Welcome to lecture 20

Like love, after the DC component fades away, the sinusoidal steady-state response continues on. Today we will move beyond simple infatuation between electrons and consider circuit relationships that have lasted for a long time. Carrying this analogy to its logical conclusion... or perhaps simply going to far, like any healthy relationship, you need phasors.

Euler's relationship:

$$e^{j\theta} = \cos\theta + j\sin\theta$$

Cast:

$j =$ why "j"?

$\cos\theta =$

$\sin\theta =$

Bonus reading, Appendix-C, p. A12-A15

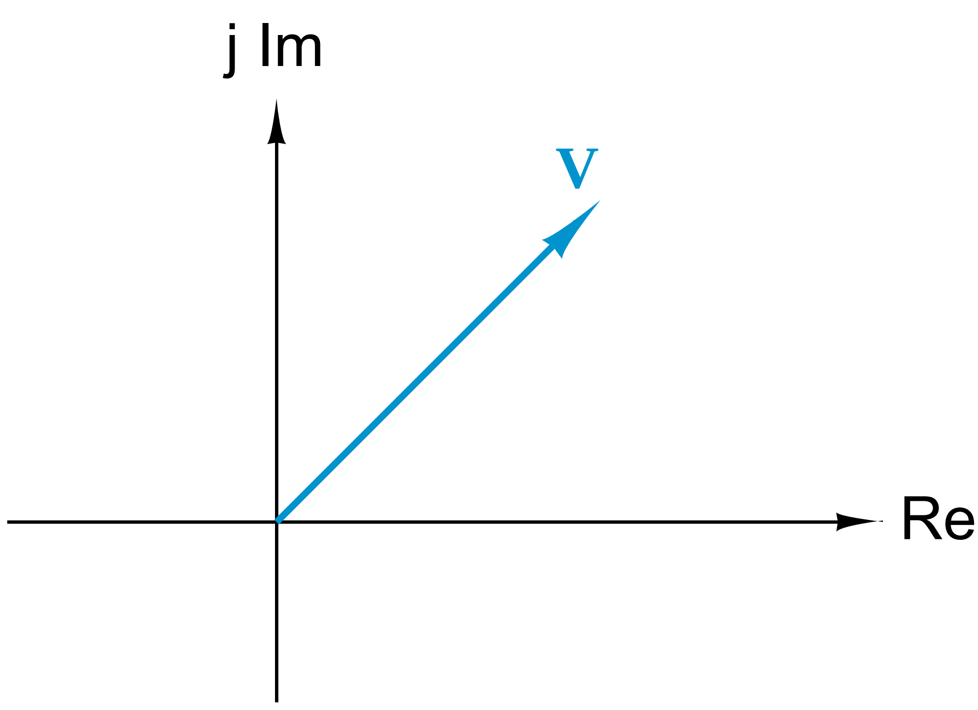
First calculus, then trigonometry and now imaginary numbers... man, this class is like a bad high school reunion!

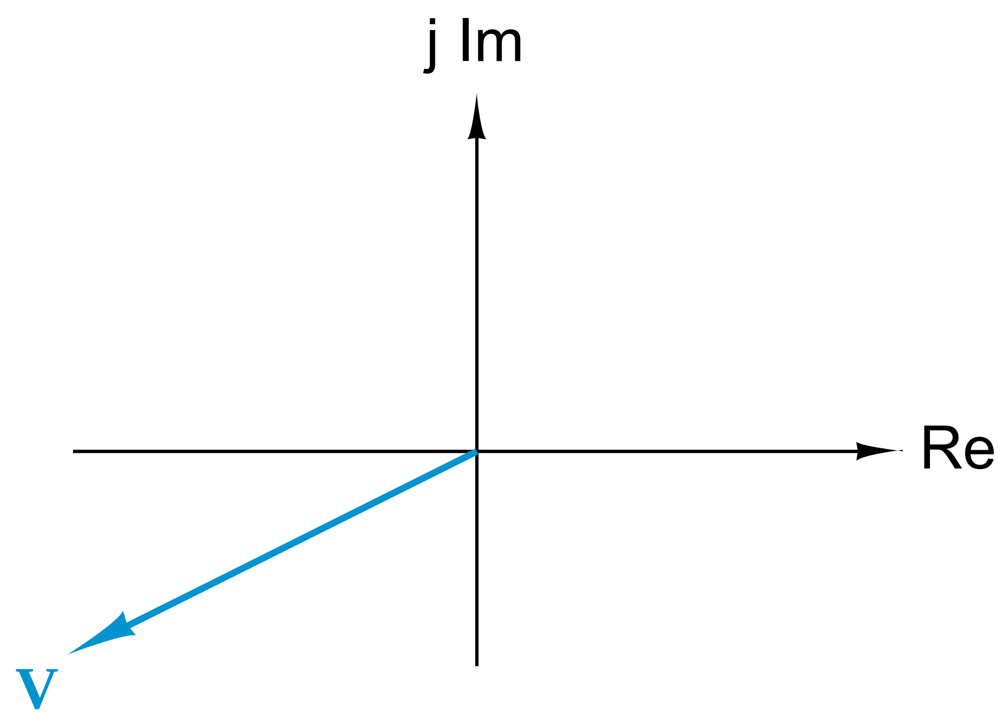
Act I, they meet

$$v(t) = V_A \cos(\omega t + \phi), \quad \forall t$$

Phasor representation of the sinusoid $v(t)$:

$\mathbf{V} =$





Important points to note about phasors

- Phasors are written in boldface type like **V** or **I** to distinguish them from signal waveforms such as $v(t)$ or $i(t)$
- A phasor is determined by amplitude and phase angle (get it, phase... phasor). <u>It does not contain any information about the frequency of the sinusoid!</u>

e.g., AC current coming into your home

So you know how to get yourself into phasors, you don't want to be stuck there forever. How do you get out?

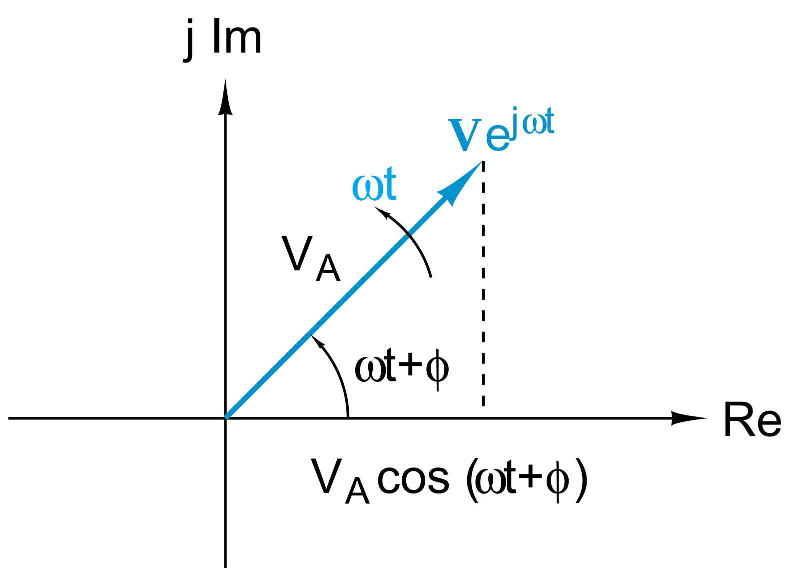
A: just retrace your steps

$$v(t) = \text{Re}\left\{\mathbf{V} \cdot e^{j\omega t}\right\} =$$

So what does this mean to me?

$$v(0) = \text{Re}\{\mathbf{V}\} = V_A \cos(\phi)$$

$$v(t) = \text{Re}\left\{\mathbf{V} \cdot e^{j\omega t}\right\} = V_A \cos(\omega t + \phi)$$



But wait, there's more, what if you have a party and all of your friends come over...

$$v(t) = v_1(t) + v_2(t) + ... + v_N(t)$$

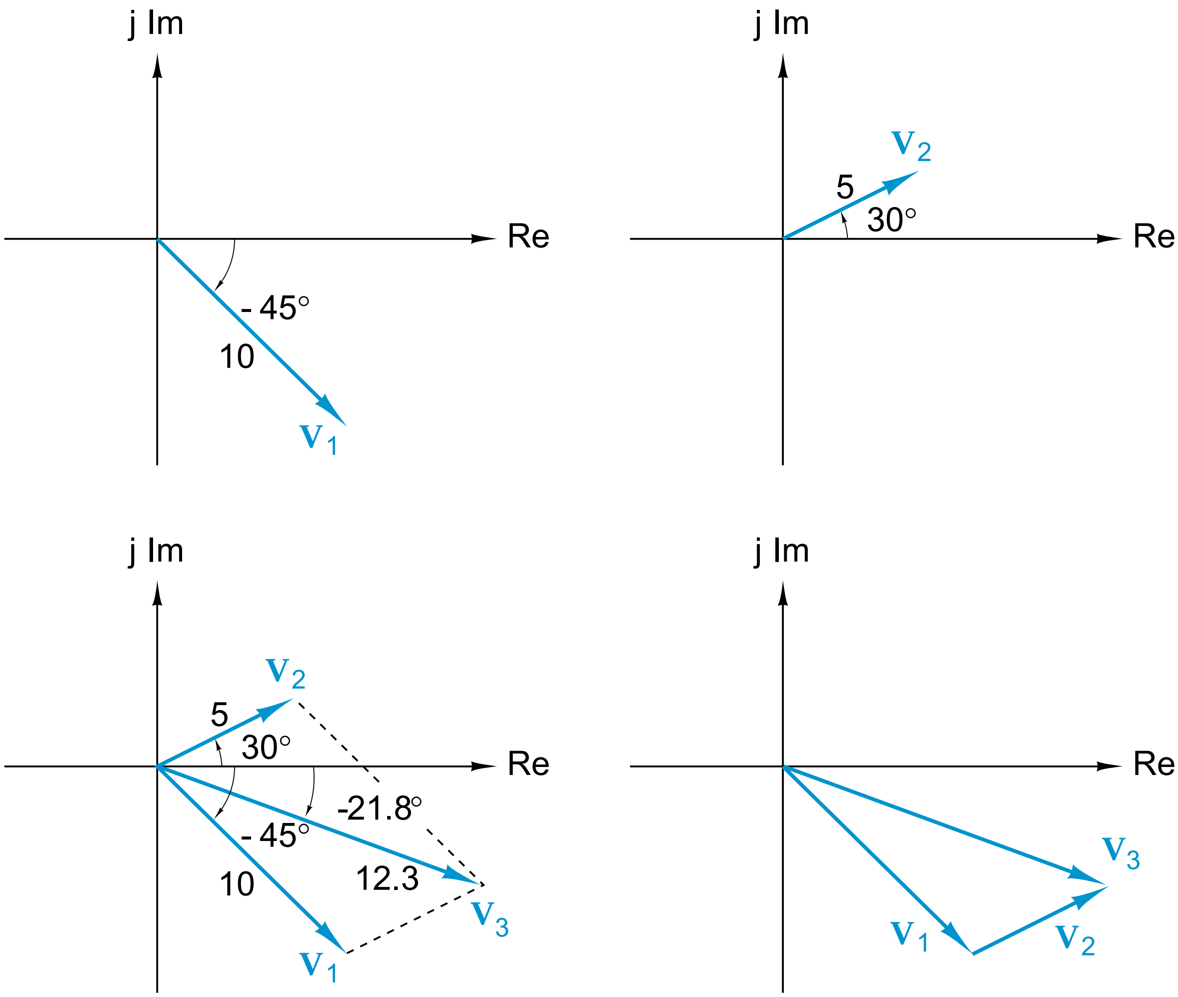
The police come and you want to get rid of the crowd quickly... take the derivative... or integrate with the cops.

$$\frac{dv(t)}{dt} = \frac{d}{dt}\text{Re}\left\{\mathbf{V} \cdot e^{j\omega t}\right\} \qquad \int v(x)dx = \int \text{Re}\left\{\mathbf{V} \cdot e^{j\omega x}\right\}dx$$

$$j\omega\mathbf{V} \qquad\qquad \frac{1}{j\omega}\mathbf{V}$$

Construct the phasors for the following signals:

$$v_1(t) = 10\cos(1000t - 45)$$
$$v_2(t) = 5\sin(1000t - 60)$$
$$v_3(t) = v_1(t) - v_2(t)$$



Using phasors, integrate the following signal:

$$v(t) = 15\cos(200t - 30)$$

Using phasors, find the forced response to the following and compare to the example in the last lecture.

$$G_N L \frac{di(t)}{dt} + i(t) = I_A \cos \omega t$$