate,  $F_s = K/T_c$  is the time sampling frequency and  $k = 0, \dots, K-1$ . So, if  $N$  samples are taken from the spectrum of the real-valued continuous-time signal,  $K = 2(N-1)$  DFT coefficients are obtained and these coefficients constitute the digital representation of the time-domain signal. The relationship in (13) is valid only if the  $K$  samples of the continuous time signal in the interval  $T_c$  comply simultaneously with Nyquist rate ( $F_s \geq 2W$ ) and Eqn. (12). Otherwise, the samples  $S(k) \big|_{k=0}^{N-1}$  provide an approximate representation of the signal, as was indicated in (2).

#### A. Example

Let us consider a stationary zero-mean Gaussian source with sample variance  $\sigma_t$  and a power spectrum density (PSD) that follows a raised cosine shape with rolloff factor  $\alpha = 1$ , center frequency  $F_c = 7$  GHz and bandwidth 4 GHz as shown in Fig.

3 (a). The signal is segmented in intervals of  $T_c = 3$  ns to be A/D converted, thus requiring a frequency space between samples of  $\Delta F = 333.33$  MHz. The bit rates are optimally distributed among the coefficients as indicated by Eqn. (6), leading to the set of curves of MSE distortion vs. average bit rate  $R$  plotted in Fig. 3 (b). The MSE distortion for PCM is also shown for comparison purposes. The orthogonal space A/D conversion gain ( $G_{OSADC}$ ) is plotted in Fig. 3 (c) against the number of coefficients  $N$  for several values of average bit rates  $R$ . These figures show the potential gain of performing the A/D conversion in the frequency domain together with optimal bit allocation, specially when the target average bit rate is low. For this example, a gain of up to 54.7 (5.2 dB) can be achieved when  $N = 17$  coefficients are implemented. Notice that although the MSE distortion in Eqn (5) is in general only valid for large bit rates, for Gaussian sources the expression holds even for small rates [8], so the curves in Fig. (3) are exact.

## V. FURTHER APPLICATIONS

A/D conversion in orthogonal spaces is the main idea of the work presented in this paper. This concept can be utilized in numerous applications including communications and signal processing problems such as signal modulation, matched filtering and space-time array processing. For example, the matched filtering problem can be carried out in the frequency domain thanks to the time-frequency duality provided by Fourier analysis.

### A. Frequency-Domain Matched Filtering

Let us assume that the frequency domain ADC provides a set of full resolution coefficients  $\mathbf{S}_m = [S_m(0), \dots, S_m(K-1)]$  every  $T_c$  seconds, where  $m = 0, \dots, M-1$  and the information symbol period  $T$  is related with the A/D conversion period as  $T = MT_c$ . Let us begin with expressing the calculation of the matched filter output  $\hat{m}$  in the time domain

$$\hat{m} = \int_0^T s(\tau)h(T-\tau) d\tau, \quad (14)$$

where  $h(t)$  is the impulse response of the matched filter and the output of this filter is sampled at  $t = T$ . In order to reflect the effect of segmenting the information symbol time  $T_s$  into  $M$  time-slots of duration  $T_c$ , let us define the following signals

$$\begin{aligned} s_m(t) &= s(t+mT_c), & 0 \leq t \leq T_c \\ h_m(T-t) &= h(T-(t+mT_c)), & 0 \leq t \leq T_c, \end{aligned} \quad (15)$$

where  $m = 0, \dots, M-1$ , and the signals  $s_m(t)$  and  $h_m(t)$  are equal to zero outside the interval  $0 \leq t \leq T_c$ . Using these definitions, the matched filter output in (14) can be expressed as

$$\begin{aligned} \hat{m} &= \sum_{m=0}^{M-1} \int_{mT_c}^{(m+1)T_c} s(\tau)h(T-\tau) d\tau \\ &= \sum_{m=0}^{M-1} \int_0^{T_c} s_m(\tau)h_m(T-\tau) d\tau, \end{aligned} \quad (16)$$

in which the integral in (14) has been segmented into  $M$  pieces of duration  $T_c$  each, such that  $T = MT_c$ .

In order to express the matched filter operations in the frequency-domain, the Fourier transform is applied to (16), leading to

$$\begin{aligned} \hat{m} &= \mathcal{F} \left\{ \sum_{m=0}^{M-1} \int_0^{T_c} s_m(\tau)h_m(T-\tau) d\tau, \right\} \\ &= \sum_{m=0}^{M-1} \mathcal{F} \left\{ \int_0^{T_c} s_m(\tau)h_m(T-\tau) d\tau \right\} \\ &= \sum_{m=0}^{M-1} \int_{-\infty}^{\infty} S_m(F)H_m^*(F) dF, \end{aligned} \quad (17)$$

where  $S_m(F) = \mathcal{F}\{s_m(t)\}$  and  $H_m(F) = \mathcal{F}\{h_m(T-t)\}$ , the second line in (17) follows from the linearity property of the Fourier transform and the third line follows from Parseval's theorem. So, the exact calculation of the matched filter output in the frequency domain requires the Fourier transforms of all the segmented received signals, namely  $S_m(F) |_{m=0}^{M-1}$ , and the Fourier transform of the segmented matched filters,  $H_m(F) |_{m=0}^{M-1}$ . However, since only  $K$  samples of the signal spectrum are provided by the frequency ADC, (17) is approximated as

$$\begin{aligned} \hat{m} &= \sum_{m=0}^{M-1} \int_{-\infty}^{\infty} S_m(F)H_m^*(F) dF \\ &\approx \sum_{m=0}^{M-1} \frac{\Delta F}{F_s} \sum_{k=0}^{K-1} S_m(k)H_m^*(k) \\ &= \frac{\Delta F}{F_s} \sum_{m=0}^{M-1} \sum_{k=0}^{K-1} S_m(k)H_m^*(k). \end{aligned} \quad (18)$$

If the number of frequency samples  $K$  is chosen such that the discrete-time signals  $s_m(n) = \text{IDFT}\{S_m(k)\}$  and  $h_m(n) = \text{IDFT}\{H_m(k)\}$ , where  $n, k = 0, \dots, K-1$ , comply with both no discrete-time aliasing and Nyquist rate, the error introduced in (18) is negligible. The next two sections illustrate these ideas in a practical application.

### B. Multi-orthogonal Transmission Techniques

Expressing a signal as in Eqn. (1), where the coefficients  $a_l |_{l=0}^{N-1}$  are information symbols to be transmitted through a communications channel, opens a general technique to transmit information using orthogonal signals. A well known example of multi-orthogonal transmissions is the multicarrier transmission technique where the orthogonal functions are the complex exponentials

$$s(t) = \sum_{l=0}^{N-1} a_l e^{j2\pi f_l t}, \quad 0 \leq t \leq T. \quad (19)$$

To this end, the analog to digital conversion techniques presented in this paper provide a simple and compact method to perform signal A/D conversion and demodulation in the same domain used to transmit the information. An example of this family of receivers is illustrated in the following, where the frequency domain is used to achieve orthogonality.

### C. A Multicarrier UWB Receiver

Let us consider a practical example application of the frequency A/D converter in which a multicarrier modulation is

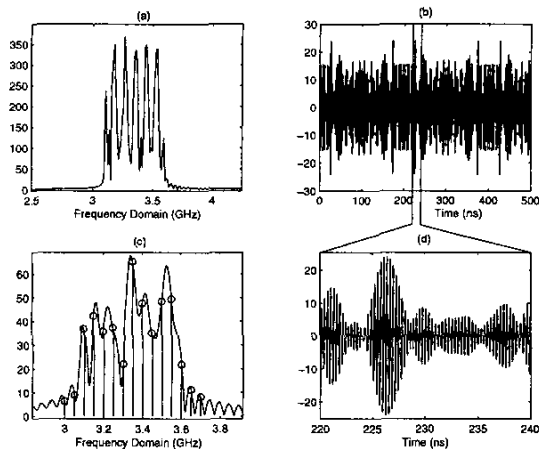


Fig. 4. Illustrative example of the frequency domain A/D converter operating over a multicarrier binary modulated signal: (a) the spectrum of the 500 ns multicarrier signal where 100 equally spaced carriers in the band  $3.1\text{GHz} \leq F \leq 3.6\text{GHz}$  are used, (b) one realization of the 500 ns transmitted signal, (c) the spectrum and the frequency samples taken by the ADC for one piece of the segmented signal, and (d) the continuous-time signal with duration  $T_c = 20$  ns whose spectrum was sampled in (c).

used to transmit information. A transmission rate of  $R = 200$  Mbps is desired, which will be achieved by transmitting 100 binary modulated carriers with frequency spacing  $\Delta F = 5$  MHz in a 500 MHz bandwidth that lies in the range 3.1 GHz - 3.6 GHz. Notice that while this is the lowest UWB spectrum sub-band allowed for the FCC regulation in February 2002, more sub-bands can be accommodated up to 10.6 GHz. So, if only this sub-band is utilized, the total duration of each signal carrying the 100 bits will be  $T_s = 500$  ns.

Let us assume that  $N = 15$  samples of the signal spectrum is a reasonable number for the implementation of the proposed ADC. The samples are taken from 3 GHz (DC) up to 3.7 GHz, where 100 MHz have been added to each side of the spectrum to capture some energy from the ripple introduced by the time domain segmentation. This frequency range, and the number of samples selected, lead to a conversion time  $T_c = \frac{N-1}{W} = 20$  ns, ensuring that no time aliasing occurs in the discrete time version of the segmented signal. Notice that the frequency samples do not match the frequency of the transmitted carriers since the received signal is being A/D converted in a much shorter time window. However, as shown in (18), matched filtering can still be carried out with the frequency samples. Consequently, the frequency ADC is not a correlator bank, unless the signal is not time-segmented and the sampling frequencies are matched to the carrier frequencies. Furthermore, the technique proposed here only requires 15 samples of the spectrum, whereas a correlator bank would need 100 samples to estimate the decision variables in this specific example.

Figure 4 illustrates the effect of doing the A/D conversion in the frequency domain by sampling the spectrum of the received signal in a time window that is much shorter than the information symbol. Figure 4(a) shows the spectrum of one

realization of the transmitted signal with duration 500 ns, 4(b) shows the time domain realization of the 500 ns transmitted signal, 4(c) shows how the spectrum of the signal is spread out when the frequency ADC takes samples in the short time window, and 4(d) illustrates the continuous-time envelope of the signal sampled in (c). It is interesting to notice that since the A/D conversion is performed in the frequency domain, no fast Fourier transform (FFT) is needed, which reduces the system complexity. On the other hand, if classical time-domain A/D conversion plus an FFT is used, as in orthogonal frequency division multiplexing (OFDM), not only the complexity is potentially higher but the quantization error due to both the quantization in the time-domain and in the frequency domain after the FFT, will degrade the system performance. We are currently studying the performance penalty associated with the approximation in (18).

## VI. CONCLUSIONS

This paper introduces analog to digital conversion in orthogonal spaces, where instead of sampling the signal in the time domain, samples of the coefficients of an orthogonal expansion are taken every  $T_c$  seconds. As a specific application, the frequency domain appears as an appealing domain to perform the A/D conversion of signals that are very wideband, such as UWB signals. Moreover, having samples of the signal spectrum encourages implementing the receiver in the frequency domain. Specifically, it was shown how the matched filter can be easily implemented, even though segmentation of the time-domain signal is used to reduce the number of basis coefficients. When a multicarrier transmission is preferred, the proposed frequency domain ADC goes together with a very simple discrete-frequency domain implementation of the correlators needed for the estimation of the information symbols.

Extensions of interest include system performance in additive noise and narrow band interference, as well as the study of other sampling domains, such as wavelets and Hadamard sequences.

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