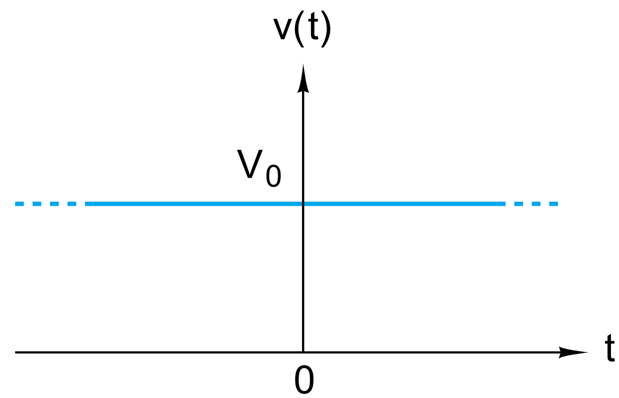
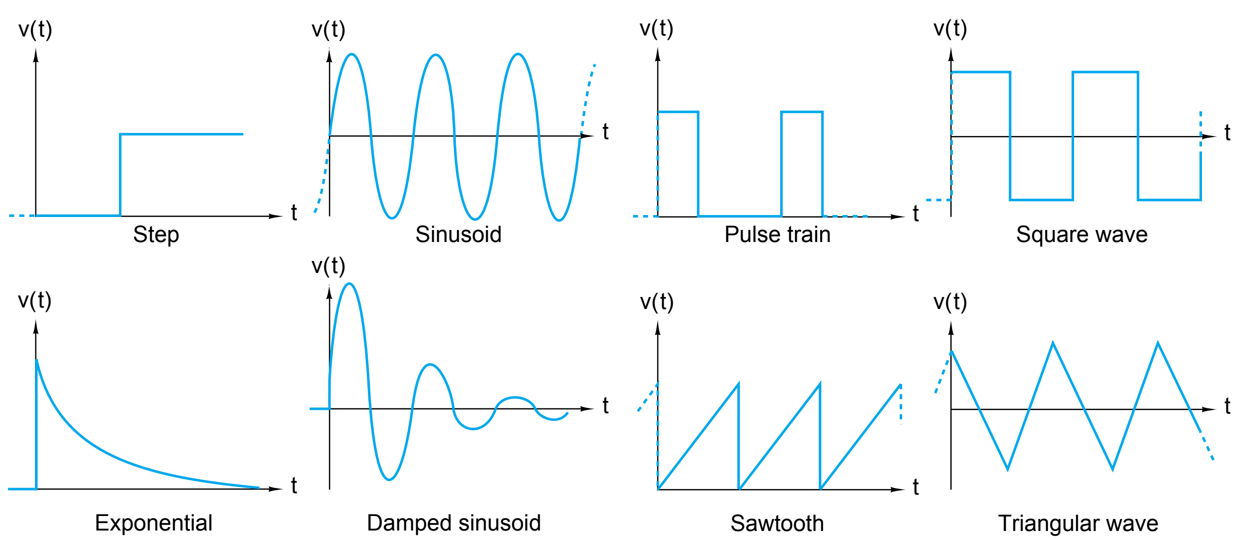
Homework set #6, due 5/7/08
4-37, 4-38, 4-49, 5-4, 5-6, 5-9, 5-12,
5-14, 5-15, 5-19, 5-22, 5-29, 5-33, 5-34

Dynamic Circuits - Signal Waveforms

$$\left.\begin{aligned} v(t) &= V_o \\ i(t) &= I_o \end{aligned}\right\} \text{for } -\infty < t < \infty$$





The Step Function

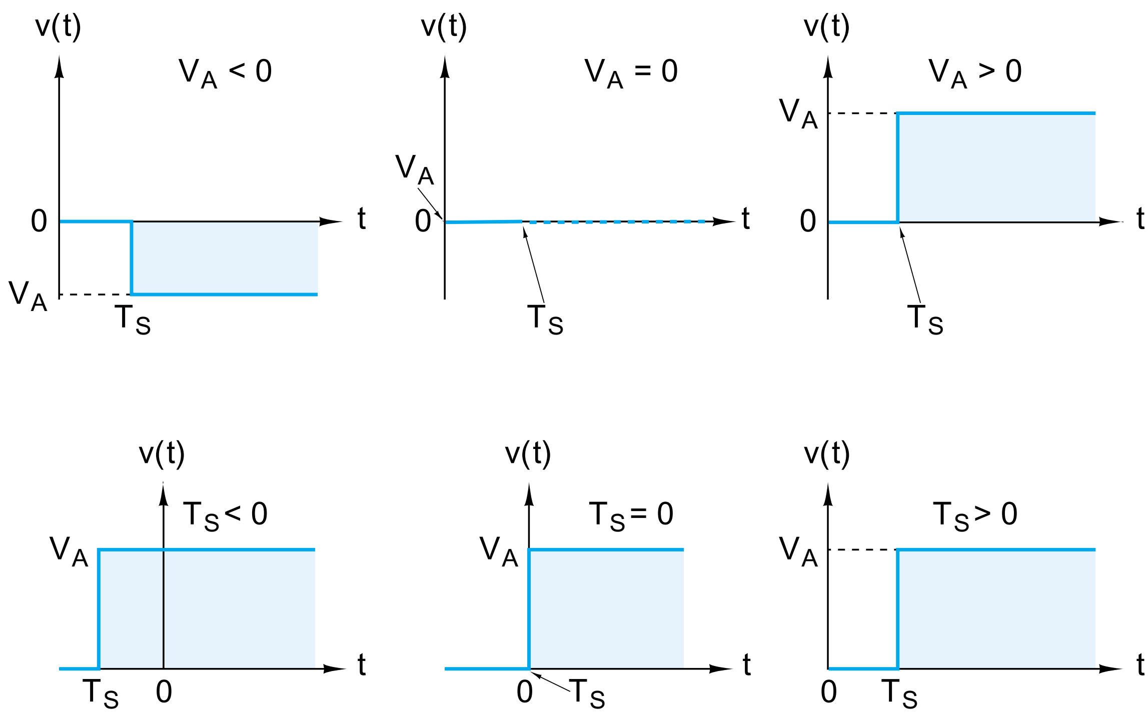
$$u(t) = \begin{cases} 0, & t < 0 \\ 1, & t \geq 0 \end{cases}$$

now change the amplitude...

$$V_A u(t) = \begin{cases} 0, & t < 0 \\ V_A, & t \geq 0 \end{cases}$$

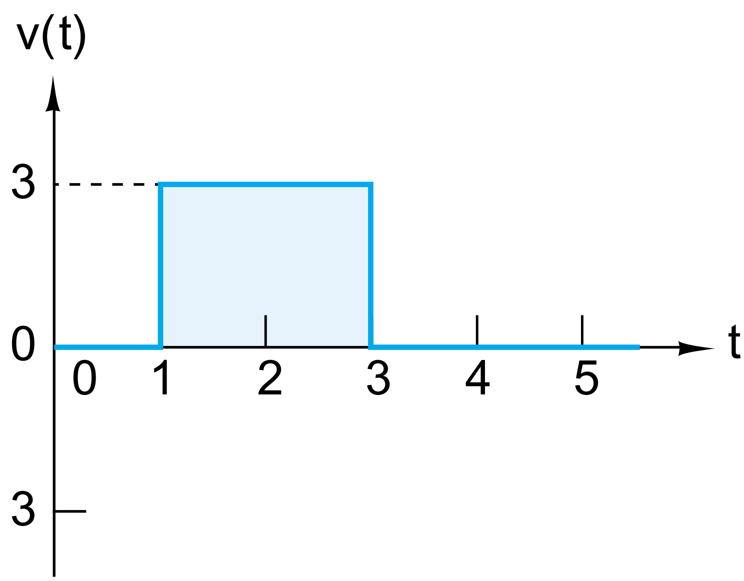
and the time-shift...

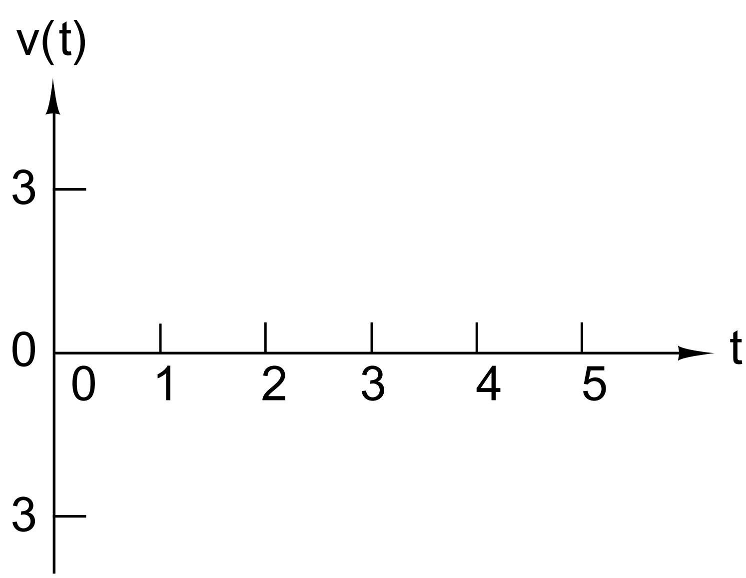
$$V_A u(t - T_S) = \begin{cases} 0, & t < T_S \\ V_A, & t \geq T_S \end{cases}$$



So what else does it do?

Show how the following pulse is simply the superposition of step functions.

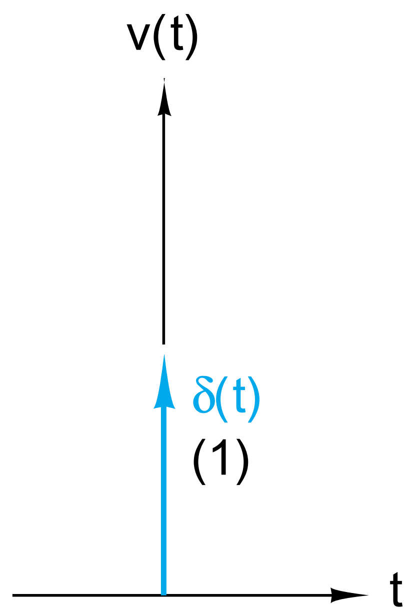




Unit Impulse Function... you gotta trust me on this one.

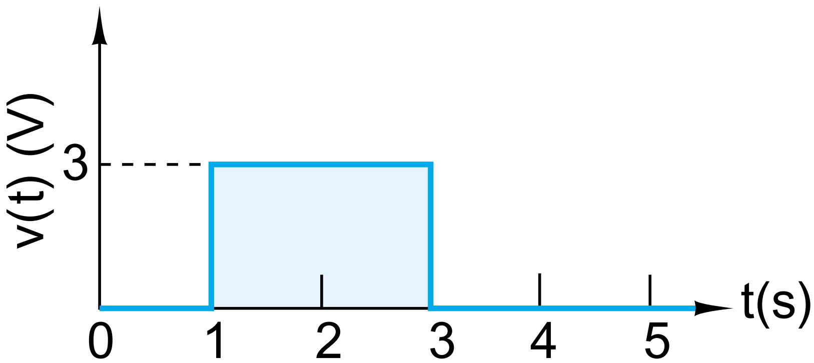
$$\delta(t) = \begin{cases} 0, & t \neq 0 \\ \infty, & t = 0 \end{cases}$$ and $$\int_{-\infty}^{t} \delta(x)dx = u(t)$$

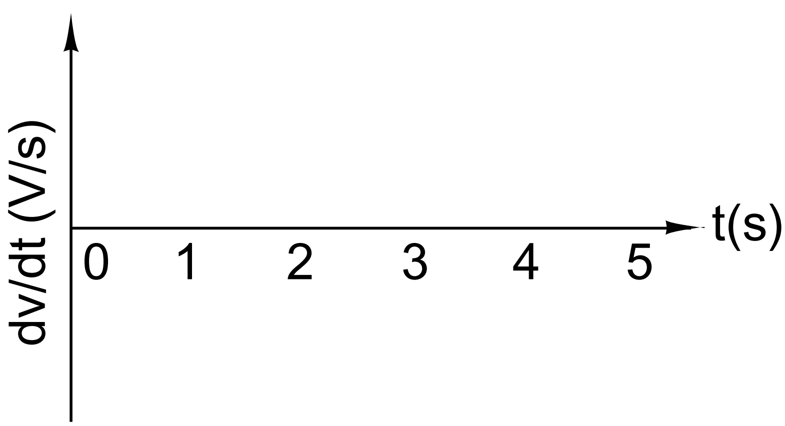
i.e., $$\delta(t) = \frac{du(t)}{dt} \quad \left[\frac{1}{\sec}\right]$$



How do you change the amplitude and the time-shift for this function?

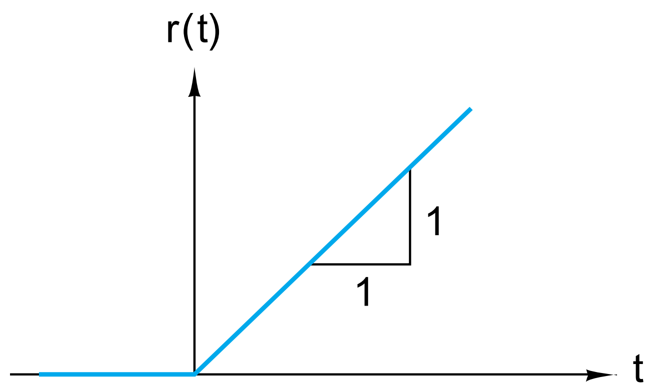
Given the fact that $\delta(t) = \frac{du(t)}{dt}$, calculate the derivative for the following pulse:

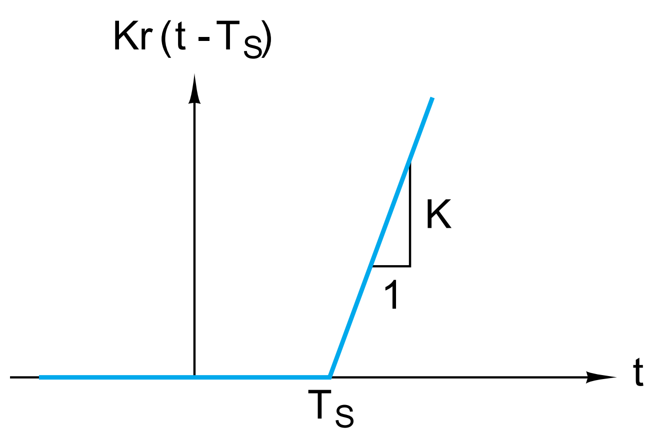




Unit Ramp Function

$$r(t) = \int_{-\infty}^{t} u(x)dx = t \cdot u(t) \quad [\text{sec}]$$





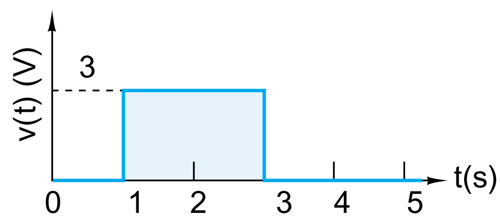
How do you change the amplitude and the time-shift for this function?

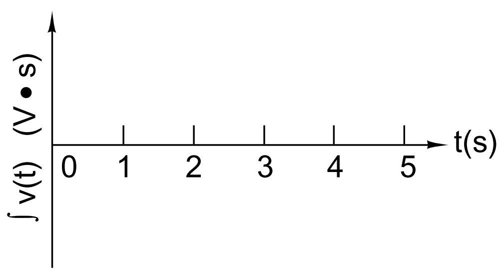
Singularity Functions

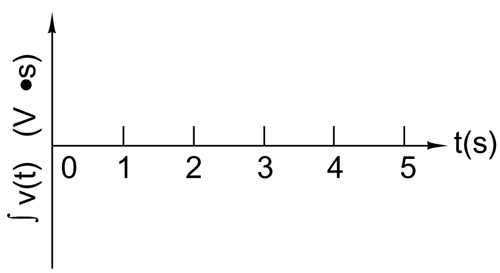
$$u(t) = \int_{-\infty}^{t} \delta(x)dx \qquad \delta(t) = \frac{du(t)}{dt}$$

$$r(t) = \int_{-\infty}^{t} u(x)dx \qquad u(t) = \frac{dr(t)}{dt}$$

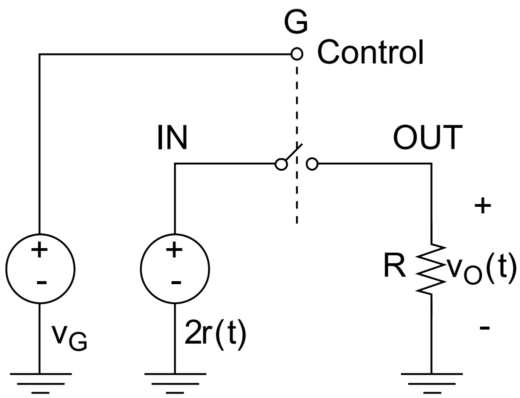
Given the fact that $r(t) = \int_{-\infty}^{t} u(x)dx$, calculate the integral for the following pulse:

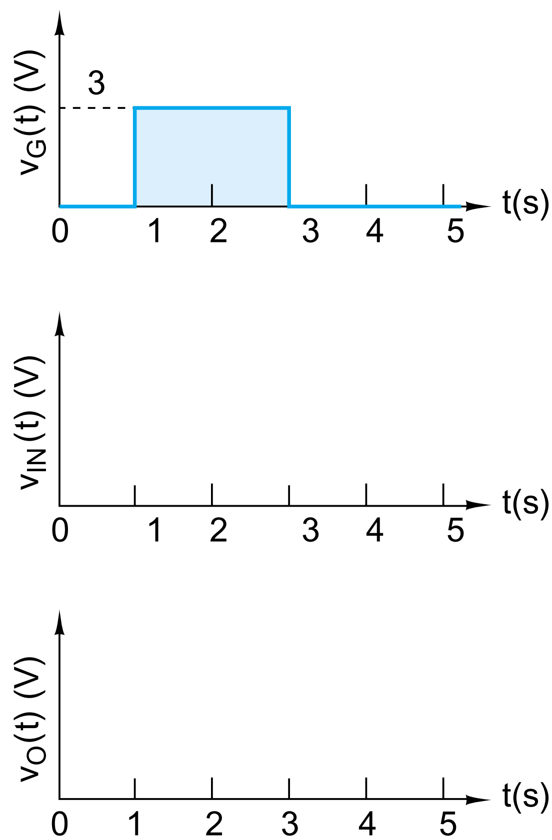






Given v_{in}, as shown in the diagram, gated by the pulse shown below, find v_{out}

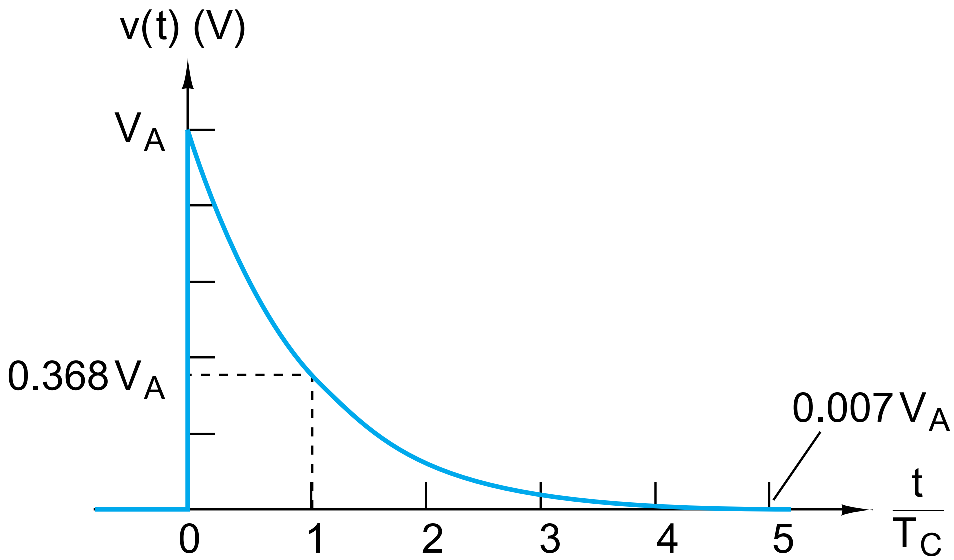


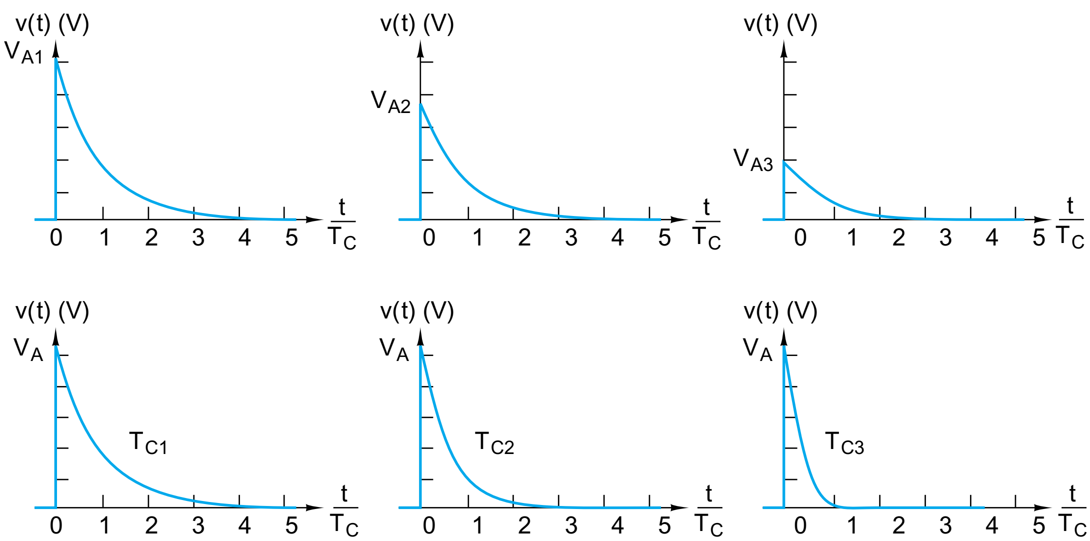


The Exponential Function

$$v(t) = \left[V_A e^{-t/T_C} \right] u(t)$$

time constant: T_C (roughly, how long will the signal last)

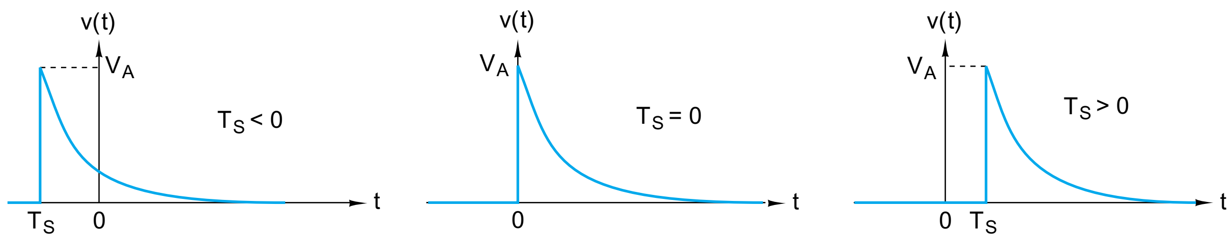




See the text about the *decrement property* and the *slope property*.

Finally, how do you change the time-shift for this function?

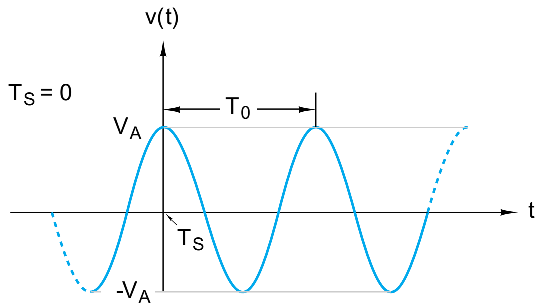
$$v(t) = \left[V_A e^{-(t-T_S)/T_C}\right] u(t - T_S)$$

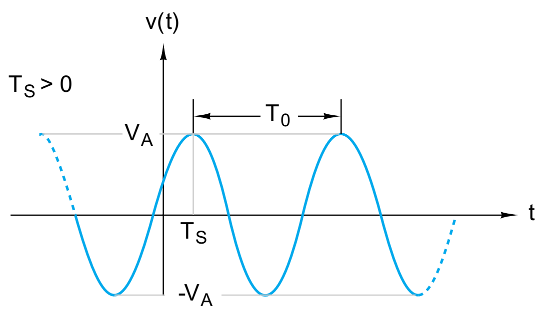


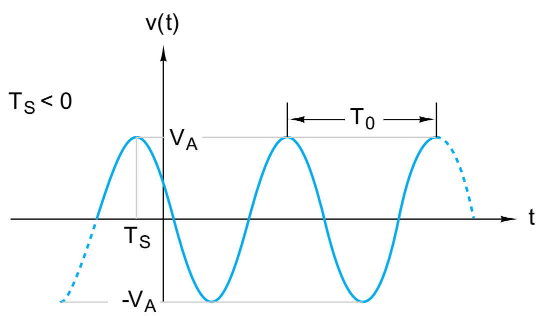
The Sinusoidal Function

$$v(t) = V_A \cos\left(2\pi t / T_O\right)$$

$$v(t) = V_A \cos\left(2\pi \left(t - T_S\right) / T_O\right) = V_A \cos\left(2\pi t / T_O + \phi\right)$$







where:
V_A = amplitude
T_O = period
and the phase angle is:

$$\phi = -2\pi \frac{T_S}{T_O}$$

Frequency:

$$f_O = \frac{1}{T_O}$$

Angular frequency:

$$\omega_O = 2\pi f_O = \frac{2\pi}{T_O}$$

Thus

$$v(t) = V_A \cos\left(\omega_O t + \phi\right) = a\cos\left(\omega_O t\right) + b\sin\left(\omega_O t\right)$$

$\cos(x+y) = \cos(x)\cos(y) - \sin(x)\sin(y)$ $\sin(x+y) = \sin(x)\cos(y) + \cos(x)\sin(y)$

Properties of sinusoids

if there exists some T_O such that, $v(t+T_O)=v(t)$, then v(t) is periodic, else aperiodic

<u>Additive Property</u>:

if

$$v_1(t)=a_1\cos(\omega_O t)+b_1\sin(\omega_O t)$$
$$v_2(t)=a_2\cos(\omega_O t)+b_2\sin(\omega_O t)$$
$$v_3(t)=v_1(t)+v_2(t)$$

then

$$v_3(t)=(a_1+a_2)\cos(\omega_O t)+(b_1+b_2)\sin(\omega_O t)$$

<u>Integrals & derivatives:</u>

$$\frac{d}{dt}V_A\cos(\omega t)=-\omega V_A\sin(\omega t)=\omega V_A\cos(\omega t+\pi/2)$$
$$\int V_A\cos(\omega t)dt=\frac{V_A}{\omega}\sin(\omega t)=\frac{V_A}{\omega}\cos(\omega t-\pi/2)$$