Bidding Strategies in Oligopolistic Dynamic Electricity Double-Sided Auctions

Ashkan R. Kian, *Member, IEEE*, Jose B. Cruz, Jr., *Life Fellow, IEEE*, and Robert J. Thomas, *Fellow, IEEE*

***Abstract*—In this paper, the problem of developing bidding strategies in oligopolistic dynamic electricity double-sided auctions is studied. We model electricity double-sided auctions as dynamic systems and use Nash-Cournot strategies for the market participants (generating firms and load serving entities). Through simulation studies, we compare the efficiency and competitiveness of electricity double-sided auctions to those of electricity supplier-only auctions (using the developed bidding strategies).**

***Index Terms*—Bidding strategies, double-sided auctions, generating firm (GF), load serving entity (LSE), Nash-Cournot games, supplier-only auctions.**

I. Introduction

SOME percentage of the electricity markets in the United States are based on auction mechanisms and the rest are based on bilateral contracts. An auction is a market institution with an explicit set of rules determining resource allocation and prices on the basis of bids from the market participants.

In electricity double-sided auctions, the participants submit supply and demand bid curves for the forward electricity markets in sealed bid format. Then, aggregated hourly supply and demand bid curves are constructed to determine market clearing prices as well as the corresponding supply and demand schedules. A marginal clearing price is set at the intersection point between the aggregated demand and supply curves in each scheduling period of the market. All power suppliers winning the auction are paid the uniform clearing price and all power consumers winning the auction pay the market clearing-price for each megawatt of electric power they purchase in the market.

In this paper the problem of developing bidding strategies in oligopolistic dynamic electricity double auctions is studied. Attention is given to strategic bidding of the generating firms (GFs) and load serving entities (LSE) in these markets. We model electricity double-sided auctions as dynamical systems and use closed-loop Nash-Cournot strategies for the market participants (GFs and LSEs). Dynamic modeling of the electricity markets provides insights about their efficiency and stability that are not available through static models.

The rest of the paper is organized as follows. Section II covers the literature review. In Section III, we formulate the problem mathematically. In Section IV, a numerical example of the proposed method is presented. This section has three subsections. Section IV-A presents an IEEE 30-bus power system one-line diagram and our market model assumptions. Section IV-B presents the double-sided auction results and Sectopm IV-C presents the supplier-only auction results. Section V concludes this paper.

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II. Literature Review

There has been a great deal of research to understand, model and analyze electric power markets. Many researches have studied the problem of developing bidding strategies for power suppliers in wholesale electricity markets. A few have studied the dynamics and stability of oligopolistic electricity markets. We will divide our literature review into two parts: 1) dynamic models of electricity markets and 2) double-sided auctions.

A. Dynamic Models of Electricity Markets

Maiorano, Song, and Trovata [1] proposed a dynamic oligopolistic market model to analyze the new competitive electricity environment. Their model was developed on the basis of the well-known Cournot model for the analysis of a noncollusive oligopolistic market. They claim that their market model is dynamic but they have solved a series of static optimization (profit-maximizing) problems for three competing GFs over ten bidding periods. They have shown how the behavior of GFs affected their rivals' profits.

Alvarado [2] has studied the stability of power system markets by means of eigenvalues technique. He studies the stability of several market structures, where only one of them is of interest to us: "A market with m suppliers and n elastic consumers". In this case, the dynamics of both supply and demand are taken into considerations. The state variables of his dynamic equations are the power quantities (generated and consumed). He mathematically shows that if all suppliers' supply function slopes are positive and all consumers' benefit function slopes are negative, market stability is assured.

In [3], Kian *et al.* developed state estimators for stochastic dynamic Nash games with constrained information about the state by each player. The assumption was that the initial state (x_0), and the initial state-estimation errors ($\tilde{x}_0^i, i = 1, \ldots, n$) are known to all players. The developed estimators could be used to model market-clearing prices and system demand in electricity markets for the players.

In stochastic dynamic environments (games) such as electricity multicommodity markets, dynamic modeling of the state variables (price or demand in electricity markets) helps us solve the problem using the dynamic programming method (refer to the Appendix). Dynamic modeling of electricity markets with stochastic price models presented in this paper are novel and have not been proposed before in the literature.

B. Electricity Double-Sided Auctions

Double-sided auctions are a better setting for thinking about price formation than one-sided auctions, both because they are often a better match to reality, and especially because they capture the essential problems of trade better than one-sided auctions [4].

In this paper, we use iterated double-sided auctions. The motive for an iterated auction is the important role of price discovery. A supplier needs to anticipate the pattern of clearing prices across the entire 24 hourly markets in order to make well-informed decisions about which generating units to start. The efficiency of the market outcome is partly dependent on a reliable and informative process of price discovery as the iterations proceed.

In [5], the authors propose using "activity rules" in order to encourage the market participants to bid sincerely from the start. The key idea is to confront traders with irreversible decisions throughout the iterative process. Activity rules must be designed carefully to minimize adverse effects on efficiency from restricting traders' bidding strategies, but if designed well then they benefit each trader by encouraging others to bid seriously in each iteration. The resulting progressive revelation of the pattern of prices across the markets enables each trader to take advantage of this information in constructing its own bids.

Nicolaisen *et al.* [6] have studied the market power and efficiency of electricity markets with discriminatory double-auction pricing. They construct an agent-based computational model of a wholesale electricity market to use for economical experiments. They study how the relative market power of the buyers and sellers varies in response to changes in market concentration and capacity. They also study the distinction between traders' market power due to the market structure and those due to buyer and seller learning. Their market simulation results confirm that active bidding by buyers will limit the ability of sellers to exercise market power.

Most of the researchers in the area of electricity auctions have reached the conclusion that electricity double-sided auctions are more efficient and competitive than supplier-only auctions, but none of them has modeled the electricity double-sided auctions as dynamic games for developing bidding strategies for suppliers and demanders in these markets. This novel approach is presented in this paper.

The aim of this paper is to develop feedback Nash-Cournot strategies for the GFs and LSEs in oligopolistic dynamic electricity double-sided auctions. Using the developed bidding strategies, we compare the efficiency and price stability of electricity double-sided auctions to those of electricity supplier-only auctions. In our dynamic stochastic model, the power quantities (generated and consumed) are the control variables; the market clearing-price is the state variable (with a stochastic linear dynamic model) estimated by each market participant.

III. Problem Formulation

Attention is given to a problem that a few LSEs and GFs play a Nash-Cournot game to maximize their profits in an electricity double-sided auction. A linear stochastic market-price predictor is used, which is known to all players. It is assumed that the players' (LSE/GF) profit function coefficients (their means and variances) can be estimated using publicly available energy databases [7]–[9].

With the above assumptions, let us formulate a dynamic Nash-Cournot game with n players ($\mathrm{n_g}$ GFs and $\mathrm{n_l}$ LSEs, $\mathrm{n = n_g + n_l}$) as follows.
The profit-to-go of GF-i at time step k is defined as

$$\pi_k^{g,i}\left(\lambda_k, q_k^i\right) = \lambda_k \cdot q_k^i - \frac{1}{2} a_g^i \left(q_k^i\right)^2 - b_g^i \cdot q_k^i, \quad i = 1, \ldots, n_g. \quad (1)$$

The profit-to-go of LSE-i at time step k is defined as

$$\pi_k^{l,i}\left(\lambda_k, d_k^i\right) = b_l^i \cdot d_k^i - \frac{1}{2} a_l^i \left(d_k^i\right)^2 - \lambda_k \cdot d_k^i, \quad i = 1, \ldots, n_l. \quad (2)$$

The market-price predictor is assumed to have a linear stochastic dynamic model as follows:

$$\lambda_{k+1} = A_k \lambda_k + \sum_{i=1}^{n_g} B_k^{g,i} \cdot q_k^i + \sum_{i=1}^{n_l} B_k^{l,i} \cdot d_k^i + w_k \quad (3)$$

where

- λ_k Market clearing price at time step k (in dollars per Megawatt);
- q_k^i Supply quantity (control variable) of the GF-i at time step k (in Megawatts);
- d_k^i Demand quantity (control variable) of the LSE-i at time step k (in Megawatts);
- a_g^i, b_g^i GF-i profit function coefficients;
- a_l^i, b_l^i LSE-i profit function coefficients;
- w_k Normally distributed process noise (with zero-mean and known variance).

Equations (1) to (3) show that each player's profit function depends not only on his own decision variable but also on his rivals' decision variables in dynamic electricity markets. A dynamic Nash-Cournot game (with n players) can be formulated as follows:

$$J_k^i\left(\lambda_k, u_k^i, u_k^{-i}\right) = \min_{u_k} \mathop{E}_{w_k} \left\{ \frac{1}{2} \lambda_N{}' Q_N^i \lambda_N + \sum_{j=k}^{N-1} \left\{ \frac{1}{2} u_j^{i'} a^i u_j^i + \alpha^i \left(\lambda_j - b^i\right) \cdot u_j^i \right\} \right\} \quad i = 1, \ldots, n \quad (4)$$

$$\lambda_{k+1} = A_k \lambda_k + \sum_{i=1}^{n} B_k^i \cdot u_k^i + w_k \quad (5)$$

where $J_k^i(\lambda_k, u_k^i, u_k^{-i})$ is the expected cost-to-go of player-i at time step k (for $\mathrm{k} = 1, \ldots, \mathrm{N}-1$) and $J_N^i(\lambda_N) = 1/2 \lambda_N{}' Q_N^i \lambda_N$ is the terminal cost at time step N. Q_N^i is a weighting factor.

Note:

Player-i is a GF if $\alpha^i = -1$, then $u_k^i = q_k^i$.
Player-i is a LSE if $\alpha^i = 1$, then $u_k^i = d_k^i$.

A set of feedback Nash-Cournot strategies $\{u_k^{i*}(\lambda_k), i = 1, \ldots, n\}$ defines the equilibrium of the game if it satisfies the following inequality conditions for $\mathrm{k} = 1, \ldots, \mathrm{N}-1$ ::

$$J_k^i\left(\lambda_k, u_k^{i*}, u_k^{-i*}\right) \leq J_k^i\left(\lambda_k, u_k^i, u_k^{-i*}\right), \quad \forall u_k^i \in U^i \quad \text{for } \mathrm{i} = 1, \ldots, \mathrm{n} \tag{6}$$

$U^i = \{u_k^i | u_k^{i,\min} \leq u_k^i \leq u_k^{i,\max}\}$, is a set of all feasible choices for player-i's control variable (strategy).

Using dynamic programming and using mathematical induction, we can show that the optimal Nash-Cournot strategies for the GFs and the LSEs satisfy the following (for a complete mathematical derivations see the Appendix):

$$u_k^* = L_k \lambda_k + \Phi_k \tag{7}$$
$$L_k = -\Psi_k^{-1} \cdot \Gamma_k \tag{8}$$
$$\Phi_k = -\Psi_k^{-1} \cdot \Omega_k \tag{9}$$

where

$$\Psi_k^{ii} = a^i + B_k^{i'} K_{k+1}^i B_k^i, \quad i = 1, \ldots, n \tag{10a}$$
$$\Psi_k^{ij} = B_k^{i'} K_{k+1}^i B_k^j, \quad i, j = 1, \ldots, n; \quad j \neq i \tag{10b}$$
$$\Gamma_k^i = B_k^{i'} K_{k+1}^i A_k + \alpha^i, \quad i = 1, \ldots, n \tag{11}$$
$$\Omega_k^i = B_k^{i'} P_{k+1}^i - \alpha^i \cdot b^i, \quad i = 1, \ldots, n. \tag{12}$$

Matrices K_k^i, P_k^i satisfy the following Riccati-like equations:

$$K_k^i = L_k^{i'} a^i L_k^i + \alpha^i \cdot L_k^i + \left(A_k + \sum_{j=1}^n B_k^j L_k^j\right)' \times K_{k+1}^i \left(A_k + \sum_{j=1}^n B_k^j L_k^j\right), \quad i = 1, \ldots, n \tag{13}$$

$$P_k^i = \alpha^i \cdot \left(\Phi_k^i - b^i L_k^i\right) + \Phi_k^{i'} a^i L_k^i + \left(A_k + \sum_{j=1}^n B_k^j L_k^j\right)' K_{k+1}^i \times \left(\sum_{j=1}^n B_k^j \Phi_k^j\right) + P_{k+1}^i \left(A_k + \sum_{j=1}^n B_k^j L_k^j\right), \quad i = 1, \ldots, n \quad \text{for } k = 1, \ldots, N-1 \tag{14}$$

with boundary conditions $K_N^i = Q_N^i; P_N^i = 0, i = 1, \ldots, n$. The optimal costs-to-go at time step k for player-i $(\mathrm{i} = 1, \ldots, \mathrm{n})$ is

$$J_k^{i*}(\lambda_k) = \lambda_k' K_k^i \lambda_k + P_k^i \lambda_k + M_k^i + \sum_{j=k}^{N-1} \underset{wj}{E}\left\{w_j' K_{j+1}^i w_j\right\} \tag{15}$$

where M_k^i satisfies the following Riccati-like equation:

$$M_k^i = \Phi_k^{i'} a^i \Phi_k^i + \left(\sum_{j=1}^n B_k^i \Phi_k^i\right)' K_{k+1}^i \left(\sum_{j=1}^n B_k^i \Phi_k^i\right) + P_{k+1}^i \left(\sum_{j=1}^n B_k^i \Phi_k^i\right) + M_{k+1}^i \tag{16}$$

with boundary conditions $M_N^i = 0$.

In order to implement the derived Nash-Cournot strategies for a real power system, the following procedure may be followed. First, (13) and (14) and then (5), and (7)–(12) must be solved backward in time starting with the final boundary conditions: $K_N^i = Q_N^i, P_N^i = 0, i = 1, \ldots, n$. For example, for step $\mathrm{N}-1$

$$\Psi_{N-1}^{ii} = a^i + B_{N-1}^{i'} K_N^i B_{N-1}^i, \quad i = 1, \ldots, n \tag{17}$$
$$\Psi_{N-1}^{ij} = B_{N-1}^{i'} K_N^i B_{N-1}^j, \quad i, j = 1, \ldots, n; \quad j \neq i \tag{18}$$
$$\Gamma_{N-1}^i = B_{N-1}^{i'} K_N^i A_{N-1} + \alpha^i, \quad i = 1, \ldots, n \tag{19}$$
$$\Omega_{N-1}^i = B_{N-1}^{i'} P_N^i - \alpha^i \cdot b^i, \quad i = 1, \ldots, n \tag{20}$$
$$L_{N-1} = -\Psi_{N-1}^{-1} \cdot \Gamma_{N-1} \tag{21}$$
$$\Phi_{N-1} = -\Psi_{N-1}^{-1} \cdot \Omega_{N-1} \tag{22}$$
$$\lambda_{N-1} = \left(A_{N-1} + \sum_{j=1}^n B_{N-1}^j L_{N-1}^j\right)^{-1} \times \left(\lambda_N - \sum_{j=1}^n B_{N-1}^j \Phi_{N-1}^j - w_{N-1}\right) \tag{23}$$
$$u_{N-1}^{i*} = L_{N-1}^i \lambda_{N-1} + \Phi_{N-1}^i, \quad i = 1, \ldots, n \tag{24}$$

Now, use (13) and (14) to calculate: K_{N-1}^i, P_{N-1}^i $(i = 1, \ldots, n)$ and then follow the above procedure to solve for optimal Nash-Cournot strategies at time step $\mathrm{N}-2$ and so on until we have calculated all the closed-loop Nash-Cournot strategies down to time step 1.

Note that λ_N is the predicted value of the market clearing price at time step N and term $(1/2)\lambda_N' Q_N \lambda_N$ is considered as a fixed cost for each player (i.e., a fee for entering the electricity double-sided auction).

For the sake of finite-time market stability, the following condition must be satisfied [11]:

$$\left|\text{eigenvalue}\left[A_k + \sum_{j=1}^n B_k^j L_k^j\right]\right| \leq 1, \quad \text{for } \mathrm{k} = 1, \ldots, \mathrm{N}-1. \tag{25}$$

IV. Numerical Example

In this section, the developed bidding strategies in Section III are used to compute the optimal bids for 12 competing power marketers (six GFs and six LSEs) in a three-zone interconnected power system (IEEE 30-bus power system) shown in Fig. 1. We will show the effect of active demand bidding on market

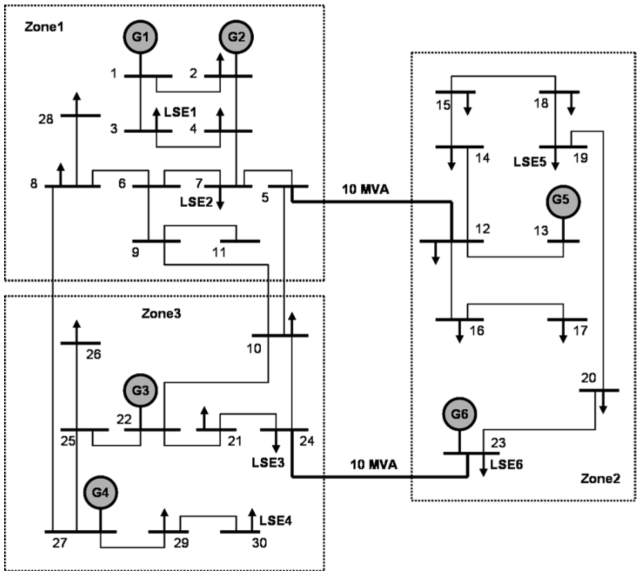

Fig. 1. IEEE 30-bus power system (with three interconnected demand and generation zones).

power by comparing the results of Section IV-B to those of Section IV-C.

A. Assumptions

It is assumed that each LSE can buy electric power from any zone (price area) and each GF can sell its electric power to any zone. The LSEs and GFs try to maximize their expected profits in the electricity double-sided auction.

We assume that GFs try to increase the market-clearing price (MCP) by withholding capacity from the market and the LSEs try to decrease the MCP by adjusting their power consumptions in response to the GFs acts in double-sided auctions. We further assume that each player is more effective in adjusting the MCP of its own zone (price area) and less effective in adjusting the MCPs of other zones. Therefore, in our numerical example the values for B_k^i were (arbitrarily) chosen as follows:

$$B_k^{i,j} = \begin{Bmatrix} 1, & \text{if } \mathrm{GF}_i \in \mathrm{Zone}_j \\ .25, & \text{if } \mathrm{GF}_i \notin \mathrm{Zone}_j \\ -1, & \text{if } \mathrm{LSE}_i \in \mathrm{Zone}_j \\ 0, & \text{if } \mathrm{LSE}_i \notin \mathrm{Zone}_j \end{Bmatrix} \tag{26}$$

Note that LSEs' B_k^i values are negative to counter those of the GFs who try to raise the market clearing prices.

Since the tie lines between demand zones 1 and 3 have unlimited capacity to exchange power between the two zones, the B_k^i values for the GFs in these zones were set to 1 in their MCP models [refer to (29)]. From (7) to (14), we can see that the B_k^i values affect the optimal Nash-Cournot strategies therefore, it will be very important for the market participants to have a reasonable estimate/measurement of these values in the electricity market. Market historical data, OASIS database in the ISO's web-pages [7]–[9], system operating conditions and market shares of the suppliers/bulk-consumers could help them estimate the B_k^i values more accurately.

TABLE I
PROBABILITY DISTRIBUTION DATA OF THE PLAYERS' PRIVATE INFORMATION

Parameter	Mean	Variance
$Q_N^i,\ i=1,\ldots,12$	**1**	**0.1**
$a_g^i,\ i=1,\ldots,6$	**0.01**	**0.002**
$b_g^i,\ i=1,\ldots,6$	**10**	**1**
$a_d^i,\ i=1,\ldots,6$	**0.01**	**0.001**
$b_d^i,\ i=1,\ldots,6$	**{75,80,90}**	**{2,2,3}**

The system parameters are given as follows:

$$A_k = \begin{bmatrix} .97 & 0 & 0 \\ 0 & .97 & 0 \\ 0 & 0 & .97 \end{bmatrix} \tag{27}$$

$$\mathrm{cov}\{w_k\} = 1, \quad k = 1, \ldots, N \tag{28}$$

$$B_k = 5e-2 \times \begin{bmatrix} 1 & .25 & 1 \\ 1 & .25 & 1 \\ 1 & .25 & 1 \\ 1 & .25 & 1 \\ .25 & 1 & .25 \\ .25 & 1 & .25 \\ -1 & 1 & 0 \\ -1 & 1 & 0 \\ 0 & 0 & -1 \\ 0 & 0 & -1 \\ 0 & -1 & 0 \\ 0 & -1 & 0 \end{bmatrix}^T. \tag{29}$$

We assume that the players' objective function coefficients $(Q_N^i, a^i, b^i, i = 1, \ldots, 12)$ are normally distributed with known means and variances to all players. For our numerical example, these values are given in Table I for all 12 players. Each GF submits a three-block optimal supply bid (priced at its marginal cost of production) and each LSE submits its optimal demand bid (with no price) to the market. Then MATPOWER 2.1 solver [10] is used to solve an AC-OPF for clearing the market in each bidding period. In this paper, we assume that each player has a learning algorithm to update its bidding strategy for maximizing its expected profit in the market. Moreover, the market operator (ISO) includes generation-load balance constraint, transmission capacity constraints, bus-voltage limit constraints and generator output limit constraints in its market clearing process (AC-OPF solver). The optimal generation schedules and nodal MCPs are the outputs of the AC-OPF solution.

There are three large penalty factors used in the market optimization process and simulation studies. Two of them are used to keep the bid quantities within the boundary limits and one is used to keep the supply quantities equal to the forecasted system demand.

B. Double-Sided Auction Results

The market simulation results including zonal: 1) generation and demand [MW]; 2) average market-clearing price (estimated and real) [$/MW]; 3) GF's profits [$]; and 4) LSE's profits [$]

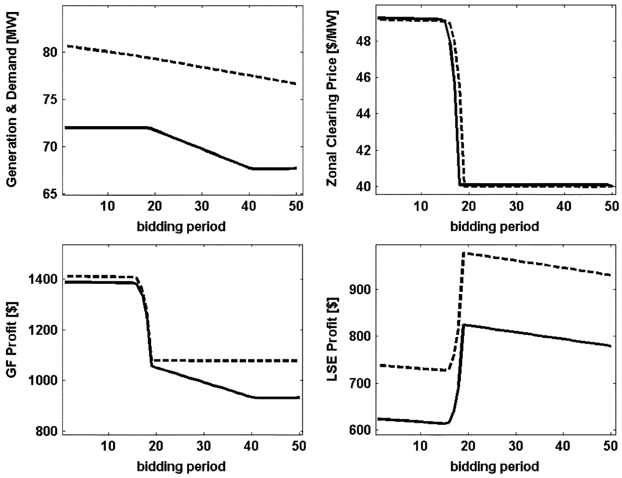

Fig. 2. Market simulation results of the electricity double-sided auctions for zone-1. (Top left): generation (solid line) and demand (dashed line) [MW]. (Top right): zonal market clearing price: predicted (solid line) and real (dash line) [$/MW]. (Bottom left): GF1 profit (solid line) and GF2 profit (dashed line) [$]. (Bottom right): LSE1 profit (solid line) and LSE2 profit (dash line) [$].

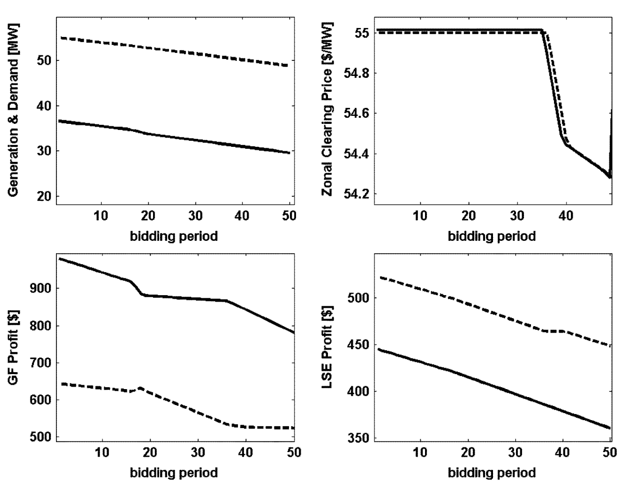

Fig. 3. Market simulation results of the electricity double-sided auctions for zone-2. (Top left): generation (solid line) and demand (dashed line) [MW]. (Top right): zonal market clearing price: predicted (solid line) and real (dash line) [$/MW]. (Bottom left): GF5 profit (solid line) and GF6 profit (dashed line) [$]. (Bottom right): LSE5 profit (solid line) and LSE6 profit (dashed line) [$].

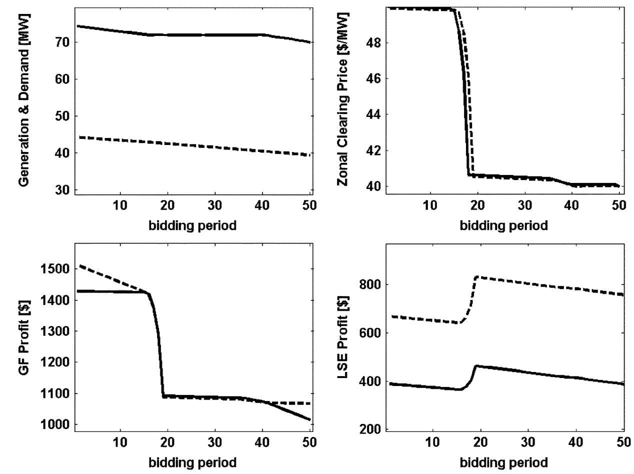

Fig. 4. Market simulation results of the electricity double-sided auctions for zone-3. (Top left): generation (solid line) and demand (dashed line) [MW]. (Top right): zonal market clearing price: predicted (solid line) and real (dashed line) [$/MW]. (Bottom left): GF3 profit (solid line) and GF4 profit (dashed line) [$]. (Bottom right): LSE3 profit (solid line) and LSE4 profit (dashed line) [$].

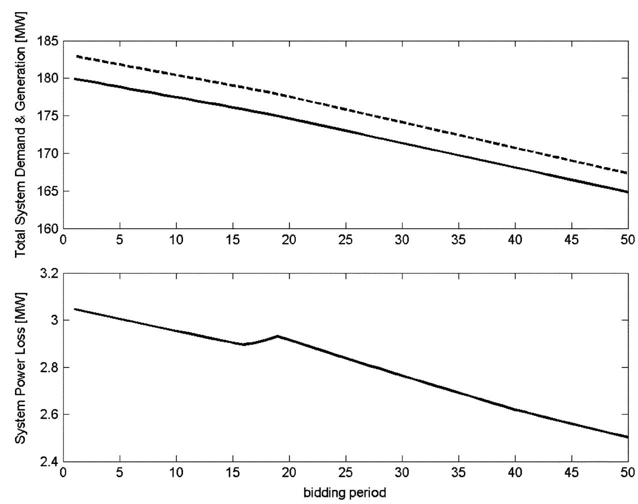

Fig. 5. (Top graph): total system demand (solid line) and generation (dashed line) [MW]. (Bottom graph): system power losses [MW].

are shown in Figs. 2 to 4 for 50 bidding periods. Total system generation, demand, and power losses are also shown in Fig. 5.

GF1, GF2, LSE1 (bus 3), and LSE2 (bus 7) are located in zone 1. GF3, GF4, LSE3 (bus 24), and LSE4 (bus 30) are located in zone 3. GF5, GF6, LSE5 (bus 19), and LSE6 (bus 23) are located in zone 2. All other demands in the system are fixed and inelastic.

The simulation results of Figs. 2 to 4 show that in competitive electricity markets demand elasticity plays an important role in reducing the GFs' market power and price volatility. GFs in zones 1 and 3 (Figs. 2 and 4) gain higher profits in bidding periods 1–20 compare to those of the rest of the bidding periods. This is because their scheduled generations after period 20 drop to the less expensive blocks offered (from the 50 $/MW block to the 40 $/MW block due to the lower demand). On the other hand, in bidding periods 20 to 50, the LSEs' profits increase in the market because they pay $10 less for each MW of their consumptions. In Fig. 3, due to the tie-line congestion and creation of a load pocket in zone 2, LSE5 and LSE6 can not purchase power from zones 1 and 3. Therefore, their only way to keep the market price at a competitive level is to control their power consumptions (although their profits are decreased).

Fig. 6 shows the offered quantities (solid line) and scheduled quantities (dash line) of the GFs for the 50 bidding periods. As we can see from the graphs of this figure, GF1 and GF4 adapt faster to the market conditions and (with a superior learning process, refer to [3]) bid closer to their market schedules. Although, the power market of zone 2 is isolated from zones 1 and 3 (because of the tie-line congestion), GF5 and Gf6 can not exploit market power due to the competitive behavior of LSE5 and LSE6. Some of our market simulation results are not at their steady-state solutions yet. The simulation time horizon will be extended in our future studies of the electricity markets to reach the steady-state solutions (e.g., market equilibrium).

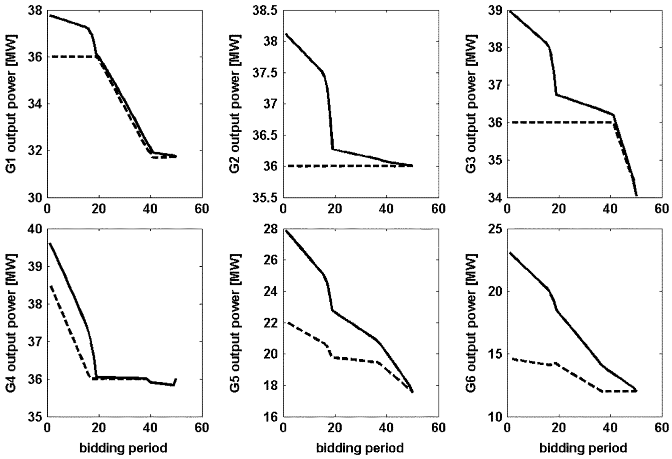

Fig. 6. GFs' offered quantities (solid line) and scheduled quantities (dashed line). (Top graph): GF1 to GF3. (Bottom graph): GF4 to GF6.

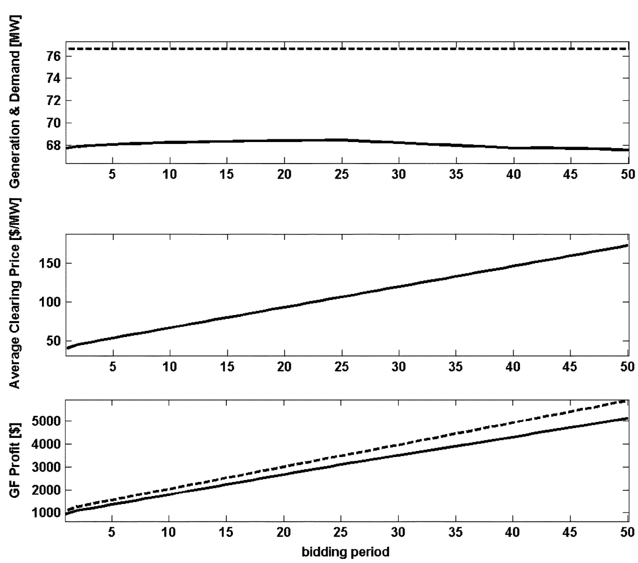

Fig. 7. Market simulation results of the electricity supplier-only auction for zone-1. (Top): generation (solid line) and demand (dashed line) [MW]. (Middle): zonal market clearing price [$/MW]. (Bottom): GF1 profit (solid line) and GF2 profit (dashed line) [$].

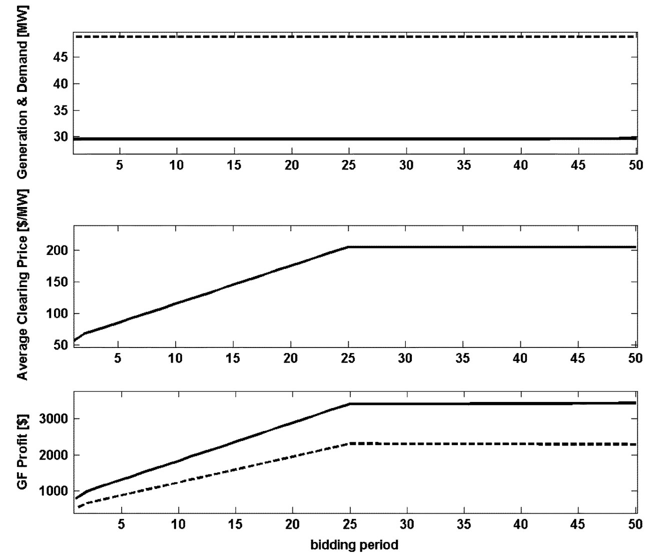

Fig. 8. Market simulation results of the electricity supplier-only auction for zone-2: (Top): generation (solid line) and demand (dashed line) [MW]. (Middle): zonal market clearing price [$/MW]. (Bottom): GF5 profit (solid line) and GF6 profit (dashed line) [$].

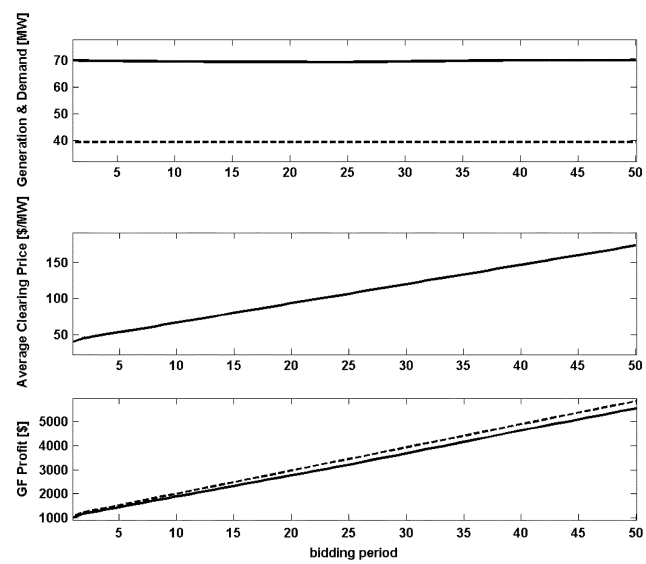

Fig. 9. Market simulation results of the electricity supplier-only auction for zone-3: (Top): generation (solid line) and demand (dashed line) [MW]. (Middle): zonal market clearing price [$/MW]. (Bottom): GF3 profit (solid line) and GF4 profit (dashed line) [$].

C. Supplier-Only Auction Results

In this section, we simulate a single-sided electricity auction using the developed Nash-Cournot strategies, and its results will be compared to those of the double-sided auction (Section IV-B). The simulation results of our supplier-only auction studies for the power system of Fig. 1 (with six competing generating firms and fixed demands in all buses) are shown in Figs. 7–9.

These simulation results show that in fixed demand markets the GFs explore and exercise their market power. In these markets, the market clearing prices could go high above their marginal values. Dynamic analysis of electricity markets gives us detailed insights about the existence and exercise of market power by the GFs and LSEs based on their profits, strategic behaviors, and market prices.

Figs. 7 and 9 show that GFs 1–4 constantly raise their price offers over the 50 bidding periods and manage to raise the MCPs up to 180 $/MW from their marginal values (40 $/MW) in zones 1 and 3. In Fig. 8, GF5 and GF6 manage to raise the MCP to its cap price (200 $/MW) in bidding period 25 and keep it at this value for the rest of the bidding periods. By comparing the single-sided auction results versus the double-sided auction results, we can see that the GFs' profits in a supplier-only auction are two to three times higher than those of a double-sided auction. By comparing Figs. 2 and 7, we can see that the double-sided auction prices move toward their competitive (marginal) values but, the supplier-only auction prices move toward the market cap-prices. These simulation results show that double-sided auctions are more competitive and efficient than single-sided auctions. This confirms the experiments by Mount *et al.* [15], and by Schuler *et al.* [13], [14] that show the same effects.

V. Conclusion

In this paper, the problem of developing bidding strategies in oligopolistic dynamic electricity double-sided auctions has been studied. Attention was given to strategic bidding of the GF and LSEs in these markets. We modeled electricity double-sided auctions as dynamical systems and used feedback Nash-Cournot strategies for the market participants (GFs and LSEs). As we showed in our market simulation results and analysis, dynamic modeling of the electricity markets is a novel approach for analyzing market efficiency, price stability, and suppliers' market power.

The developed bidding strategies were used to compute the optimal bids for 12 competing power marketers (six GFs and six LSEs) in a three-zone interconnected power system (Fig. 1).

Our market simulation results of the dynamic electricity double-sided and supplier-only auctions showed the following (a confirmation of the results in [13]–[15]).

1) Double-sided auctions are more efficient than single-sided auctions because of the demand side management in response to the GFs' strategic behaviors,
2) Market clearing-prices of double-sided auctions are more stable and competitive than those of single-sided auctions because of price responsive demands,
3) Market power of the GFs are limited and controlled in the double-sided auctions.

In our future electricity market studies, we plan to develop mathematical algorithms for estimating the B_k^i values using the market historical data and system operating conditions.

Appendix

In this section, we derive (7) to (16).

Note that in the following derivation, $(-j)$ means the set of all players except player (j). Applying the dynamic programming (DP) algorithm for player (j), we have

$$J_N^j(\lambda_N) = \frac{1}{2}\lambda_N{}' Q_N^j \lambda_N \tag{A1}$$

$$J_k^j(\lambda_k) = \min_{u_k} \mathop{E}_{w_k} \left\{ \frac{1}{2} u_k^{j'} a^j u_k^j + \alpha^j(\lambda_k - b^j)\cdot u_k^j + J_{k+1}^j(\lambda_{k+1}) \right\} \tag{A2}$$

$$\lambda_{k+1} = A_k\lambda_k + B_k^j u_k^j + B_k^{-j} u_k^{-j} + w_k. \tag{A3}$$

For $k = N-1$, we have

$$J_{N-1}^j(\lambda_{N-1}) = \min_{u_{N-1}} \mathop{E}_{w_{N-1}} \left\{ \frac{1}{2} u_{N-1}^{j'} a^j u_{N-1}^j + \alpha^j(\lambda_{N-1} - b^j)\cdot u_{N-1}^j + J_N^j(\lambda_N) \right\}. \tag{A4}$$

Using A1 and A4, we have

$$\begin{aligned} J_{N-1}^j(\lambda_{N-1}) = \min_{u_{N-1}} \mathop{E}_{w_{N-1}} \Big\{ & \frac{1}{2} u_{N-1}^{j'} a^j u_{N-1}^j \\ & + \alpha^j(\lambda_{N-1} - b^j)\cdot u_{N-1}^j \\ & + \frac{1}{2}\Big(A_{N-1}\lambda_{N-1} + B_{N-1}^j u_{N-1}^j \\ & + B_{N-1}^{-j} u_{N-1}^{-j} + w_{N-1}\Big)' \\ & \times Q_N^j \Big(A_{N-1}\lambda_{N-1} + B_{N-1}^j u_{N-1}^j \\ & + B_{N-1}^{-j} u_{N-1}^{-j} + w_{N-1}\Big)\Big\}. \end{aligned} \tag{A5}$$

Let us derive the optimal control law for player (j) as follows:

$$\begin{aligned} \frac{\partial J_{N-1}^j}{\partial u_{N-1}^j} = & \left(B_{N-1}^{j'} Q_N^j B_{N-1}^j + a^j\right) \\ & \cdot u_{N-1}^j + \left(B_{N-1}^{j'} Q_N^j B_{N-1}^{-j}\right) \\ & \times u_{N-1}^{-j} + \left(B_{N-1}^{j'} Q_N^j A_{N-1} + \alpha^j\right) \\ & \cdot \lambda_{N-1} - \alpha^j b^j = 0 \end{aligned} \tag{A6}$$

$$\begin{aligned} \frac{\partial J_{N-1}^{-j}}{\partial u_{N-1}^{-j}} = & \left(B_{N-1}^{-j'} Q_N^{-j} B_{N-1}^j\right) \\ & \cdot u_{N-1}^j + \left(B_{N-1}^{-j'} Q_N^{-j} B_{N-1}^{-j} + a^{-j}\right) \\ & \times u_{N-1}^{-j} + \left(B_{N-1}^{-j'} Q_N^{-j} A_{N-1} + \alpha^{-j}\right) \\ & \cdot \lambda_{N-1} - \alpha^{-j} b^{-j} = 0. \end{aligned} \tag{A7}$$

From (A6) and (A7) and some matrix algebra, we can write the optimal expressions for u_{N-1}^{j*} *and* u_{N-1}^{-j*}, as shown in (A8), at the bottom of the page. In a simple expression, we have

$$\begin{aligned} u_{N-1}^* &= -\Psi_{N-1}^{-1}\cdot(\Gamma_{N-1}\cdot\lambda_{N-1} + \Omega_{N-1}) \\ &= L_{N-1}\cdot\lambda_{N-1} + \Phi_{N-1}. \end{aligned} \tag{A9}$$

By substituting (A9) into (A5) and some matrix algebra, we can derive the optimal cost expression for player (j) as follows:

$$\begin{aligned} J_{N-1}^{j*}(\lambda_{N-1}) = & \lambda_{N-1}{}' K_{N-1}^j \lambda_{N-1} + P_{N-1}^j \lambda_{N-1} \\ & + M_{N-1}^j + E\left\{w_{N-1}{}' Q_N^j w_{N-1}\right\} \end{aligned} \tag{A10}$$

where

$$\begin{aligned} K_{N-1}^j = & L_{N-1}^{j'} a^j L_{N-1}^j + \alpha^j L_{N-1}^j \\ & + \left(A_{N-1} + B_{N-1}^j L_{N-1}^j + B_{N-1}^{-j} L_{N-1}^{-j}\right)' \\ & \times Q_N^j \left(A_{N-1} + B_{N-1}^j L_{N-1}^j + B_{N-1}^{-j} L_{N-1}^{-j}\right) \end{aligned} \tag{A11}$$

$$\begin{aligned} \begin{bmatrix} u_{N-1}^{j*} \\ u_{N-1}^{-j*} \end{bmatrix} = - & \begin{bmatrix} B_{N-1}^{j'} Q_{N-1}^j B_{N-1}^j + a^j & B_{N-1}^{j'} Q_{N-1}^j B_{N-1}^{-j} \\ B_{N-1}^{-j'} Q_{N-1}^{-j} B_{N-1}^j & B_{N-1}^{-j'} Q_{N-1}^{-j} B_{N-1}^{-j} + a^{-j} \end{bmatrix}^{-1} \\ & \times \left(\begin{bmatrix} B_{N-1}^{j'} Q_N^j A_{N-1} + \alpha^j \\ B_{N-1}^{-j'} Q_N^{-j} A_{N-1} + \alpha^{-j} \end{bmatrix} \cdot \lambda_{N-1} + \begin{bmatrix} -\alpha^j b^j \\ -\alpha^{-j} b^{-j} \end{bmatrix} \right) \end{aligned} \tag{A8}$$

$$P_{N-1}^{j} = \alpha^{j}\left(\Phi_{N-1}^{j} - b^{j}L_{N-1}^{j}\right) + \Phi_{N-1}^{j'}a^{j}L_{N-1}^{j} + \left(A_{N-1} + B_{N-1}^{j}L_{N-1}^{j} + B_{N-1}^{-j}L_{N-1}^{-j}\right)'Q_{N}^{j} \times \left(B_{N-1}^{j}\Phi_{N-1}^{j} + B_{N-1}^{-j}\Phi_{N-1}^{-j}\right) \quad \text{(A12)}$$

$$M_{N-1}^{j} = \Phi_{N-1}^{j'}a^{j}\Phi_{N-1}^{j} + \left(B_{N-1}^{j}\Phi_{N-1}^{j} + B_{N-1}^{-j}\Phi_{N-1}^{-j}\right)' \times Q_{N}^{j}\left(B_{N-1}^{j}\Phi_{N-1}^{j} + B_{N-1}^{-j}\Phi_{N-1}^{-j}\right). \quad \text{(A13)}$$

For k = N − 2 we have

$$J_{N-2}^{j}(\lambda_{N-2}) = \min_{u_{N-2}} \mathop{E}_{w_{N-2}}\left\{\frac{1}{2}u_{N-2}^{j'}a^{j}u_{N-2}^{j} + \alpha^{j}(\lambda_{N-2} - b^{j})\cdot u_{N-2}^{j} + J_{N-1}^{j}(\lambda_{N-1})\right\}. \quad \text{(A14)}$$

Using (A10) and (A14), we have

$$J_{N-2}^{j}(\lambda_{N-2}) = \min_{u_{N-2}} \mathop{E}_{w_{N-2}}\left\{\frac{1}{2}u_{N-2}^{j'}a^{j}u_{N-2}^{j} + \alpha^{j}(\lambda_{N-2} - b^{j})\cdot u_{N-2}^{j} + \frac{1}{2}\left(A_{N-2}\lambda_{N-2} + B_{N-2}^{j}u_{N-2}^{j} + B_{N-2}^{-j}u_{N-2}^{-j} + w_{N-2}\right)' \times K_{N-1}^{j}\left(A_{N-2}\lambda_{N-2} + B_{N-2}^{j}u_{N-2}^{j} + B_{N-2}^{-j}u_{N-2}^{-j} + w_{N-2}\right)\right\}. \quad \text{(A15)}$$

By going through similar procedures as (A6) and (A7) and using some matrix algebra, we have (A16), shown at the bottom of the page. In a simple expression, we have

$$u_{N-2}^{*} = -\Psi_{N-2}^{-1}\cdot(\Gamma_{N-2}\cdot\lambda_{N-2} + \Omega_{N-2}) = L_{N-2}\cdot\lambda_{N-2} + \Phi_{N-2}. \quad \text{(A17)}$$

By substituting (A17) into (A15) and some matrix algebra, we can derive the optimal cost expression for player (j) as follows:

$$J_{N-2}^{j*}(\lambda_{N-2}) = \lambda_{N-2}'K_{N-2}^{j}\lambda_{N-2} + P_{N-2}^{j}\lambda_{N-2} + M_{N-2}^{j} + \sum_{i=N-2}^{N-1} E\left\{w_{i}'K_{i+1}^{j}w_{i}\right\} \quad \text{(A18)}$$

where

$$K_{N-2}^{j} = L_{N-2}^{j'}a^{j}L_{N-2}^{j} + \alpha^{j}L_{N-2}^{j} + \left(A_{N-2} + B_{N-2}^{j}L_{N-2}^{j} + B_{N-2}^{-j}L_{N-2}^{-j}\right)' \times K_{N-1}^{j}\left(A_{N-2} + B_{N-2}^{j}L_{N-2}^{j} + B_{N-2}^{-j}L_{N-2}^{-j}\right) \quad \text{(A19)}$$

$$P_{N-2}^{j} = \alpha^{j}\left(\Phi_{N-2}^{j} - b^{j}L_{N-2}^{j}\right) + \Phi_{N-2}^{j'}a^{j}L_{N-2}^{j} + \left(A_{N-2} + B_{N-2}^{j}L_{N-2}^{j} + B_{N-2}^{-j}L_{N-2}^{-j}\right)'K_{N-1}^{j} \times \left(B_{N-2}^{j}\Phi_{N-2}^{j} + B_{N-2}^{-j}\Phi_{N-2}^{-j}\right) + P_{N-1}^{j}\left(A_{N-2} + B_{N-2}^{j}L_{N-2}^{j} + B_{N-2}^{-j}L_{N-2}^{-j}\right) \quad \text{(A20)}$$

$$M_{N-2}^{j} = \Phi_{N-2}^{j'}a^{j}\Phi_{N-2}^{j} + \left(B_{N-2}^{j}\Phi_{N-2}^{j} + B_{N-2}^{-j}\Phi_{N-2}^{-j}\right)' \times K_{N-1}^{j}\left(B_{N-2}^{j}\Phi_{N-2}^{j} + B_{N-2}^{-j}\Phi_{N-2}^{-j}\right) + P_{N-1}^{j}\left(B_{N-2}^{j}\Phi_{N-2}^{j} + B_{N-2}^{-j}\Phi_{N-2}^{-j}\right) + M_{N-1}^{j}. \quad \text{(A21)}$$

If we proceed sequentially for every timestep k (k = 1, ..., N − 1), we have

$$\begin{bmatrix} u_{k}^{j*} \\ u_{k}^{-j*} \end{bmatrix} = -\begin{bmatrix} B_{k}^{j'}K_{k+1}^{j}B_{k}^{j} + a^{j} & B_{k}^{j'}K_{k+1}^{j}B_{k}^{-j} \\ B_{k}^{-j'}K_{k+1}^{-j}B_{k}^{j} & B_{k}^{-j'}K_{k+1}^{-j}B_{k}^{-j} + a^{-j} \end{bmatrix}^{-1} \times \left(\begin{bmatrix} B_{k}^{j'}K_{k+1}^{j}A_{k} + \alpha^{j} \\ B_{k}^{-j'}K_{k+1}^{-j}A_{k} + \alpha^{-j} \end{bmatrix}\cdot\lambda_{k} + \begin{bmatrix} B_{k}^{j'}P_{k+1}^{j} - \alpha^{j}b^{j} \\ B_{k}^{-j'}P_{k+1}^{-j} - \alpha^{-j}b^{-j} \end{bmatrix}\right) \quad \text{(A22)}$$

or simply

$$u_{k}^{*} = -\Psi_{k}^{-1}\cdot(\Gamma_{k}\cdot\lambda_{k} + \Omega_{k}) = L_{k}\cdot\lambda_{k} + \Phi_{k}. \quad \text{(A23)}$$

The optimal cost-to-go for player (j) at time step k is determined as follows:

$$J_{k}^{j*}(\lambda_{k}) = \lambda_{k}'K_{k}^{j}\lambda_{k} + P_{k}^{j}\lambda_{k} + M_{k}^{j} + \sum_{i=k}^{N-1} E\left\{w_{i}'K_{i+1}^{j}w_{i}\right\} \quad \text{(A24)}$$

where

$$K_{k}^{j} = L_{k}^{j'}a^{j}L_{k}^{j} + \alpha^{j}L_{k}^{j} + \left(A_{k} + B_{k}^{j}L_{k}^{j} + B_{k}^{-j}L_{k}^{-j}\right)' \times K_{k+1}^{j}\left(A_{k} + B_{k}^{j}L_{k}^{j} + B_{k}^{-j}L_{k}^{-j}\right) \quad \text{(A25)}$$

$$P_{k}^{j} = \alpha^{j}\left(\Phi_{k}^{j} - b^{j}L_{k}^{j}\right) + \Phi_{k}^{j'}a^{j}L_{k}^{j} + \left(A_{k} + B_{k}^{j}L_{k}^{j} + B_{k}^{-j}L_{k}^{-j}\right)'K_{k+1}^{j} \times \left(B_{k}^{j}\Phi_{k}^{j} + B_{k}^{-j}\Phi_{k}^{-j}\right) + P_{k+1}^{j}\left(A_{k} + B_{k}^{j}L_{k}^{j} + B_{k}^{-j}L_{k}^{-j}\right) \quad \text{(A26)}$$

$$\begin{bmatrix} u_{N-2}^{j*} \\ u_{N-2}^{-j*} \end{bmatrix} = -\begin{bmatrix} B_{N-2}^{j'}K_{N-1}^{j}B_{N-2}^{j} + a^{j} & B_{N-2}^{j'}K_{N-1}^{j}B_{N-2}^{-j} \\ B_{N-2}^{-j'}K_{N-1}^{-j}B_{N-2}^{j} & B_{N-2}^{-j'}K_{N-1}^{-j}B_{N-2}^{-j} + a^{-j} \end{bmatrix}^{-1} \times \left(\begin{bmatrix} B_{N-2}^{j'}K_{N-1}^{j}A_{N-2} + \alpha^{j} \\ B_{N-2}^{-j'}K_{N-1}^{-j}A_{N-2} + \alpha^{-j} \end{bmatrix}\cdot\lambda_{N-2} + \begin{bmatrix} B_{N-2}^{j'}P_{N-1}^{j} - \alpha^{j}b^{j} \\ B_{N-2}^{-j'}P_{N-1}^{-j} - \alpha^{-j}b^{-j} \end{bmatrix}\right). \quad \text{(A16)}$$

$$M_k^j = \Phi_k^{j'} a^j \Phi_k^j + \left(B_k^j \Phi_k^j + B_k^{-j} \Phi_k^{-j}\right)' \times K_{k+1}^j \left(B_k^j \Phi_k^j + B_k^{-j} \Phi_k^{-j}\right) + P_{k+1}^j \left(B_k^j \Phi_k^j + B_k^{-j} \Phi_k^{-j}\right) + M_k^j. \quad \text{(A27)}$$

This concludes our derivation of (7) to (16) in the text. Note that in an n-player game the sizes of matrices: $u_k^*, \Psi_k, \Gamma_k, \Omega_k, L_k$ *and* Φ_k are $n\times 1, n\times n, n\times 1, n\times 1, n\times 1$ and $n\times 1$, respectively.

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