

Set your phasors on stun...

it's time for phasor circuit analysis

In other words, finding the steady state sinusoidal steady state response without touching differential equations.

How do the following translate into the phasor domain:

Connection constraints (Kirchhoff's laws)

Device constraints (element equations)

If a circuit is being driven by sinusoidal inputs with frequency  $\omega$ , what will the following elements do to this frequency:

- a capacitor
- an inductor
- an Op Amp

Homework set #8, due 5/24/04

8-1, 8-2, 8-3, 8-6, 8-10, 8-12, 8-18, 8-22, 8-26, 8-29, 8-30

Connection constraints, consider KCL at a node with N inputs:

$$i_1(t) + i_2(t) + \dots + i_N(t) = 0$$

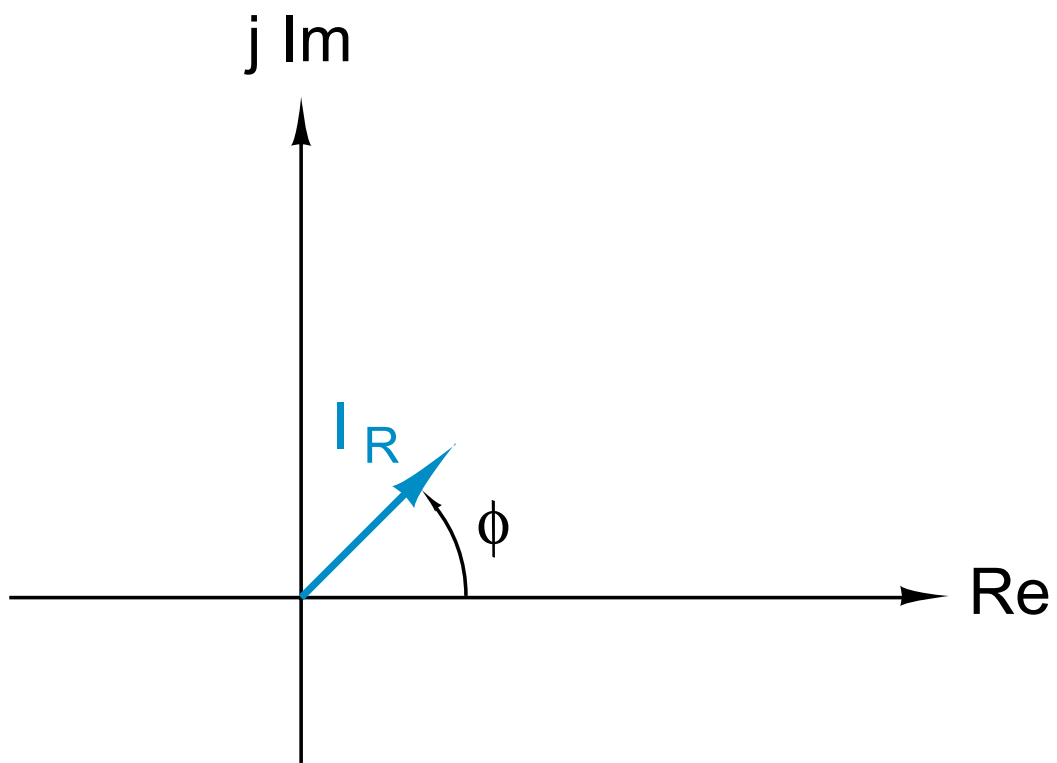
$$I_1 \cos(\omega t + \phi_1) + I_2 \cos(\omega t + \phi_2) + \dots + I_N \cos(\omega t + \phi_N) = 0$$

Using duality, extend to KVL

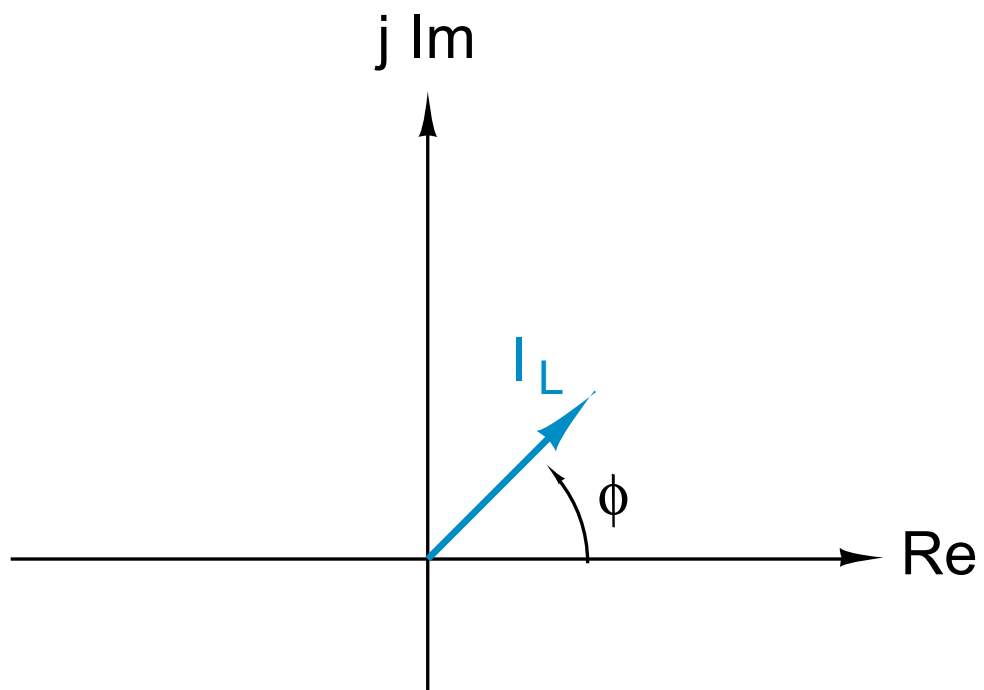
- KVL: The algebraic sum of phasor voltages around a loop is zero
- KCL: The algebraic sum of phasor currents at a node is zero

## Device constraints in phasor phorm

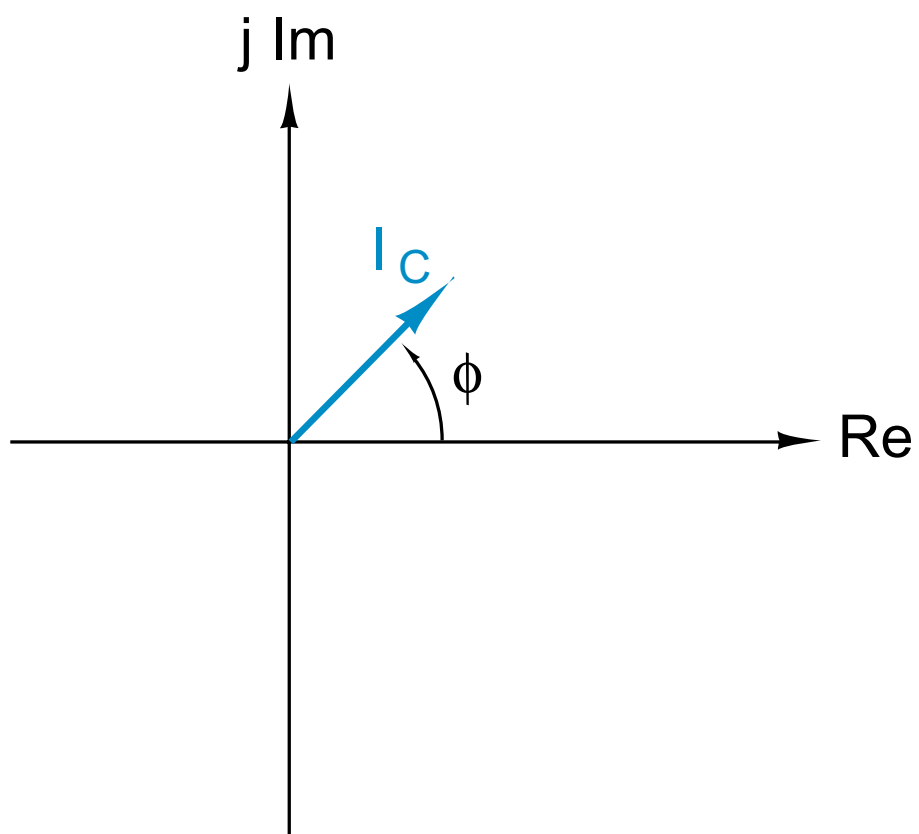
Resistor:  $v_R(t) = Ri_R(t)$



Inductor:  $v_L(t) = L \frac{di_L(t)}{dt}$



Capacitor:  $i_C(t) = C \frac{dv_C(t)}{dt}$



To resist is human, to impede is divine (or at least complex)

$$\mathbf{V} = \mathbf{Z}\mathbf{I}$$

$Z$  = impedance, the proportionality constant relating phasor voltage and phasor current in a linear, two terminal element (again, assuming passive sign convention)

Resistor:  $Z_R = R$

Inductor:  $Z_L = j\omega L$

Capacitor:  $Z_C = \frac{1}{j\omega C} = \frac{-j}{\omega C}$

What are the units of impedance?

$$Z = \frac{\mathbf{V}}{\mathbf{I}}$$

Is impedance a complex number?

Is impedance a phasor?

Breaking things apart...

$$Z = R + jX$$

R=resistance, X=reactance

and flipping them over...

$$Y = \frac{1}{Z} = G + jB$$

Y=admittance, G=conductance, B=susceptance

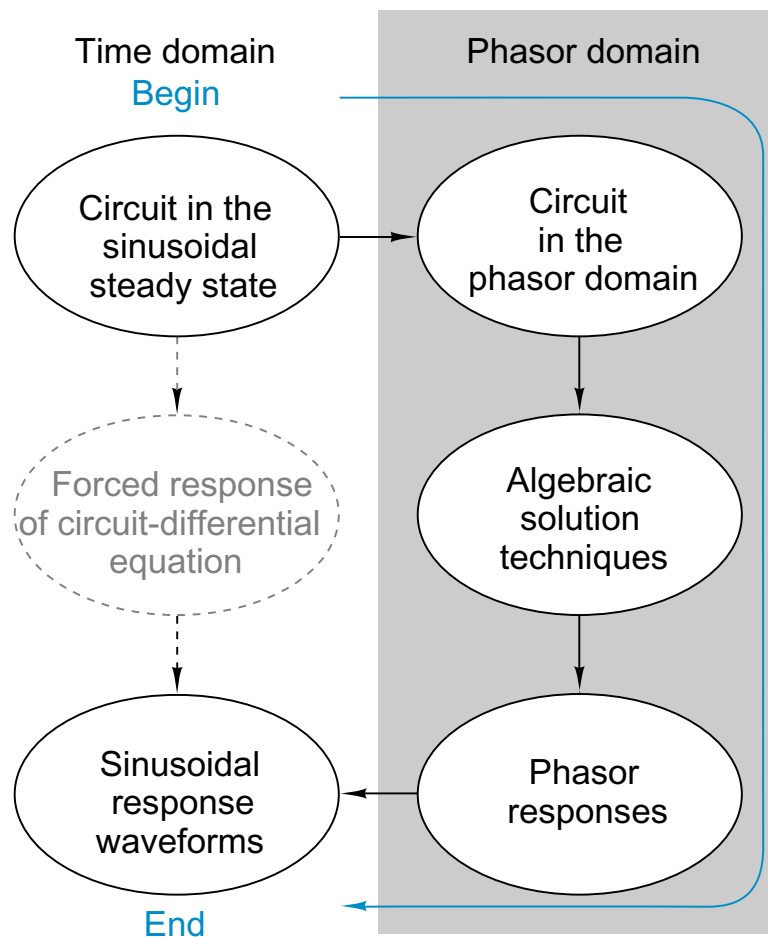
(alternatively, B>0 is inductance, B<0 is capacitance)

... and putting them back together

All of the tools we used for DC resistor circuits (including nodal and mesh anal) apply to AC phasors, except the signals and elements are described using complex numbers rather than real numbers.

## How to analyze AC circuits:

- 1) Transform circuit to phasor domain (input sinusoids become phasors, passive elements are replaced by their impedances)
- 2) Use the existing tools to solve for desired output phasors
- 3) Transform back into time-domain sinusoids



What happened to the good old days when we always went out and did examples?

Given  $v_s(t) = V_A \cos(\omega t + \phi)$ , find  $v_o(t)$ :

