

PROBLEM SET 2
ECE 816

Assigned: April 14th
Quiz: April 23rd in class

Instructor: Joel Johnson

Project Topic (Turn in by April 23rd):

Turn in a description of the topic for your course computer project. Discuss the general problem you will consider (discrete, continuous, or rough surface random medium) and the numerical method you will use to obtain exact results (DDA, method of moments, FDTD, other methods.) Also describe a particular physical medium that your chosen problem could approximately model (snow, sea ice, ocean surface, atmosphere, etc.) If you are not sure, please discuss with the instructor before hand so that you can complete this assignment.

Problem 1

(a) Derive the second, third, and fourth rows of the Stokes' matrix using the coordinate system defined in the handout "Phase matrix for Rayleigh scattering." Verify that your answers match those on page 3 of the handout.

(b) Derive the phase matrix for Rayleigh scattering using this coordinate system. Verify that your answers match those on page 4 of the handout.

(c) Simplify the Rayleigh phase matrix for backscattering. Which terms remain? Be sure to check the incident and scattering directions to verify you are defining backscattering correctly.

(d) Simplify the Rayleigh phase matrix for "in plane" scattering, i.e. $\phi = \phi'$. Which terms remain?

Problem 2

Consider 3 GHz backscattering from rain. A radar with a 2 m diameter parabolic transmitting antenna illuminates a thick layer of rain at a distance of 2 km. Assume the antenna is directed at the rain region, and has an aperture efficiency of 50% and α illumination factor of 1.5. Polarization effects can be neglected and the "narrow beam" approximation can be used.

(a) Calculate and plot the average values of $\rho \langle \sigma_b \rangle$ and $\rho \langle \sigma_t \rangle$ versus rain rate from 1 to 50 mm/hr assuming Rayleigh scattering and the Marshall-Palmer drop size distribution. A relative permittivity of $77+i24$ can be used for water at this frequency.

(b) Plot the ratio of received to transmitted power for this radar as a function of rain rate from 1 to 50 mm/hr.

(c) Discuss the use of microwave radar to measure scattering from rain. How could this system be improved? Discuss any limitations of the Rayleigh approximation also for studying microwave backscattering from rain.

Useful integral:
$$\int_u^{\infty} \frac{e^{-px}}{x^2} dx = p Ei(-pu) + \frac{e^{-pu}}{u}, \text{ where } Ei(x) \text{ is related to Matlab's "expint".}$$

Problem 3

Consider the same radar system as in Problem 2, but now the radar illuminates a 50 m thick random layer containing a 1% volume fraction of spheres with radius 0.5 mm and relative permittivity $\epsilon = 3 + i0.1$. The layer is 2 km away and illuminated at normal incidence; since the layer is relatively thin, variations in R in the layer can be neglected. All of the spheres in this medium are moving with a fluctuating velocity along the line of sight of the radar. The velocity is a Gaussian random variable with mean 10 and standard deviation 2 m/s.

- (a) Find the temporal correlation function for voltages output from the receiver. Interpret your results. Assume c (a calibration factor) = 1.
- (b) Find the temporal frequency spectrum for voltages output from the receiver. Interpret your result.
- (c) How would your part (a) and (b) answers change if the mean velocity increased to 20 m/s? Interpret these changes.
- (d) How would your part (a) and (b) answers change if the standard deviation of velocity fluctuations increased to 5 m/s. Interpret these changes.

Problem 4

In this problem we will perform a Monte Carlo simulation to estimate the pdf of backscattered fields obtained under the independent scattering approximation. Consider 1 GHz backscattering from a random medium containing small spheres, each with radius 1 mm and relative permittivity $\epsilon = 3 + i0.1$. The spheres are distributed randomly (i.e. positions are independent and have a uniform distribution) inside an origin centered, cubical volume 15 cm on each side. A plane wave propagating in the x direction illuminates this volume. We will perform the Monte Carlo simulation by generating a realization of the random medium, then adding up the scattering from each sphere (including phase information) assuming independent scattering to obtain a sample of the scattered field. Repeating this process for many realizations we can generate a histogram of the results which should approximate the pdf of scattered fields as the number of realizations considered becomes very large.

There are several possible ways to generate a realization of the random medium, but for low fractional volumes it is reasonable to neglect the possibility that particles overlap. Then each particle position simply can be taken as a uniformly distributed quantity from -7.5 cm to 7.5 cm in x, y, and z. The number of particles to place is determined by the fractional volume.

- (a) Generate a histogram for real and imaginary parts of backscattered scattering amplitudes with sphere fractional volumes of 0.01% and 0.1%. Interpret your results and discuss any analytical curves you think these histograms resemble. Also discuss the mean and variances of your histograms. Include a sufficient number of realizations to obtain a relatively smooth curve.
- (b) Repeat part (a) but generate a histogram for backscattering cross sections.
- (c) Discuss attenuation effects if the first order multiple scattering approximation were used. Would you expect significant changes in this problem? What changes in histograms might occur when attenuation becomes important?