1 2 3

Quantitative Analysis of the Ankle Strategy Under Translational Platform Disturbance

Hooshang Hemami, *Life Fellow, IEEE* Kamran Barin,*member, IEEE* and Yi-Chung Pai

***Abstract*— The ankle strategy is one of the postural adjustment maneuvers humans utilize when the support platform is disturbed. This work presents a quantitative analysis of the ankle strategy. A three-link sagittal biped model is considered. The first link represents the two legs locked together. The second link represents the two thighs locked together. The third link represents the hip, the torso, the upper limbs, the neck and the head. The dynamics, control and stability of the three-link biped, under platform translation, are considered. The disturbance of the platform is represented as an input and the effect of the muscular system is reduced to a set of torques applied to the joints and across the joints.**

Two digital computer simulations are presented to demonstrate the behavior of the biped under backward or forward platform disturbance. The simulations are compared with experimental measurements of humans subjected to postural disturbances.

It is shown that the effect of a horizontal disturbance at the ankle appears to be about 40 times that of the effect of the disturbance at the knees and at least a few hundred times larger than the effect of a disturbance at the hip. This means that, under translational platform disturbance, the ankle angle is subjected to the largest excursion. The knee and the hip angle excursions are relatively minor. Consequently, the biped, as a whole, appears to move as a single inverted pendulum. Major postural corrections are initiated by the ankle excursion. Further, when the available ankle torque is limited or nonexistent, the stability requires resorting to the knee or hip strategies.

***Index Terms*— keywords: sagittal biped dynamics, stability, platform disturbance, postural adjustments, postural strategies, muscle synergies balance control, slipping, balance recovery,**

I. Introduction

THE PURPOSE of this study is to develop a quantitative framework to study the biomechanics and neural basis of the ankle strategy for maintaining postural stability. The human postural system has been the subject of theoretical, empirical and experimental studies in the past fifty years. The control of motion by the central nervous system (CNS) and how the different components of the CNS take part in such control have produced novel experiments and control hypotheses, e.g., that of Nashner and McCollum [1], Barin et al [2-4] and others. Several quantitative models have been considered in the past. One is based on Newtonian mechanics [5]. Another is the empirical model of Nashner and colleagues [1]. Certain neural models have been proposed, e.g., that of Katayama [6-7]. It is not known what internal models [8] of the self and the environment are utilized by the CNS. Further, it is not known how such internal models are neurally implemented.

From a theoretical point of view, studies of human and animal movement fall in the class of system identification [1–3] where the system is subjected to specific disturbances [4]. Translational and/or rotational [5], [6] motion of the supporting platform or other well-defined disturbances [7–9] have been applied to human and animal subjects. Disturbances that induce stumbling, slipping or falling have also been applied [10]. The subject is observed during the disturbance and a variety of physical attributes of the subject are measured. The physical and physiological parameters of the model are tuned so that the behavior of the model and the subject are close. Typical measurements are EMGs of the muscular system [11], motion of the center of gravity, ground reaction forces, actual trajectories of motion, other synthesized curves such as phase plane trajectories [12], and traces in empirical spaces [11]. Other studies are based on the anatomical and morphological attributes of the CNS [13].

This process involves reduced and simplified [14], [15] neuro-musculo-skeletal models of different dimensions and levels of complexity. The musculo-skeletal system is gradually enlarged to include sensory apparatus, neural networks [16], learning capability [17], tactile apparatus [18], and visual and vestibular subsystems [1], [19], [20]. Such models also allow studies of control strategies, pattern generation [21], function of the cerebellum [22], and coordination of movement.

The quantitative model here has the following attributes:
1. The inputs are disturbances of the base.
2. The outputs are the trajectories of motion.
3. The model is a three - link sagittal biped with torque generators at and across the joints.

The dynamics of a three-link sagittal biped subject to horizontal platform disturbance are formulated. The stability of the system is analyzed based on five torques at and across the joints. The effect of these torques on stability is studied by linearizing the biped near the vertical stance, and identifying the locations of the system poles in the complex S domain. The biped is subjected to horizontal platform disturbance in two computer simulations: forward and backward platform motion. The behavior of the biped is compared with recorded human behavior under similar platform disturbance.

The three-link biped is presented in section 2. The platform disturbances and their effects are discussed in section 3. Stability issues are considered in section 4. Computer simulations are presented in section 5, and discussions and conclusions in section 6.

manuscript received January 2005; revised January and June 2006.

H. Hemami and K. Barin are with the Ohio State University. Yi-Chung Pai is with the University of Illinois at Chicago.

II. The Three-link Biped

The dynamics of a three-link rigid body system, hinged at the joints and equipped with a sensor-less and mass-less foot, shown in Fig. 1, are formulated first. The physical parameters are given in the Appendix. The system is stabilized by position and velocity feedback as a result of co-activation of agonist-antagonist pair of muscles. This model does not allow study of stepping strategies which will be presented separately. We also exclude inclusion of the mass and dynamics of the platform [8], [23].

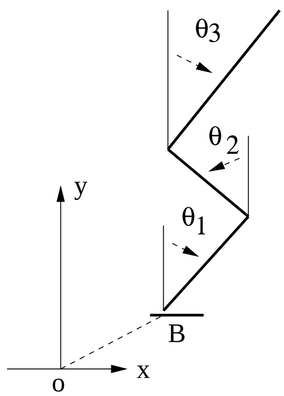

Fig. 1. The Three-link biped on a mobile platform. The arrows indicate that all angles are measured relative to the vertical line.

A. The Dynamics with no joint constraints

The general procedure for deriving the equations of motion for the biped is described in reference [24], and is not repeated here. The two constraints that connect the biped to the ground involve the vector of disturbance $[x_d(t), y_d(t)]'$ in both the horizontal and vertical directions. The accelerations of the platform in the horizontal and vertical directions appear in the equations of motion.

Let vector Θ be the angles of the leg, the thigh, and the torso with respect to the vertical:

$$\Theta = [\theta_1, \theta_2, \theta_3]'.$$

Let

$$\dot{\Theta} = [\dot{\theta}_1, \dot{\theta}_2, \dot{\theta}_3]'.$$

be the angular velocities of the three links.

The equations of the biped, in symbolic form, [22] are:

$$J(\Theta)\ddot{\Theta} + B(\Theta, \dot{\Theta})\dot{\Theta} + (\ddot{y}_d - g)G(\Theta) + \ddot{x}_d H(\Theta) = U + U_d \quad (1)$$

The variable U is the 3-vector of muscular torques. The variable U_d is the torque disturbance due to platform rotation, and is set to zero. We assume there is no platform rotation. Gravity effect and vertical translation disturbance are given by the vector $G(\Theta)$. The effect of the horizontal disturbance is given by the term $H(\Theta)$. The expressions for all symbols are given in the Appendix. The vectors G and H have the same constants in their components. The difference between them is that the sine of the segment angles appear in G and the cosine of the segment angles in H.

B. Physiological Space

Nashner and McCollum propose a physiologically based position space for studies of postural control [11]. Their position space is the three-dimensional space of ankle angle, hip angle, and the radial distance of the center of gravity of the body from the ankle joint. Here we develop a transformation from the space of the link angles, i.e., Θ to the physiological space of Nashner and McCollum. For this purpose, we first derive the position and velocity vectors of the center of gravity of the three-link biped.

Let x_c and y_c be the coordinates of the center of gravity of the biped relative to an origin at the base of the biped, i.e., the point of its contact with the floor. The coordinate axes are horizontal for x and vertical for y. let α be a row vector of three ones.

$$\alpha = [1, 1, 1].$$

It can be shown that

$$\begin{aligned} x_c &= (1/(m_1 + m_2 + m_3))\alpha G \\ y_c &= (1/(m_1 + m_2 + m_3))\alpha H \end{aligned} \quad (2)$$

where G and H have already appeared in equation 1 and are described in detail in the Appendix. Similarly, velocities of the center of gravity are:

$$\begin{aligned} \dot{x}_c &= +(1/(m_1 + m_2 + m_3))(\dot{\Theta})' H \\ \dot{y}_c &= -(1/(m_1 + m_2 + m_3))(\dot{\Theta})' G \end{aligned} \quad (3)$$

Let the distance of the center of gravity from the ankle joint be r_c, and the rate of change of this distance with respect to time be $\dot{r_c}$. It follows that

$$r_c{}^2 = x_c^2 + y_c{}^2,$$

and

$$\dot{r_c} = (\partial r_c / \partial \Theta)\dot{\Theta}.$$

The above mentioned physiological space can be described by the state space of

$$[\theta_1, \theta_3, r_c, \dot{\theta_1}, \dot{\theta_3}, \dot{r_c}].$$

With this transformation, it is possible to formulate mathematically the bipedal postural control, stability and movement issues in the physiological state space. This transformation will be explored in future work.

III. Platform Disturbance

A. Translational Disturbance

The effect of translational disturbance can be assessed from the structure of vectors G and H in the equations of motion (see equation 1). The two components of the translational disturbance, namely, the horizontal component $\ddot{x}_d(t)$ and the vertical component $\ddot{y}_d(t)$, affect the system differently as can be seen in equation 1.

1) Vertical Translation: The vertical acceleration disturbance adds to the gravity constant g. Since the body is normally equipped to deal with gravity almost all the time, vertical acceleration is not a problem. A positive acceleration upwards, namely, a positive $\ddot{y}_d(t)$ requires an increase in the co-activation of muscles involved in anti-gravity activity in order to keep relative stability the same. A negative acceleration requires a corresponding reduction in muscular activity.

2) Horizontal Translation: The horizontal translation, as seen in vector H, is dependent on the cosines of the angles θ. It is a nonlinear effect that cannot be compensated by linear feedback. From a theoretical point of view, the effect of a horizontal disturbance is reduced when all θ's tend towards 90 degrees, i.e., $\pm 0.5\pi$, because the cosines tend toward zero in $H(\Theta)$. The strategy of reducing the disturbance effect is also effective when a subset of the angles are quickly increased, i.e., the system is partially flexed.

Three observations are in order here.

1. The ankle actuations are usually mediated by automatic stretch responses and require shorter delays compared to actuation of other joints. Therefore, the ankle strategy becomes important here [1]. In fact, the CNS may employ the ankle strategy as a generalized motor command across a range of responses that may not solely depend on precise and timely inflow of the local sensory receptors. For instance, it has been noted that in response to forward support surface perturbations similar EMG patterns were elicited during gait, just as those that have been shown to be in response to stance perturbations, [25], [26]. Moreover, forward perturbations during the heel strike phase of the gait cycle resulted in early onset (70-120 ms), large magnitude and long duration proximal to distance activation patterns on the perturbed side leg and thigh muscles such as tibialis anterior and rectus femoris [27] resembling the ankle strategy observed in response to small support surface perturbations [25], [26]. Such responses aid in generating the observed ankle planter flexor moment ([28]) for restoring the ankle dorsiflexion trajectory.

2. An additional requirement in all such flexion strategies is to coordinate the motion such that the projection of the center of mass remains in the base of support. From a control point of view, such a strategy could be brought about by an ordered and programmed reduction of all extensor forces such that the body collapses to the floor in a squatting state. If this ordering is not pre-programmed, chances of a fall increase [6]

3. The disturbance magnitude is much larger in the leg equation and it gradually decreases for the higher links. This again points out to why an ankle strategy is important.

The strategy of sudden squatting as a postural strategy can be further justified by the following argument. Sudden squatting will reduce the vertical ground support forces [29]. This reduction, in turn, will reduce the frictional horizontal component of support force, and, therefore, the maneuver has the tendency to decouple the biped somewhat from the moving platform.

Cases of an ordered squatting rather than falling can be observed in the absence of any platform disturbance. It has been observed occasionally in people under sudden and severe emotional shock. People who are afraid of heights, and are suddenly exposed to an unacceptable frightening environment, may squat without volition. Similar behavior has been observed in animals. Collapse of the foreleg or the hind legs as a result of ventroflexion or dorsiflexion of the head in decerebrate cats has been described by Henneman [30]. It is stated in [11] that the postural strategies along the stooping or suspensory axis are rare. If the simultaneous increase of all three angles is not adopted, then increasing a subset of angles is possibly the best choice. Nashner and McCollum argue that the CNS has adopted the two single axis strategies of ankle and hip.

For simpler living organisms, the detection of disturbances and the repertoire of responses are much simpler. Ewert [31] has discussed the existence and functioning of certain inter neurons for escape in swimming behavior of certain species. He further elaborates on the role of neural networks for this kind of goal directed behavior, and also the neural mechanisms needed for detection of the conditions that induce the escape behavior. The study of the mechanisms of detecting disturbances, and issuing the commands for adoption of the appropriate postural strategy are beyond the scope of this paper. One may conjecture that sequential image matching and sequential adaptive resonance theory [32] could apply here.

IV. Stability

Stability can be achieved by control of the stiffness of agonist-antagonist pairs at joints and across the segments. The simultaneous co-activation of agonist-antagonist pairs of muscles at least by a pair at every joint achieves the same end [33]. The co-activation produces sufficient position and velocity feedback to bring about stability in some vicinity of the vertical equilibrium position.

One can analytically show that the co-activation of a pair of agonist-antagonist actuators results in negative angular position and angular velocity feedback. The stiffness and viscosity, i.e., the position and the velocity feedback gains, can be programmed [15], [34]. The stiffness is proportional to the neural excitation level. The position and velocity feedback gains are given in the Appendix. Thus the system is activated by five quantities: stiffness and viscosity at the ankle, at the knee, at the hip, across the thigh, and across the leg. We consider the stability of the system about the vertical stance.

The values of the feedback matrices are given in the Appendix.

V. Computational Experiments

A number of computational experiments are performed here in order to illustrate some of the findings of [11] with quantitative results.

A. Muscle Synergies

We assume five pairs of muscle-like actuators are available as shown in Fig. 2.

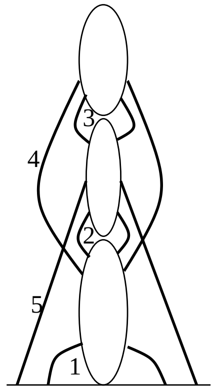

Fig. 2. The sagittal Three-link biped with five pair of muscle-like actuators: at the ankle (1), the knee (2), the hip (3), spanning the thigh (4), and spanning the leg (5)

These five pairs correspond, respectively, to an ankle pair, e.g., the tibialis anterior and the soleus, a knee pair corresponding to the vasti and a hamstring short head, a pair corresponding to paraspinal and abdominal pair, a quadricep-hamstring two-joint pair, and a gastrocnemius-tibialis pair spanning the leg.

Seven sets of values are selected for the vector K that determine the stiffness of the muscle pairs as shown in Table I. The values for the stiffness are selected such that, in single pendulum tilts from the vertical stance, the elastic energy of the muscles exceeds the loss of potential energy due to gravity. The postural disturbances, as a consequence, dissipate in less than three seconds. The following argument can clarify the choice for K1, i.e., the ankle stiffness. In normal standing, the ankle stiffness must compensate for gravity. A biped of weight 70 Kg, with a center of gravity at about 1.1 meters above the ground, and a gravity constant of gr = 9.81 m/sec2 needs a minimum ankle stiffness equal to the product of the above three numbers in order not to fall. Loram and Lakie [35] estimate intrinsic stiffness of about 91 N-m for the ankle joint, which is far less than the amount required for stability. We have taken this stiffness to be 1000 because higher stiffness levels provide more stability. Similar arguments can be forwarded for the choice of the knee and hip feedback gains. Ideally these stiffnesses must be measured, and they could very well be not constant throughout the ankle strategy maneuver.

The nature of the control mechanisms for control of quiet stance is still an object of controversy. The model proposed by Winter et al. [36] attributes muscle stiffness as the single factor involved in solving the problem based on the argument that the controlled variable, center of mass (COM), was virtually in phase with the controlling motor variable, center of pressure (COP). According to this theory, the intervention of the CNS is limited to the selection of an appropriate muscle tone at the ankle joint, in order to establish an ankle stiffness that stabilizes the otherwise unstable inverted pendulum system. Thus according to Winter et al. [36], [37] the stabilization of quiet standing is a fundamentally passive process without any significant active or reactive component, except for the background setting of the stiffness parameters. The authors aruge that if a normal reactive control were present, the short- and long- reflex loop latencies combined with the biomechanical delays at the neuromuscular junction would result in a COP delay of over 100 ms after the COM. On the other hand, Morasso and Schieppati [38] have argued that a simple spring stiffness control does exist but that ”there must be something in the control circuitry that compensates for the original delays” and that ”the phase-lock between COP and COM is a necessary consequence of physical laws.” They propose an alternative computational scheme can be outlined that is based on the indirect estimate of a postural state vector obtained from the complex combination of a variety of sway-related sensory signals. Further, they strongly argue for a central computational process to integrate the multisensory information into a unifying estimate of the state vector and to compensate the transmission delays with an anticipatory action, i.e., a short-time prediction of the postural time series. The ankle stiffness values used was 674 Nm/rad by Winter et al. [36] and 1050 Nm/rad by Morasso and Schieppati [38].

With the mentioned seven set of stiffnesses, the stability of the biped can be determined near the vertical stance by computing the poles. The biped equations are linearized (see Appendix B) about the erect posture, i.e., when

$$\Theta = 0, \dot{\Theta} = 0,$$

The poles of the linearized system are given in table II for seven cases. When the pole is negative or the real part of the complex pole is negative, the corresponding natural mode of the system is stable. We briefly discuss each of the seven cases here.

Case 1 corresponds to five active agonist - antagonist pairs of actuators. Table 2 shows that the linearized system is stable in this case.

Case 2 corresponds to a disabled gastrocnemius pair. When the knees are locked, the gastrocnemius effects can be integrated in the ankle stiffness through the use of tibialis anterior, etc. Nashner and McCollum [11] appear to adopt this condition, namely, when the ankle and thigh angles are the same, and when the feet are firmly on the ground, the soleus and gastrocnemius muscles essentially act as generators of torque at the ankle.

Case 2 shows that the system is unstable near the vertical stance because the third pole is positive. Physically this case corresponds to a situation when the platform is moved forward and the stiffness across the leg is zero.

Case 3 is the same situation as case 2, but the stiffness of the ankle joint is doubled. Table 2 shows that the system becomes stable - all poles are in the left half of the complex plane.

Case 4 corresponds to case 1 except that the knee pair is made inactive. Table 2 shows that the system is stable. This confirms the hypothesis that the knee muscles may not be needed in balance as has been observed by Nashner and McCollum [11].

Case 5 demonstrates that the three pair of ankle, hip and quad pair are not sufficient for stability.

Case 6 shows that the ankle pair is essential for stability. Two other variations of case 6 are also presented. In case

6a, the hip gains were doubled. In case 6b, the quad pair gains were doubled, but, in both cases, the system remained unstable.

Case 7 similarly demonstrates that the ankle pair and the two-jointed pairs of quads and the gastrocnemius are sufficient for stability.

Indeed, if one may generalize and extrapolate the results here, they imply that smaller subsets of muscles are adequate for stability as well as the recovery of balance as has been stated above [11], [39].

B. Backward Platform Motion

Suppose the magnitude of a backward acceleration is one unit of impulse and suppose the biped is standing with a slight flexion at all joints. The platform induces an initial velocity at all the joints, i.e., the initial state is set to

$$[0.1, -0.1, 0.1, 2.5580, -0.0654, 0.0027].$$

The first three states are angles and the numbers are in radians. The next three states are angular velocities in rad/sec. Simulation of the three link nonlinear system with the above initial condition and case 1 are represented in four figures. Fig. 3 shows the trajectories of motion, i.e., three angles and three angular velocities, as functions of time.

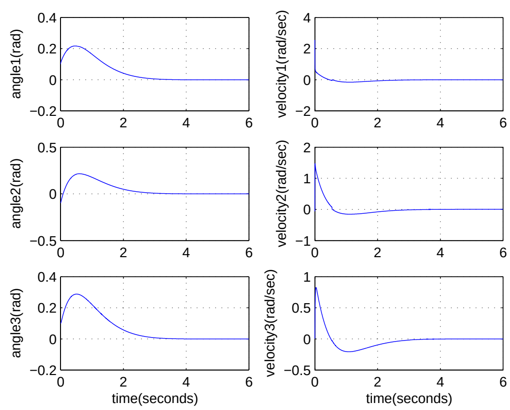

Fig. 3. The trajectories of motion: the three angles and the three angular velocities as functions of time in a backward platform motion.

Fig. 4 shows the angles θ_2 and θ_3 as functions of θ_1. This figure appears to confirm the finding [11] that the knees remain locked during the backward platform motion, i.e., a forward subject sway. Initially the biped starts from a slightly stooped position, but the knee angle increases and the knee joint locks.

Fig. 5 shows the torques of the muscular system at the three joints and the activity of the gastrocnemius muscle. Fig. 6 shows the activity of the two-jointed quadricep as a function of time. All five muscular pair activities seem to be of the same order of magnitude.

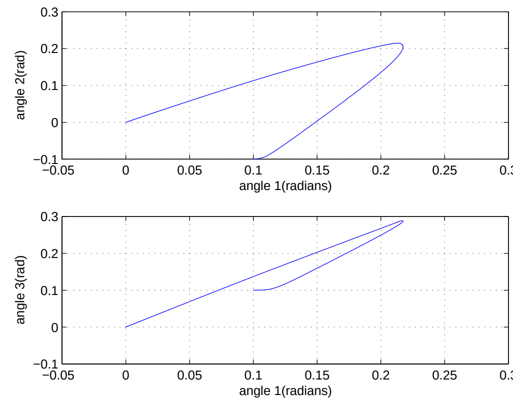

Fig. 4. The spatial trajectories of motion: the angles θ_2 and θ_3 as functions of θ_1 in a backward platform motion - a forward subject sway. All angles seem to linearly depend on one another, and the knee gets locked at the point of maximum excursion forward.

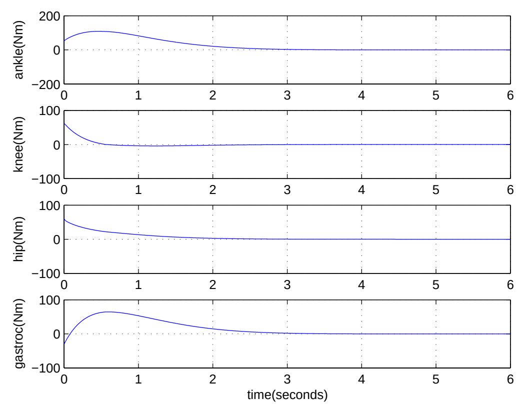

Fig. 5. The torque activity at the three joints and the gastrocnemius as functions of time - all torques are in Newton-meters

Fig. 7 shows the horizontal motion of the center of gravity in the phase plane.

C. Forward Platform Motion

When a forward platform disturbance of the same magnitude is introduced, the response of the system is almost identical to the backward platform disturbance provided that K_5 is included in the dynamics. The simulation of the system confirms this. The phase plane for the horizontal component of the motion of the center of gravity is shown in Fig. 8.

The function of the gastrocnemius pair is partially performed by the tibialis anterior, because the knees are locked. Whenever the knees are locked, the following equation is in effect:

$$\theta_1(t) - \theta_2(t) = 0.$$

When the action of the gastrocnemius is eliminated the biped is unstable and falls down. This is shown in the trajectory of the center of gravity diverging from the origin as shown in Fig. 9.

TABLE I
MUSCLE SYNERGIES (K) FOR SEVEN EXPERIMENTS

	ankle	knee	hip	quad pair	gastroc pair
case 1	500	300	300	300	300
case 2	500	300	300	300	000
case 3	1000	300	300	300	000
case 4	1000	000	300	300	300
case 5	l000	000	300	300	000
case 6	000	000	300	300	1000
case 6 a	000	000	600	300	1000
case 6 b	000	000	300	600	1000
case 7	1000	000	000	300	1000

TABLE II
POLES OF THE LINEARIZED SYSTEM FOR THE SEVEN CASES

case 1	-554.04	-88.94	-1.60 +.79 i	-1.60 - 0.79 i	-3.3	-3.62
case 2	-481.02	-87.25	+1.17*	-3.61	-3.01 + 0.11 i	-3.01 - 0.11
case 3	-575.417	-90.56	-0.90	-3.64	-2.95 + 0.14 i	-2.95 - 0.14 i
case 4	-391.20	-93.36	-3.64	-2.75	-2.23+1.39 i	-2.23-1.39 i
case 5	-320.97	-90.05	-3.64	+1.86*	-3.12 +0.76 i	-3.12 -0.76 i
case 6	-390.68	-77.24	-3.64	+2.11*	-3.23 +0.81 i	-3.23 -0.8 i
case 6a	-513.52	-91.56	-3.77	+1.55*	-3.19 +0.83 i	-3.19 -0.83 i
case 6b	-418.34	-132.57	-3.26	+1.75*	-3.26 +0.69 i	-3.26 -0.69 i
case 7	-469.46	-53.90	-3.46	-2.67	-3.18 +2.00 i	-3.18 - 2.00 i

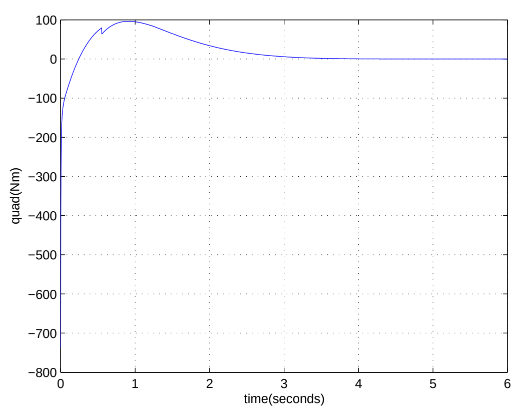

Fig. 6. The quadricep two-jointed muscle activity as a function of time

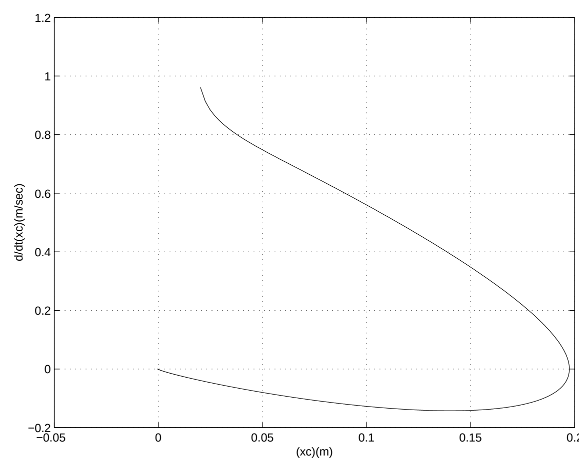

Fig. 7. The phase plane portrait of the horizontal motion of the center of gravity - velocity versus position - in a backward platform motion, i.e., in a forward sway.

This result shows the instability of the biped in Case 3. In the last simulation, it is assumed that the knees are locked and, therefore, the action of the gastrocnemius can be compensated for by higher ankle torques. The phase plane of the resulting stable motion is shown in Fig. 10.

A more physiological and experimentally observed phenomenon is at work here. In all of these simulations, the knee is, relatively speaking, extended. This means that

$$\theta_2 = \theta_1$$

Therefore one could have resorted to a larger force by the tibialis anterior, and this larger force would represent the artificial counter gastrocnemius.

In the second simulation, a larger tibialis anterior force, could have compensated for the ankle torque needed to prevent the biped from falling. It has been observed that the tibialis anterior dominates at the ankle by the dorsi flexor moment and prevents falling in forward platform motions.

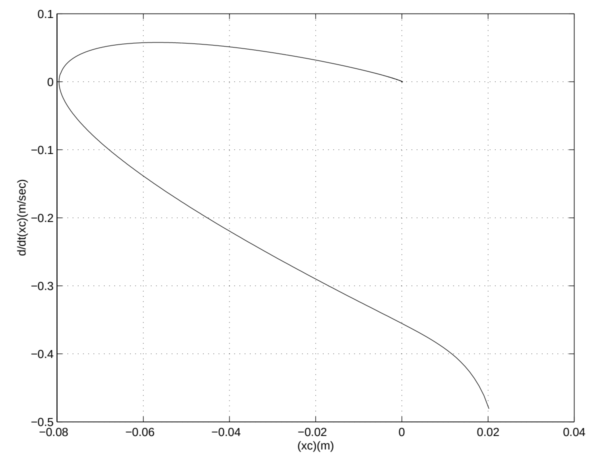

Fig. 8. The phase plane portrait of the horizontal motion of the center of gravity - velocity versus position in forward platform disturbance. Artificial antagonistic gastrocnemius is assumed. In reality, the tibialis anterior would compensate for the artificial gastrocnemius muscle introduced here, because the knees are locked and θ_1 and θ_2 are equal.

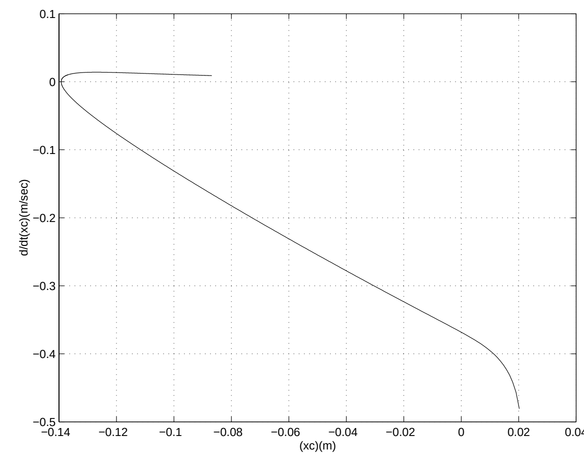

Fig. 10. The phase plane portrait of the horizontal motion of the center of gravity - velocity versus position in forward platform disturbance with no gastrocnemius, but larger ankle stiffness. The system is recovering from the disturbance but more slowly. A larger tibialis anterior - soleus force pair would implement this case.

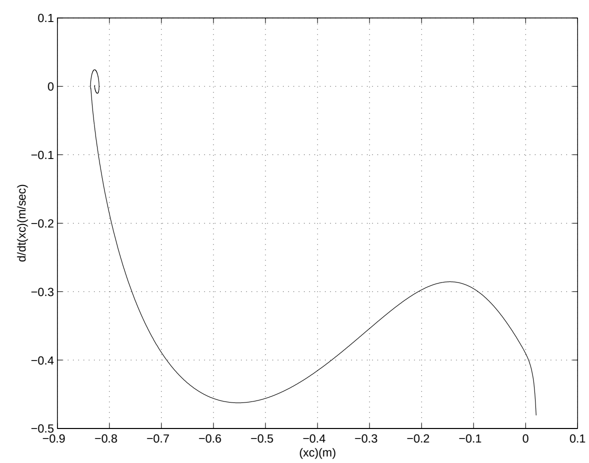

Fig. 9. The phase plane portrait of the horizontal motion of the center of gravity - velocity versus position - in forward platform disturbance. The system is unstable, and fall is inevitable. It is possible that a much larger tibialis anterior force would have prevented the fall.

VI. Discussion and Conclusions

Figure 11 shows actual motion trajectories of a subject in response to postural disturbances. These results are very similar to the simulated angles in Fig.3. There are minor differences in the absolute amplitude of the angles mainly because the disturbance was not exactly the same for the experimental and simulated responses. However, the ratio and the timing of the angles in both cases are surprising similar and provide support for the simulation results. An analytical method was developed to study the ankle strategy and some of its attributes. The effect of a horizontal or vertical platform disturbance was represented in the equations of the biped and was quantified. If one computes the effect of the horizontal disturbance on the three angles, the severity of the effect can be described by vector

$$J(\Theta)^{-1}H(\Theta).$$

The numerical value of this vector is

$$[2.5580, -.0654, 0.0027]'.$$

The effect of a horizontal disturbance at the ankle appears to be about 40 times that of the effect at the knee and, at least by a factor of several hundreds, larger than that at the hip. This means, for relatively equal stiffness of the musculature at all and across the joints, the torque at the ankle is, relatively, much larger than at the other joints. Therefore it is not surprising that, with all other conditions being equal, the ankle strategy would be a first choice or priority. When the ankle torque is limited or nonexistent, the stability necessitates the knee or hip torque.

Nashner and McCollum [11] interpret the preference for the ankle strategy as follows. Humans keep the knees locked, and, therefore, use fewer muscles. Once a joint is locked, the system loses some degrees of freedom, and hence fewer actuators are needed to control it. One can conjecture that the evolution of the knee joint and its locking mechanism obviated the need for an antagonist gastrocnemius. The tibialis anterior would perform the function as long as the knee was locked, and the foot was firmly on the ground.

A transformation from the physical to the physiological space was articulated. If this physiological space is indeed utilized by the CNS, and a minimal number of muscles are utilized, then construction of neural systems that control the ankle strategy or identification of these neural modules in the CNS should become easier. The internal (CNS) models for motion could possibly be based on the physiological space.

A discussion of how to implement a postural response to disturbances is in order here. Suppose a system has been designed to detect the disturbance, and issue commands to select a desired postural trajectory response from a repertoire of stored trajectories. The dynamics of this process are sketched in Fig. 12.

Once the command for recovery is issued, and, under the assumption that a specific trajectory is selected, the ensuing

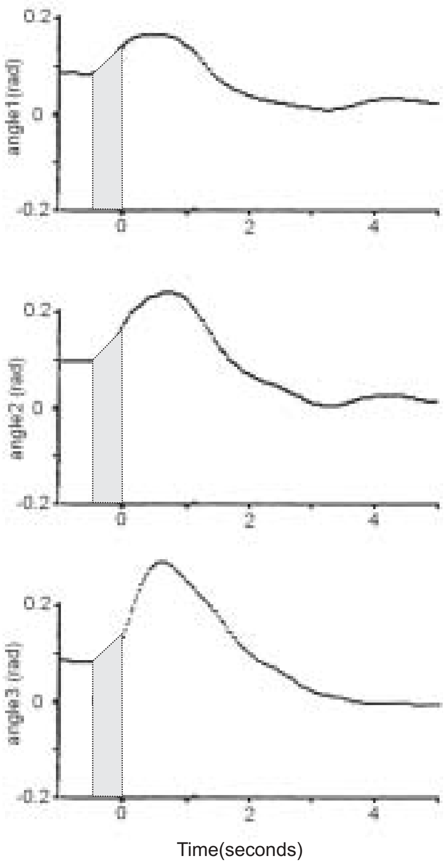

Fig. 11. The trajectories of motion for a subject following a postural disturbance. The figure is from Barin [40], modified here, to reflect the direction and units of angles used in this paper. Shaded area shows the duration of the disturbance.

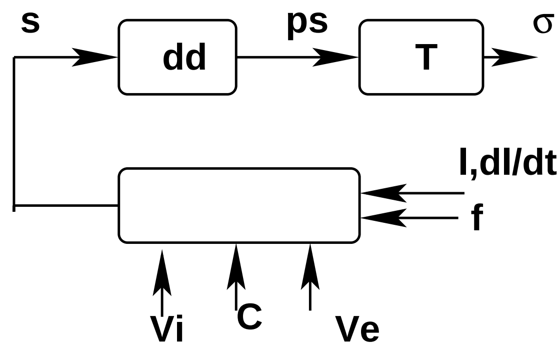

Fig. 12. The neural mechanism for detection of the disturbance ps, and selection of the appropriate desired trajectory σ_d. The variables l ,$\partial l/\partial t$ and f are respectively, the vectors of length, velocity, and forces of muscles. The variables Vi, C, and Ve are the sensory modalities of vision, contact and vestibular apparatus. The variable s *describes the state of self and the environment. The processor* dd *detects the disturbance ps, and an appropriate desired trajectory is selected from the repertoire of stored trajectories in* T.

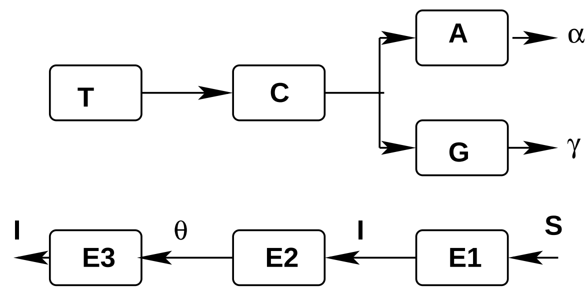

Fig. 13. The seven neural network system for estimating position from the somesthetic and sensory data, and for generating desired trajectories. The interconnections between the two subsystems above are not shown.

neural system can be modeled as a seven-neural network system as shown in Fig. 13 and elaborated on in [17]. The seven network is comprised of three feedback subsystems and four feed-forward subsystems.

The three networks estimate the position of the biped in external physical space, and the vector l of the lengths of all muscles. The four feed-forward networks generate trajectories T, compensate for delays C, and provide the α and γ neural inputs to the muscular system. The network $E1$ estimates the length of muscles from the spindle and other outputs. The network $E2$ estimates the angles. The network $E3$ is the inverse of $E2$. The network T is the repertoire of stored trajectories. The network C compensates for latencies and delays. The networks G and A, respectively, synthesize γ and α signals to the muscular system. Eventually, such systems are needed to implement the ankle strategy in the higher levels of the CNS.

The present paper is devoted to the ankle strategy. The hip and the squatting strategies should be studied next. Taking a step to modify the support surfaces, and also addressing issues of limb collapse and hip failure that contribute to loss of balance are important problems that have not yet been analytically formulated.

Finally, the analysis presented here is relevant to applications in diagnostics, physical therapy, and monitoring human performance. Combined with measurement of the trajectories, and optimization tools, the model presented here could be used to estimate the stiffness matrix, its adequacy, and its evolution in time for those recovering from injuries, mastering other tasks or developing strength. The methodology here could be used to quantify how accurate the supposition of minimal muscle use is.

The human CNS is complex, and much more experimental, analytical, and computational work is needed to unravel its resources, beauties and mysteries.

ACKNOWLEDGMENTS

The authors gratefully acknowledge the assistance of the Computer Facilities personnel of the Department of Electrical Engineering at the Ohio State University, specifically Vincent Juodvalkis, Donald S. Gibb and Aaron Aufderheide. The authors are also indebted to the reviewers and the Associate Editors for their critical reading of the manuscript, and numerous valuable suggestions.

VII. APPENDICES

A. Appendix A, Physical Parameters

The physical parameters of the biped are selected as follows. Link1 is the leg, link 2 is the thigh, and link 3 is the torso. For each link four parameters are specified. Parameter I refers to the moment of inertia in Kgm^2 about the center of gravity of the link. Parameter m refers to the mass of the link in Kg. Parameter l refers to the length of the link in m. Parameter k refers to the distance of the center of mass of the link from the lower joint.

$$I1 = .17;$$
$$I2 = .5;$$
$$I3 = 2.1;$$
$$m1 = 5.05;$$
$$m2 = 18.67;$$
$$m3 = 41.87;$$
$$l1 = .401;$$
$$l2 = .412;$$
$$l3 = .619;$$
$$k1 = .21;$$
$$k2 = .26;$$
$$k3 = .24;$$
$$g = 9.81;$$

B. Appendix B, Equations of Motion

The matrices and vectors appearing in equation 1 are given below.

$$J(\Theta) = \begin{bmatrix} I_1 + m_1k_1^2 + (m_2+m_3)l_1^2 & (m_2k_2+m_3l_2)l_1cos(\theta_2-\theta_1) & m_3k_3l_1cos(\theta_3-\theta_1) \\ (m_2k_2+m_3l_2)l_1cos(\theta_2-\theta_1) & I_2+m_2k_2^2+m_3l_2^2 & m_3k_3l_2cos(\theta_3-\theta_2) \\ m_3k_3l_1cos(\theta_3-\theta_1) & m_3k_3l_2cos(\theta_3-\theta_2) & I_3+m_3k_3^2 \end{bmatrix}.$$

The equation for B is

$$B(\Theta,\dot{\Theta}) = \begin{bmatrix} 0 & (m_2k_2+m_3l_2)l_1\dot{\theta}_2sin(\theta_1-\theta_2) & m_3k_3l_1\dot{\theta}_3sin(\theta_1-\theta_3) \\ (m_2k_2+m_3l_2)l_1\dot{\theta}_1sin(\theta_2-\theta_1) & 0 & m_3k_3l_2\dot{\theta}_3sin(\theta_2-\theta_3) \\ m_3k_3l_1\dot{\theta}_1sin(\theta_3-\theta_1) & m_3k_3l_2\dot{\theta}_2)sin\theta_3-\theta_2) & 0 \end{bmatrix}.$$

The equation for the gravity vector is:

$$G(\Theta) = \begin{bmatrix} [m_1k_1+(m_2+m_3)l_1]sin(\theta_1) \\ (m_2k_2+m_3l_3)sin(\theta_2) \\ m_3k_3sin(\theta_3) \end{bmatrix}.$$

The vector coefficient of the horizontal disturbance is:

$$H(\Theta) = \begin{bmatrix} [m_1k_1+(m_2+m_3)l_1]cos(\theta_1) \\ (m_2k_2+m_3l_3)cos(\theta_2) \\ m_3k_3cos(\theta_3) \end{bmatrix}.$$

When the equations of motion are linearized about the origin, i.e., zero angles and zero andular velocities, one obtains the three coupled linear diffrential equations:

$$J(0)\ddot{\Theta} - g\partial G/\partial\Theta(0)\Theta = U.$$

and the control U is given by the last equation in Appendix C.

C. Appendix C, Feedback Effect of Co-activation

Consider the three-link sagittal biped with three pairs of muscle-like actuators crossing single joints, and a pairs of two-joint muscles. One pair is analogous to rectus femoris-hamstring pair that spans the thigh. The other pair spans the leg. The latter pair is analogous to the gastrocnemius and part of the tibialis anterior that controls thigh motion. The assumption made here is that when the ankle strategy is in effect, $\theta_1 = \theta_2$, and hence, the tibialis, under this condition, shares part of the thigh load. When the pair of muscles is co-activated, the net effect is the production of a torque at that joint. Let the five torques be, respectively, elements of a vector M. Let $K_d$be a diagonal $5x5$ matrix whose diagonal elements are, respectively, the stiffnesses at the joints and across the joints. Similarly, let L_d be a diagonal matrix of joint and across joint viscosities.

$$M = [M_1, M_2, M_3, M_4, M_5]'.$$

where the symbol $'$ indicates the transpose of a vector or matrix. Let $d\Theta$ be defined as follows:

$$d\Theta = [d\theta_1, d\theta_2, d\theta_3]'.$$

Let E be the following matrix

$$E = \begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ 0 & -1 & 1 \\ -1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}. \quad (4)$$

The incremental work, dW, of these torques on the system is

$$dW = -(d\Theta)E'M.$$

Therefore

$$U = -E'M.$$

It also follows that

$$M = K_dE\Theta + L_dE\dot{\Theta}.$$

Therfore

$$U = -E'K_dE\Theta - E'L_dE\dot{\Theta}.$$

In order to include the effect of the gastrocnemius muscle, it is assumed that it responds to angle θ_2 only, and only when

$$\theta_2 \geq 0.$$

For analysis purposes, an artificial antagonist is added to the dynamics in order to add gastrocnemius action for negative range of θ_2 as well. The latter pair's effect is represented by K_5. The effect of co-activation of the five pairs can be represented as

$$-K\Theta - L\dot{\Theta}$$

$$K = \begin{bmatrix} K_1+K_2+K_4 & -K_2, & -K_4 \\ -K_2 & K_2+K_3+K_5 & -K_3 \\ -K_4 & -K_3 & K_3+K_4 \end{bmatrix} \quad (5)$$

This matrix allows one to do quantitative analysis by selecting the joint and biarticular stiffnesses K_i for

$$i = 1, 2, 3, 4, 5.$$

The velocity feedback matrix L is constructed by assuming that the diagonal $5x5$matrix L is a fraction of the diagonal matrix K. The fraction is taken to be 0.25 here.

REFERENCES

[1] K. Barin, "Evaluation of a generalized model of human postural dynamics and control in the sagittal plane," *Biological Cybernetics*, vol. 61, pp. 37–50, 1989.

[2] L. Ljung, *System Identification, Theory for the User*. Prentice Hall, 1999.

[3] R. Kearney and R. Kirsch, "System identification and neuromuscular modeling," in *Biomechanics and Neural Control of Posture and Movement* (J. Winters and P. Crago, eds.), pp. 134 – 147, Springer, 2000.

[4] F. Horak and A. Kuo, "Postural adaptation for altered envirnments, tasks, and intensions," in *Biomechanics and Neural Control of Posture and Movement* (J. Winters and P. Crago, eds.), pp. 267 – 281, Springer, New York, 2000.

[5] F. B. Horak and L. M. Nashner, "Central programming of postural movements: adaptation to altered support surface configurations," *J. Neurophisiology*, vol. 55, pp. 1369–81, 1986.

[6] Y.-C. Pai, "Induced limb collapse in a sudden slip during termination of sit to stand," *J. Biomechanics*, pp. 1377–1382, 1999.

[7] B. J. McFadyen, *Anticipatory control for safe fast trajectories during obstructed walking*. PhD thesis, University of Waterloo, Waterloo, Ont, 1988.

[8] Y.-C. Pai and K. Iqbal, "Simulated movement termination for balance recovery: can movement strategies be sought to maintain stability in the presence of slipping or forced sliding?," *J. Biomechanics.*, pp. 779 – 786, 1999.

[9] C.-Y. Pai, M. Rogers, J. patton, T. Cain, and T. Hanke, "Static versus dynamic predictions of protective stepping following waist-pull perturbations in ypung and older adults," *J. Biomechanics*, pp. 1111–1118, 1998.

[10] W. Berger, V. Dietz, and J. Quintern, "Corrective reactions to stumbling in man: Neuronal coordination of bilateral leg muscle activity during gait," *Journal of Physiology*, vol. 357, pp. 109–125, 1984.

[11] L. Nashner and G. McCollum, "The organization of human postural movements: a formal basis and experimental synthesis," *Behavior and Brain Sciences,*, vol. 8, pp. 135–172, 1985.

[12] C. Basdogan, Y. Hurmuzlu, and C. Carollo, "Presenting joint kinematics of human locomotion using phase plane portraits and Poincare maps," *Journal of Biomechanics*, vol. 27, no. 11, pp. 217–234, 1994.

[13] V. Brooks, *The neural basis of motor control*. Oxford University Press, 1986.

[14] I. Brown and G. Loeb, "A reductionist approach to creating and using neuromuscul oskeletal models," in *Biomechanics and Neural Control of Posture and Movement* (J. Winters and P. Crago, eds.), pp. 148 – 163, Springer, 2000.

[15] S. L. Delp and J. Loan, "A software system to develop and analyze models of musculo skeletal structures," *Computers in Biology and Medicine*, vol. 25, pp. 21–34, 1995.

[16] K. Kawato, M. Furukawa and R. Suzuki, "A hierarchical neural network model for control and learning of voluntary movement," *Biological Cybernetics*, vol. 57, pp. 169–185, 1987.

[17] J. Kim and H. Hemami, "Coordinated three-dimensional motion of the head and torso by dynamic neural networks," *IEEE Transactions on Systems, Man and Cybernetics, part B*, no. 5, pp. 653–666, 1998.

[18] H. Buchner, M. Hines, and H. Hemami, "A dynamic model for finger interphalangeal coordination," *Journal of Biomechanics*, no. 6, pp. 459–468, 1988.

[19] K. Barin and C. Stockwell, "Parameter estimation of a model of human postural control," in *Proc. of the Eighth Annual Conference of the Engineering in Medicine and Biology Society, Fort Worth, Texas, Nov 7 -10, 1986*, pp. 1571–1574, IEEE, 1986.

[20] K. Barin and J. Durrant, "Applied physiology of the vestibular system," in *The Ear: Comprehensive Otology* (R. Canalis and P. Lambert, eds.), pp. 113–140, Lippincott, Williams and Wilkins, 2000.

[21] J. Bay and H. Hemami, "Modeling of a neural pattern generator with coupled nonlinear oscillators," *IEEE Trans. Biomedical Engineering*, vol. 34, pp. 297– 306, 1987.

[22] H. Hemami and H. Ozbay, "Modeling and control of biological systems with multiple afferent and efferent transmission delays," *J. Robotic Systems*, vol. 17, no. 11, pp. 609–622, 2000.

[23] B. Kawarizadeh, "Microprocessor control of an inverted pendulum with a movable base," Master's thesis, The Ohio State University, Columbus, Ohio, 1982.

[24] H. Hemami, "Modeling, control, and simulation of human movement," *CRC Critical Reviews in Biomedical Engineering*, vol. 13, no. 1, pp. 1–34, 1985.

[25] L. Nashner, "Fixed patterns of rapid postural responses among leg muscles during stance," *Experimental Brain Research*, vol. 30, pp. 13–24, 1977.

[26] L. Nashner, "Balance adjustments of humans perturbed while walking," *Journal of Neurophysiology*, vol. 44, pp. 650–664, 1980.

[27] P.F.Tang, M. Woollacott, and R. Chong, "Control of reactive balance adjustments in perturbed human walking: roles of proximal and distal postural muscle activity," *Experimental Brain Research*, vol. 119, pp. 141–152, 1998.

[28] R. Ferber, L. Osternig, M. L. Woollacott, N. Wasielewsk, and J. Lee, "Reactive balance adjustments to unexpected perturba tions during human walking," *Gait and Posture*, vol. 16, pp. 238 – 248, 2002.

[29] K. Iqbal, B. D. Clymer, and H. Hemami, "Squatting maneuver of a three-link nine-muscle biped model in the sagittal plane," *J. of Automatic Control, University of Belgrade*, no. 1, pp. 113 –129, 1997.

[30] E. Henneman, "Motor functions of the brain stem and basal ganglia," in *Medical Physiology* (V. Mountcastle, ed.), vol. 1, pp. 787–812, The C.V. Mosby Company, 1980.

[31] J.-P. Ewert, "Command neurons and command systems," in *Handbook of Brain Theory and Neural Networks* (M. Arbib, ed.), pp. 215 – 220, M.I.T. Press, Cambridge, 1995.

[32] G. Carpenter and S. Grossberg, "The ART of adaptive pattern recognition by a self- organizing neural network," *Computer*, pp. 77 – 88, 1988.

[33] B. Dariush, M. Parnianpour, and H. Hemami, "Stability and control strategy of a multi-link musculoskeletal model with applications in FES," *IEEE Trans. Biomedical Eng.*, no. 1, pp. 3 – 14, 1998.

[34] S. L. Delp and W. Maloney, "Effect of hip center location on the moment-generating capacity of the muscles," *Journal of Biomechanics*, vol. 26, no. 4/5, pp. 485–499, 1993.

[35] D. Loram and M. Lakie, "Direct measurement of human ankle stiffness during quiet standing: the intrinsic mechanical stiffness is insufficient for stability," *Journal of Physiology*, vol. 545, pp. 1041–1053, 2002.

[36] D. Winter, A. Patla, F. Prince, M. Ishac, and K.Gielo-Perczak, "Stiffness control of balance in quiet standing," *J. Neurophysiol*, vol. 80, pp. 1211–1221, 1998.

[37] D. Winter, A. Patla, S. Rietdyk, and M. Ishac, "Ankle muscle stiffness in the control of balance during quiet standing," *J. Neurophysiology*, vol. 85, pp. 2630–2633, 2001.

[38] P.G.Morasso and M.Schieppati, "Can muscle stiffness alone stabilize upright standing?," *J Neurophysiol.*, vol. 82, pp. 1622–1626, 1999.

[39] G. McCollum, F. Horak, and L. Nashner, "Parsimony in neural calculations for postural movements," in *Cerebellar Functions* (B. et al., ed.), pp. 52 – 66, Sprringer-Verlag, 1984.

[40] K. Barin, *An Experimental Study of Multi-link Models of Human Postural Dynamics and Control*. PhD thesis, The Ohio State University, 1983.

PLACE
PHOTO
HERE

Hooshang hemami H. Hemami received his B.S. in electrical and mechanical engineering from the University of Teheran. He studied wit Y. W. Lee and A. G. Bose at the Massachusetts Institute of Technology and with R. L. Cosgriff at The Ohio State University. He is currently professor of electrical and computer engineering and biomedical engineering at The Ohio State University.

His research interests are control, animation and dynamics of human, movement, bio-robotic systems, robots and marionettes. He is a Life - Fellow of IEEE.

PLACE
PHOTO
HERE

Kamran Barin, Ph.D. Kamran Barin received his Bachelor's degree in Electrical Engineering from Sharif University of Technology in Tehran, Iran, his Master's and Ph.D. degrees also in Electrical Engineering from The Ohio State University in Columbus, Ohio. He is currently the Director of Balance Disorders Clinic and Assistant Professor in the Department of Otolaryngology-Head and Neck Surgery at the Ohio State University Medical Center. He also holds joint appointments in the Department of Speech and Hearing Science and Biomedical Engineering Program.

Dr. Barin's current research interests include human postural control, vestibular function testing, and diagnosis and rehabilitation of dizziness.

PLACE
PHOTO
HERE

Yi-Chung (Clive) Pai, Ph.D., M.P.T. Dr. Pai received his B.S. (1978) and M.S. (1981) degrees from Shanghai Institute of Physical Education. In 1987, he was awarded the Ph.D. in Biomechanics from the University of Iowa. While on the faculty of Northwestern University, as Assistant Professor and then Research Associate Professor, he completed his Master's degree in Physical Therapy (1996). Dr. Pai joined the University of Illinois at Chicago (UIC) as Associate Professor of Physical Therapy in 2000; he became Professor in 2004. He currently has joint appointments in the Departments of Bioengineering, Movement Sciences, and Mechanical and Industrial Engineering. He is also the Director of Clinical Gait and Movement Analysis Laboratory at UIC.

Dr. Pai is interested in mathematical models of dynamic stability control during human locomotion. His research also includes mechanisms of motor adaptation and learning, with applications for retarding rapid progression of knee osteoarthritis and reducing fall incidence among older adults