Performance of Random Access Scheduling Schemes in Multi-hop Wireless Networks

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***Abstract*— The performance of scheduling schemes in multi-hop wireless networks has attracted significant attention in the recent literature. It is well known that optimal scheduling solutions require centralized information and lead to impractical implementations due to their enormous complexity (high-degree polynomial or NP-hard, depending on the interference scenario). Further, an important characteristic in many multi-hop scenarios is the need for distributed algorithms that operate on local information. Thus, in this paper we develop a constant-time random access distributed algorithm for scheduling in multi-hop wireless networks. An important feature of this scheme is that it achieves a 2-approximation efficiency ratio and can be implemented in an entirely distributed manner with low overhead.**

I. Introduction

In the recent literature, it has been shown that the optimal solution to maximize the aggregate utility in multi-hop wireless networks can be decomposed into two components: congestion control and scheduling, with minimal information exchange among the components [1], [2], [3], [4], [5]. While the congestion control component can be optimized using techniques from *convex programming*, the scheduling problem cannot be easily solved as it is a complex non-convex optimization problem caused due to the interplay between the link schedule and interference. While there are known optimal scheduling algorithms, they require centralized information and intensive computations, which are major obstacles to their practical use [2], [5], [6], [7], [8].

The difficulties in the implementation of optimal scheduling algorithms emphasize the need for imperfect (but simpler) scheduling scheme that may achieve a fraction of the overall optimal performance. It has been shown in [5] that designing appropriate imperfect scheduling schemes along with congestion control can achieve graceful degradation and performance bounds that are better than the layered approach. Our goal will be to develop simple imperfect scheduling solutions that can at least obtain a certain fraction of the network capacity.

We consider a multi-hop wireless network with N nodes and L links. Let λ_l and c_l denote the offered load to link l and the capacity of the link, respectively. We also define a load vector $\vec{\lambda} = [\lambda_1, \lambda_2, \dots, \lambda_L]$, and a capacity region as a set $\{\vec{\lambda}\}$. A scheduling policy is said to *achieve the capacity region* if it supports any $\vec{\lambda}$ in the capacity region while keeping the queues of all links finite. A *throughput-optimal* scheduling policy is defined as the scheduling policy that achieves the largest capacity region. If a scheduling policy achieves a capacity region, the throughput-optimal scheduling policy can achieve the same capacity region. The capacity region achieved by the throughput-optimal scheduling policy is called as the optimal capacity region Ω. If an imperfect scheduling policy can support $\gamma\vec{\lambda}$ for all $\vec{\lambda} \in \Omega$, it is said to have an *efficiency ratio* of γ.

We examine the performance of scheduling policies under the so-called *node-exclusive model*. It is a good model for Bluetooth or FH-CDMA networks [5], [9]. Under the node-exclusive model, a node cannot simultaneously transmit or receive, and cannot simultaneously communicate with two or more nodes in the network. Let $E(i)$ be the set of links connected to node i. Under the constraints of the model and the fact that the offered load cannot exceed the link capacity, the optimal capacity region Ω is bounded by

$$\Omega \subseteq \Psi,$$

where

$$\Psi = \left\{ \vec{\lambda} \,\middle|\, \sum_{l \in E(i)} \frac{\lambda_l}{c_l} \leq 1, \text{ for all nodes } i \right\}. \tag{1}$$

In this model, the optimal scheduling policy, i.e., Maximal Weighted Matching (*MWM*) has $O(N^3)$ complexity [10] and is thus difficult to implement. A simpler sub-optimal policy like Greedy Maximal Matching (*GMM*) [11] can be used at the expense of performance. It is known that the centralized *GMM* can support up to $\frac{1}{2}\vec{\lambda}$ for any $\vec{\lambda} \in \Psi$ [6], which means that it has the efficiency ratio of at least $\frac{1}{2}$ while its algorithmic complexity is $O(L \log L)$. Since both *MWM* and *GMM* are centralized algorithms requiring global information, they are difficult to implement in multi-hop networks. Resource restrictions on computation complexity, radio power, and bandwidth demand a distributed algorithm using local information.

Recent studies [5], [6], [7], [8] focus on distributed scheduling policies that have a provable efficiency ratio. However, most of these policies have a non-constant time requirement to compute a single schedule: the computation time increases with the network size. For instance, the maximal matching scheduling policy (*MM*), whose efficiency ratio is at least $\frac{1}{2}$ [5], requires $O(\log^4 N)$ computations [12]. Lin and Rasool have recently proposed a constant-time scheduling policy that completes its scheduling after M rounds [6]. This policy is proven to achieve an efficiency ratio of at least $\left(\frac{1}{3} - \frac{1}{M}\right)$. However, simulations demonstrate that there is very often a significant performance gap between it and *GMM*.

The paper is organized as follows. We begin with the description of our system model and overview previous constant-time scheduling policies in Section II. Motivated by these, we develop a new scheduling policy and analyze it in Section III, and conclude our paper in Section IV.

II. System Model

In multi-hop wireless networks with N nodes and L links, a link l consists of two nodes $s(l)$ and $r(l)$. Let $E(i)$ denote the set of links connected to node i, and $N(l)$ denote the set of neighboring links sharing a common node with link l, that is, $N(l) = E(s(l)) \cup E(r(l)) \backslash \{l\}$. In addition, let $N(l)^+$ denote the union of $N(l)$ and $\{l\}$, and n_l and n_l^+ denote the number of links in $N(l)$ and $N(l)^+$, respectively.

We assume that all links are synchronized and start a frame at the same time. Each frame consists of a contention period of M slots and a transmission period for actual packet transmission. We reserve the term *slot* to indicate a unit time in the contention period. Each link can *contend* in a frame or remain idle, skipping to the next frame. If a link participates in contention in a frame, it *attempts transmission* at a slot in a probabilistic manner.

We consider a simple interference model in this paper; the node-exclusive interference model. The node-exclusive model does not allow any two links sharing a common node to transmit at the same time. If that happens, a collision is said to occur and the frame is discarded.

A. Policy P

We refer to the constant-time distributed scheduling scheme proposed in [6] as policy P. This policy is proven to have an efficiency ratio of at least $\left(\frac{1}{3} - \frac{1}{M}\right)$. Under policy P, each link l contends in a frame with a probability x_l, which is given by

$$x_l = \frac{Q_l/c_l}{\max\left(\sum_{k\in E(s(l))} Q_k/c_k, \sum_{k\in E(r(l))} Q_k/c_k\right)}, \qquad (2)$$

where Q_l is the queue length of link l and c_l is the number of packets can be transmitted in a frame time. Let $\vec{x}$ denote the vector $[x_1c_1, x_2c_2, \ldots, x_Lc_L]$. Once a link decides to contend in a frame, it randomly picks a slot and attempts transmission at the slot if no neighboring link is already transmitting. In case that more than two neighboring links attempt transmission at the same slot, a collision occurs and the frame is discarded.

Since the conditional probability of successful transmission provided a link l attempts is at least $\left(\frac{1}{3} - \frac{1}{M}\right)$, the successful transmission probability P_S^l can be written as

$$P_S^l \geq x_l \left(\frac{1}{3} - \frac{1}{M}\right). \qquad (3)$$

It is proven in [6] that if $\vec{\lambda}$ lies strictly inside the set $\left(\frac{1}{3} - \frac{1}{M}\right)\Psi$, the network remains stable. In particular, $\vec{x}\left(\frac{1}{3} - \frac{1}{M}\right)$ itself is within the stability region of policy P. Thus the lower bound on the *efficiency ratio* γ_p is given by

$$\gamma_p \geq \frac{1}{3} - \frac{1}{M}. \qquad (4)$$

B. An equivalent modification

Now, let us consider a policy $\acute{P}$ which results in an equivalent bound on the efficiency ratio of P. This policy will be used to obtain more efficient scheduling policies.

Policy $\acute{P}$: All links contend in every frame and attempt transmission at each slot with probability $\frac{x_l}{M}$ if no neighboring link is already transmitting.

The difference from policy P is that links under policy $\acute{P}$ contend in *every frame* and attempt transmission at *each slot* with probability $\frac{x_l}{M}$. In contrast, links under policy P contend with probability x_l and attempt transmission at a randomly chosen slot among M slots.

Proposition 1: The *efficiency ratio* $\gamma_{\acute{P}}$ is no less than $\left(\frac{1}{3} - \frac{1}{M}\right)$.

Proof: Let $P_s^l[m]$ denote the successful transmission probability of link l at time slot m. $P_s^l[0]$ is the product of link l's attempt probability and the probability that neighboring links will not attempt transmission at the first slot, that is,

$$P_s^l[0] = \frac{x_l}{M} \cdot \prod_{k\in N(l)} \left(1 - \frac{x_k}{M}\right).$$

At $m = 1$, $P_s^l[1]$ is the product of the probability that no link in $N(l)^+$ attempts transmission at $m = 0$, the attempt probability of link l at $m = 1$, and the probability that other neighbors $k \in N(l)$ do not attempt transmission at $m = 1$. This will be the same as $P_s^l[0]$ with the additional product of "non-attempt probabilities" of all links in $N(l)^+$ at $m = 0$, that is,

$$P_s^l[1] = \frac{x_l}{M} \cdot \prod_{k\in N(l)} \left(1 - \frac{x_k}{M}\right) \cdot \left[\prod_{k\in N(l)^+} \left(1 - \frac{x_k}{M}\right)\right].$$

Similarly, $P_s^l[m]$ can be written as

$$P_s^l[m] = \frac{x_l}{M} \cdot \prod_{k\in N(l)} \left(1 - \frac{x_k}{M}\right) \cdot \left[\prod_{k\in N(l)^+} \left(1 - \frac{x_k}{M}\right)\right]^m$$
$$= \frac{x_l/M}{1 - x_l/M} \cdot \left[\prod_{k\in N(l)^+} \left(1 - \frac{x_k}{M}\right)\right]^{m+1}. \qquad (5)$$

Using $\frac{1}{1-x_l/M} \geq 1$ and $\left(1 - x_k\frac{1}{M}\right)^{m+1} \geq \left(1 - x_k\frac{m+1}{M}\right)$, we obtain

$$P_s^l[m] \geq \frac{x_l}{M} \prod_{k\in N(l)^+} \left(1 - x_k\frac{m+1}{M}\right). \qquad (6)$$

Since (6) is exactly the same equation as in [6], we apply the same analysis for obtaining the successful transmission probability P_S^l, i.e.,

$$\begin{aligned} P_S^l &= \sum_{m=0}^{M-1} P_s^l[m] \\ &\geq \sum_{m=0}^{M-1} \left[\frac{x_l}{M} \prod_{k\in N(l)^+} \left(1 - x_k \frac{m+1}{M}\right)\right] \\ &= \sum_{m=0}^{M} \left[\frac{x_l}{M} \prod_{k\in N(l)^+} \left(1 - x_k \frac{m}{M}\right)\right] - \frac{x_l}{M}. \end{aligned}$$

Comparing with $\prod_{k\in N(l)^+}(1 - x_k u)$, which is a monotonic decreasing function of $u \in [0, 1]$, the summation is bounded by the following integral,

$$P_S^l \geq x_l \int_0^1 \prod_{k\in N(l)^+} (1 - x_k u)\, du - \frac{x_l}{M}.$$

We also apply the following inequality, which can be validated by comparing the derivatives,

$$\prod_{k\in N(l)^+} (1 - x_k u) \geq (1 - u)^H,$$

where $H = \sum_{k\in N(l)^+} x_k$, we obtain

$$\begin{aligned} P_S^l &\geq x_l \int_0^1 (1-u)^H\, du - \frac{x_l}{M} \\ &\geq x_l \left(\frac{1}{H+1} - \frac{1}{M}\right). \end{aligned}$$

From (2),

$$H = \sum_{k\in N(l)^+} x_k \leq \sum_{k\in E(s(l))} x_k + \sum_{k\in E(d(l))} x_k \leq 2. \qquad (7)$$

Hence, we obtain

$$P_S^l \geq x_l \left(\frac{1}{3} - \frac{1}{M}\right).$$

Now, since $\vec{x}\left(\frac{1}{3} - \frac{1}{M}\right)$ is within the stability region of policy $\acute{P}$, we can bound $\gamma_{\acute{p}}$ as

$$\gamma_{\acute{p}} \geq \frac{1}{3} - \frac{1}{M}, \qquad (8)$$

which is the same as γ_p. ■

Before proceeding, let us consider a natural random access scheduling policy.

Policy U: All links contend in every frame. Each link picks a slot at random and attempts transmission in this slot if no neighboring link is already transmitting.

We consider the performance of policy U in two extreme cases; if M is fixed and the number of neighboring links increases to infinity, none of the links are able to successfully transmit. Whenever a link attempts transmission at a slot, there exists a neighboring link attempting transmission at the same slot causing a collision. On the other hand, if the number of neighbors is fixed and M infinitely increases, it can achieve an efficiency ratio bounded by $\frac{1}{2}$. The long contention period makes the probability of collision close to zero, and policy U eventually results in a maximal matching, which has an efficiency ratio of at least $\frac{1}{2}$.

The intuition obtained from policy U is that a random access scheduling can bound the efficiency ratio up to $\frac{1}{2}$ with large M. This also implies that policy P can be enhanced to achieve a better efficiency ratio than $\frac{1}{3}$.

III. Controlling the Attempt Probability

We propose an enhanced scheduling policy that is not affected by the link degree (like policy P and $\acute{P}$) and also achieves an efficiency ratio of $\frac{1}{2}$ for large M (like policy U).

Policy V: Policy V is identical to policy $\acute{P}$, except that each link attempts transmission with probability of $\alpha\frac{x_l}{M}$ instead of $\frac{x_l}{M}$, where

$$\alpha = \frac{\sqrt{M} - 1}{2}. \qquad (9)$$

A detailed algorithmic description of policy V is illustrated in Fig. 1. Since it is a distributed algorithm, all links have the same procedure. At the beginning of each frame, each link initializes a variable $sched$ to 0, which means that the schedule of the link is not determined. At each contending slot, if the link's schedule in this frame is not determined yet, it attempts transmission with the probability that depends on queue length of links in its neighborhood. If it attempts, it changes the variable $sched$ to 1 (scheduled), and all links that detects the transmission set their variable $sched$ to -1 (not scheduled). This procedure repeats until the contention period ends. After the contention period, those scheduled links can make successful transmissions.

Proposition 2: The *efficiency ratio* γ_v of policy V is bounded by

$$\gamma_v \geq \frac{1}{2} - \frac{1}{\sqrt{M}}. \qquad (10)$$

Proof: We begin with the analysis of policy $\acute{P}$ except using attempting probability $\alpha\frac{x_l}{M}$ instead of $\frac{x_l}{M}$. We assume that α is an arbitrary real number in $(0, \frac{M}{2})$. We will later confirm that the α obtained to guarantee the efficiency ratio in the proposition will lie in $(0, \frac{M}{2})$.

From (5), the successful transmission probability of link l can be written as,

$$\begin{aligned} P_S^l &\geq \sum_{m=0}^{M-1} \frac{\alpha x_l/M}{1 - \alpha x_l/M} \left[\prod_{k\in N(l)^+} \left(1 - \alpha\frac{x_k}{M}\right)\right]^{m+1} \\ &\geq \sum_{m=0}^{M-1} \alpha\frac{x_l}{M} \left[\prod_{k\in N(l)^+} \left(1 - \alpha\frac{x_k}{M}\right)\right]^{m+1} \end{aligned}$$

because $\alpha\frac{x_l}{M} \leq 1$. Letting $H = \sum_{k\in N(l)^+} x_k$, we have

$$\prod_{k\in N(l)^+} \left(1 - \alpha\frac{x_k}{M}\right) \geq 1 - \frac{\alpha H}{M}.$$

At each frame, each link does the following.

$sched \leftarrow 0$.
for each contending slot **do**
  if $(sched = 0)$ **do**
    attempt transmission with probability $x_l \frac{\sqrt{M}-1}{2M}$.
    if attempted, $sched \leftarrow 1$.
  end if
  if (overhear neighbors' attempt) **do**
    $sched \leftarrow (-1)$.
  end if
end for
if $(sched = 1)$ **do**
  transmit data packets during transmission period.
end if

Fig. 1. Distributed algorithm of Policy V.

Note that $\left(1 - \frac{\alpha H}{M}\right) \geq 0$ from (7) and $\alpha \in (0, \frac{M}{2})$.

Letting $G(\alpha, M) = \left(1 - \frac{\alpha H}{M}\right)$, the inequality can be rewritten as

$$\begin{aligned} P_S^l &\geq \alpha \frac{x_l}{M} \sum_{m=0}^{M-1} G(\alpha, M)^{m+1} \\ &= \alpha \frac{x_l}{M} \left(\sum_{m=0}^{M} G(\alpha, M)^m - 1 \right) \\ &= \alpha \frac{x_l}{M} \left(\frac{1 - G(\alpha, M)^{M+1}}{1 - G(\alpha, M)} - 1 \right) \\ &\geq \alpha \frac{x_l}{M} \left(\frac{M}{\alpha H + 1} - 1 \right). \end{aligned} \tag{11}$$

For the last inequality in (11), we need to prove that

$$\frac{1 - \left(1 - \frac{\alpha H}{M}\right)^{M+1}}{1 - \left(1 - \frac{\alpha H}{M}\right)} \geq \frac{M}{\alpha H + 1}.$$

Multiplying $\frac{\alpha H}{M}$ on both sides and rearranging the equations, we obtain the following equivalent inequality:

$$\left(1 - \frac{\alpha H}{M}\right)^{M+1} \leq \frac{1}{\alpha H + 1}. \tag{12}$$

The following provides the details to show the inequality (12).

Defining a function $F(z)$ with $0 < z < M$ as

$$F(z) = (z+1) \left(1 - \frac{z}{M}\right)^{M+1},$$

and differentiating it by z, we obtain

$$\frac{dF(z)}{dz} = - \left(1 - \frac{z}{M}\right)^{M} \left(z + 2\frac{z}{M} + \frac{1}{M}\right).$$

Since $\frac{dF(z)}{dz} < 0$ for $z \in (0, M)$ and $F(0) = 1$, we have $F(z) \leq 1$ for all feasible z. After replacing z with αH, we obtain inequality (12).

Now, from (7) and (11), we obtain

$$P_S^l \geq x_l \left(\frac{\alpha}{2\alpha + 1} - \frac{\alpha}{M} \right).$$

Since this inequality holds for an arbitrary $\alpha \in (0, \frac{M}{2})$, we can maximize the lower bound by differentiating it by α,

$$\begin{aligned} P_S^l &\geq x_l \frac{(\sqrt{M}-1)^2}{2M} \\ &\geq x_l \left(\frac{1}{2} - \frac{1}{\sqrt{M}} \right) \end{aligned} \tag{13}$$

when

$$\alpha = \frac{\sqrt{M}-1}{2}.$$

Since $\alpha = \frac{\sqrt{M}-1}{2}$, we confirm $\alpha \in (0, \frac{M}{2})$ for any integer $M > 1$. Since $\vec{x}\left(\frac{1}{2} - \frac{1}{\sqrt{M}}\right)$ is within the stability region of policy V, the efficiency ratio γ_v is bounded by $\left(\frac{1}{2} - \frac{1}{\sqrt{M}}\right)$. ■

Policy V cannot be implemented in the same manner as policy P. In policy P, each link contends within a frame with probability x_l, and links participating in the contention choose a slot with probability $\frac{1}{M}$. To use the framework of policy P, each link contends with probability αx_l or contending links pick a slot with probability $\frac{\alpha}{M}$. The former method is not possible because αx_l can be larger than 1, which is meaningless. The latter method cannot accomplish an efficiency ratio of (10) either because it reduces the probability of successful transmission by restricting the number of slots to $\frac{M}{\alpha}$ rather than M.

Essentially, policy V achieves the performance improvement by controlling the attempt probability through the parameter α. However, under policy P, such control is not possible because it has a uniform distribution of links over slots, which means that the attempt probability is predetermined by M. By changing the uniform distribution to a geometric distribution, policy V has the ability of controlling the attempt probability.

IV. Conclusion

For multi-hop wireless networks, it has been shown that a cross-layer approach even with an *imperfect scheduling* component usually outperforms the layered approach [5]. However, in order to guarantee good performance, it is important for the imperfect scheduling component to achieve a provable *efficiency ratio*. A higher efficiency ratio implies better performance. However, practical requirements demand that the scheduling algorithm operate in a distributed manner with local information and complete a schedule within a constant time.

A recent constant-time distributed scheduling policy has been developed and shown to have an efficiency ratio bounded by $\frac{1}{3}$ for the node-exclusive interference model. One of its advantages is that the performance bound is not affected by the number of contending links in its neighborhood. However, it has a performance bound significantly lower than the $\frac{1}{2}$ of a maximal matching, and empirically achieves a much smaller capacity region than the centralized *GMM*.

In this paper, we propose a new constant-time distributed scheduling policy that achieves a provable efficiency ratio that is unaffected by the number of contending links and can increase up to $\frac{1}{2}$ under the node-exclusive model. Its gain essentially comes from the control of links' attempt probability separated from M. The scheduling policy enables the control using the geometric distribution of links' attempts over contention slots.

Our policy can be used to find the optimal M accounting for the overhead of a contending slot, and also can be extended to the two-hop interference model [13].

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