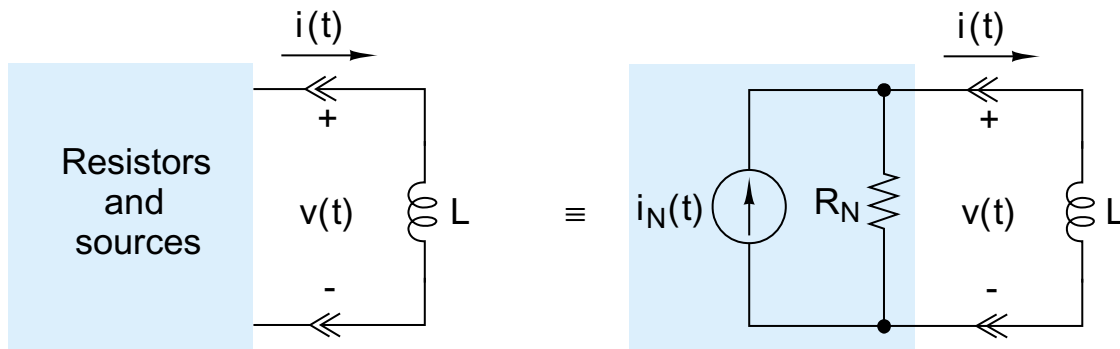


## First Order circuits



$$G_N L \frac{di(t)}{dt} + i(t) = i_N(t)$$

Three factors contribute to our solution:

- 1) The input(s) driving the circuit (e.g.,  $v_T(t)$ )
- 2) The values of the circuit parameters (e.g.,  $R_T$  and  $C$ )
- 3) The initial stored energy (initial condition) (e.g.,  $v(0)$ )

We've done part 3, now go back to part 1 and consider sinusoidal inputs (why sinusoids?), this is the "Forced response" (why?)

e.g.,

$$i_N(t) = I_A [\cos \omega t] u(t)$$

note, this is a "causal" sinusoid

For the moment, assume we start with no stored charge  
(then, we will use superposition to combine the Forced and Natural responses)

$$G_N L \frac{di(t)}{dt} + i(t) = I_A \cos \omega t, \quad t \geq 0$$

Thus, LHS must be a sinusoid of frequency  $\omega$ , so,

$$i(t) = I_F \cos(\omega t + \phi)$$

or converting to Fourier coefficients,

$$i(t) = a \cos \omega t + b \sin \omega t$$

now quit looking ahead...

$$i(t) = \frac{I_A}{1 + (\omega GL)^2} (\cos \omega t + \omega GL \sin \omega t), \quad t \geq 0$$

and adding in the natural response...

$$i(t) = Ke^{-t/GL} + \frac{I_A}{1 + (\omega GL)^2} (\cos \omega t + \omega GL \sin \omega t), \quad t \geq 0$$

Kids these days...

$$i(t) = \left[ I_o - \frac{I_A}{1 + (\omega GL)^2} \right] e^{-t/GL} + \frac{I_A}{1 + (\omega GL)^2} (\cos \omega t + \omega GL \sin \omega t), \quad t \geq 0$$

finally, let's clean up that forced response...

Sam Can't Tell, Oscar Has A Heap Of Apples

sin      cos      tan      opposite      hypotenuse      adjacent      hypotenuse      opposite      adjacent  
 $\sin\theta = \text{opp/hyp}; \cos\theta = \text{adj/hyp}; \tan\theta = \text{opp/adj};$

In sum,

$$i(t) = \left[ I_o - \frac{I_A}{1 + (\omega GL)^2} \right] e^{-t/GL} + \frac{I_A}{\sqrt{1 + (\omega GL)^2}} (\cos \omega t + \theta)$$

- 1) Natural response (effectively) decays in (approximately) 5 time constants
- 2) Forced response is a sinusoid with the same frequency as the input, but with a different phase angle and magnitude.
- 3) The forced response is proportional to  $I_A$ , thus, there is a proportionality property in the circuit.

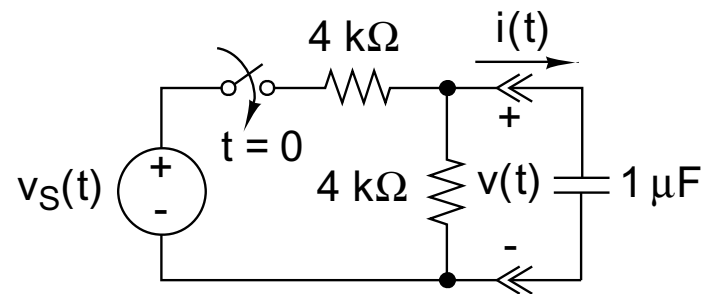
sinusoidal steady-state response

ac steady-state response

ac response

zero frequency response

dc steady-state response



The switch has been open for a long time and is closed at  $t=0$ , find the voltage  $v(t)$  for  $t \geq 0$  when  $v_S(t) = 20\sin 1000t$