Problem 1 (8 parts, 6 points each)

1) A specific intensity I_0 is produced by a source at the origin and propagates in free space. At a distance r from the origin, this specific intensity will be

(a) $\frac{I_0}{r^2}$ (b) $\frac{I_0}{r}$ (c) I_0 (d) rI_0 (e) $r^2 I_0$

2) The first order iterative solution of the radiative transfer equation is used to predict the back-scattering coefficient σ_{hh} of a layer containing spherical Rayleigh scatterers. As the optical thickness of the layer increases

(a) the backscattering coefficient will decrease significantly
(b) the backscattering coefficient will remain approximately the same
(c) the backscattering coefficient will increase significantly
(d) the backscattering coefficient may increase or may remain approximately the same
(e) none of the above

3) For a layer existing between $\tau = 0$ and $\tau = \tau_0 > 0$, the boundary condition $I_d(\tau_0, \mu) = 0$ for $-1 < \mu < 0$ can be interpreted as

(a) there is no reverse going diffuse intensity entering the back boundary of the layer
(b) there is no forward going diffuse intensity entering the back boundary of the layer
(c) there is no reverse going diffuse intensity entering the front boundary of the layer
(d) there is no forward going diffuse intensity entering the front boundary of the layer
(e) none of the above

4) In continuous random medium theory, the radar cross section per unit volume of a random medium measured at frequency f_0 is σ_0. The radar cross section per unit volume at frequency $2f_0$ is then

(a) $2\sigma_0$
(b) $4\sigma_0$
(c) $8\sigma_0$
(d) $16\sigma_0$
(e) insufficient information to answer this question

5) Which of the following statements is true regarding models of continuous random media?

(a) A medium modeled with a Gaussian correlation function will contain refractive index fluctuations in a wide range of length scales

(b) A medium modeled with a exponential correlation function will contain refractive index fluctuations in only a narrow range of length scales

(c) A medium modeled with a Komolgorov spectrum will contain refractive index fluctuations in a wide range of length scales

(d) the amount of scattering observed will decrease as the variance of the refractive index fluctuations increases

(e) none of the above

6) The average coherent reflection coefficient of a Gaussian random process rough surface is 0.15 at incidence angle $\theta_i = 30°$. Using the physical optics approximation, as θ_i is increased to angles greater than $30°$

(a) the coherent reflection coefficient will decrease

(b) the coherent reflection coefficient will increase

(c) the coherent reflection coefficient will remain the same

(d) the coherent reflection coefficient may increase, decrease, or remain the same depending on the surface rms height

(e) the coherent reflection coefficient may increase, decrease, or remain the same depending on the surface correlation function

7) The first order SPM predicts a Bragg scattering response where the scattered power at a particular angle is directly proportional to the surface power spectral density at the Bragg wavenumber. This Bragg wavenumber

(a) depends upon the propagation angles of the incident and scattered waves

(b) depends upon the polarization of the incident and scattered waves

(c) does not depend upon the electromagnetic frequency

(d) depends upon the surface statistics

(e) none of the above

8) When computing the physical optics prediction of the incoherent radar cross section per unit area of a Gaussian random process surface,

(a) it is required to assume that the surface has a Gaussian correlation function

(b) it is required to assume that the surface rms height is very large compared to the electromagnetic wavelength

(c) it is required to assume that the surface correlation length is small compared to the electromagnetic wavelength

(d) it is required to assume that the surface slope variance is large

(e) none of the above

Problem 2 (4 parts, 52 points).

Choose an appopriate theory for evaluating each of the following requests.

(a) a radar system operating at 14 GHz measures VV backscattering at $\theta_i = 60°$ from a PEC surface with a power law spectrum:

$$W(k_x, k_y) = \frac{1}{10^4}\left(\frac{1}{k_\rho^4}\right) \text{ for } k_\rho > 100 \text{ rads/m}$$

$$W(k_x, k_y) = 0 \text{ otherwise.}$$

$$\text{where } k_\rho = \sqrt{k_x^2 + k_y^2}.$$

These definitions result in a surface rms height of 0.3 mm. Find the radar cross section per unit area in this problem.

This surface obviously has a small height compared to the EM wavelength $\lambda = 2.14$ cm. Therefore use the SPM:

$$\sigma_{VV} = 4\pi k^4 (1 + (\sin\theta_i)^2)^2 W(2k\sin\theta_i, 0) = 4\pi(293.215)^4\left(1 + \frac{3}{4}\right)^2 W(507.86, 0)$$

$$\sigma_{VV} = 4.276 \times 10^{-4} = -33.69 \text{ dB}$$

(b) a radar system operating at 2 GHz measures HH backscattering at $\theta_i = 15°$ from a PEC Gaussian random process surface with a Gaussian correlation function. The surface has rms height 2 m, correlation length 10 m, and slope variance $s^2 = 0.08$. Find the radar cross section per unit area in this problem.

In this case, the rms height is large compared to $\lambda = 15$ cm, so we must use PO, correlation length is long enough for this to be reasonable. Since it is hard to evaluate the true PO series, it seems more reasonable to use GO here, note

$$|k_{dz}h| = 2k\cos\theta_i h = \frac{4\pi}{0.15}(2)\cos 15° = 161.84 \text{ is >>1 clearly this is the GO limit.}$$

Now

$$\sigma_{hh} = \frac{k^2}{\pi}\left[\frac{(1+\cos\theta_i\cos\theta_s)\cos\phi_s - \sin\theta_i\sin\theta_s}{\cos\theta_i + \cos\theta_s}\right]^2 D_I = \frac{k^2}{\pi}\left(\frac{1}{\cos\theta_i}\right)^2 \frac{2\pi}{k_{dz}^2 s^2}\exp\left(-\frac{k_{d\rho}^2}{2k_{dz}^2 s^2}\right)$$

$$\text{Note } k_{dz}^2 s^2 = \left(\frac{4\pi}{0.15}\cos 15°\right)^2 0.08 = 523.86 \text{ (rads/m)^2.}$$

$$k_{d\rho} = 2k\sin\theta_i = 21.683 \text{ rads/m. Thus}$$

$$\sigma_{hh} = \frac{2(41.89)^2}{(\cos 15°)^2}\left(\frac{1}{523.86}\right)\exp(-0.4487) = 4.58 = 6.61 \text{ dB.}$$

(c) a radar system operating at 4 GHz measures the backscattering coefficient of a layer of Rayleigh scatterers at $\theta_i = 45°$. The scatterers have albedo 0.3 and the optical thickness of the layer is $\tau_0 = 0.3$. Find the backscattering coefficient of the layer.

Use the first order iterative approximation since the optical thickness is small, even though the albedo is large:

$$\sigma_{HH} = \frac{3}{4}\left(\frac{\sigma_S}{\sigma_t}\right)\cos\theta_i(1 - e^{-2\tau_0 \sec\theta_i}) = \frac{3}{4}(0.3)\left(\frac{1}{\sqrt{2}}\right)(1 - e^{-2(0.3)\sqrt{2}}) = 0.091 = -10.41 \text{ dB.}$$

(d) a radar system operating at 2 GHz measures the radar cross section per unit volume of a continuous random medium described by the Booker-Gordon formula with $\langle n_1^2 \rangle = 10^{-4}$ and $l = 2$ m. Find the radar cross section per unit volume in this problem.

Use the continuous medium theory:

$$\sigma = 2\pi k^4 (\sin\chi)^2 \Psi_n(k_s)$$

For backscattering, $k_s = 2k\sin\frac{\theta}{2} = 2k = \frac{4\pi}{0.15} = 83.776$ rads/m. Also $\chi = \frac{\pi}{2}$. We need

$$\Psi_n(k_s) = \frac{\langle n_1^2 \rangle l^3}{\pi^2}\left[\frac{1}{1 + (k_s l)^2}\right]^2 = \frac{10^{-4}(8)}{\pi^2}\left(\frac{1}{1 + ([2]83.776)^2}\right)^2 = 1.028 \times 10^{-13} \text{ meters cubed.}$$

Finally

$$\sigma = 2\pi\left(\frac{2\pi}{0.15}\right)^4 (1.028 \times 10^{-13}) = 1.99 \times 10^{-6} = -57 \text{ dB.}$$