

PROBLEM SET 3
ECE 816

Assigned: April 23rd
Quiz: None

Instructor: Joel Johnson

ANNOUNCEMENT: The course midterm examination will take place in class on Tuesday, May 5th. The exam will be open book and notes, and will cover the material up to 4/30.

Project Update (Turn in in class, May 7th)

Turn in a document describing progress on your course project to date. By May 7th, there are only four weeks remaining, so you should by now have implemented and be starting to test the numerical code you will use. If you are having problems please discuss with the instructor.

Problem 1

In section 7-2 of the text, Ishimaru finds the relationship between incident, reflected, and transmitted specific intensities at a planar dielectric interface. However this derivation neglects some polarization effects. A specific intensity modified Stokes vector can be defined as

$\vec{I} = [I_v \ I_h \ U \ V]^t$, where the t superscript indicates transpose, I_v and I_h are specific intensities in vertical and horizontal polarizations respectively, U and V measure correlations between vertical and horizontal specific intensities analogous to the standard modified Stokes vector, and all of these quantities have units of watts per meter squared per steradian per Hertz.

(a) Define vertical and horizontal polarizations similar to those of the handout “Phase matrix for Rayleigh scattering” for the incident, reflected, and transmitted waves. Choose your coordinate system so that the \hat{z} direction is normal to the interface and the incident wave propagates in the $\hat{x} \sin \theta_i - \hat{z} \cos \theta_i$ direction, with $0^\circ < \theta_i < 90^\circ$. Note each field will have its own definition for vertical and horizontal polarizations due to differing propagation directions.

(b) If a boundary condition matrix for the reflected specific intensity is defined through $\vec{I}_r = [R] \cdot \vec{I}_i$, find the 4x4 matrix $[R]$. This will require use of the field solution for reflection from a planar dielectric boundary (not necessary to re-derive, a reference can be used), and specific intensities can be related to field Poynting vectors or correlations.

(c) If a boundary condition matrix for the transmitted specific intensity is defined through $\vec{I}_t = [T] \cdot \vec{I}_i$, find the 4x4 matrix $[T]$. Assume the transmit medium is lossless. Again use of the field solution for reflection from a planar dielectric interface is required, but follow Ishimaru’s derivation to include changes in differential solid angles.

Problem 2

Consider radiative transfer in a non-scattering layer, i.e. $p(\hat{s}, \hat{s}') = 0$, under plane wave incidence. Use the coordinate system shown in Figure 8-2, p. 170, and do not assume that the extinction properties of the medium are constant through the layer. Use the Rayleigh scattering approximation to estimate extinction cross sections.

(a) If regions one through three all are free space so that interface reflections can be neglected, find the ratio of the specific intensity at $z = d$ to that at $z = 0$.

(b) If the layer is a 20 km thick rain layer with a rain rate which increases linearly from 0 mm/hr at $z = 0$ to 20 mm/hr at $z = 20$ km, find the ratio of the specific intensity at $z = 20$ km to that at $z = 0$ for a 5 GHz plane wave incident at 45 degrees. Use $71 + i30$ as the permittivity of water. (This could perhaps model attenuation through an extremely thick rain cloud for satellite systems if we neglect scattering).

(c) Plot the attenuation through the cloud as a function of the incidence angle from 0 to 89 degrees, using the same parameters as in part (b). Interpret your result.

Problem 3

Consider 3 GHz scattering from and propagation in a 100 m thick layer containing a 1% fractional volume of 0.5 mm radius spheres with $\epsilon = 3 + i0.1$. The incident intensity is a plane wave at $\theta_i = 45^\circ$, $\phi_i = 0^\circ$.

(a) Find the ratio of the specific intensity at $z = 100$ m to that at $z = 0$ m using the first order iterative solution. Plot your result for observation angle θ from 0 to 90 degrees with $\phi = 0^\circ$ assuming a horizontally polarized incident field. Discuss the difference between this result and results when scattering was neglected.

(b) Find the backscattering coefficients in VV, HH, VH, and HV polarizations (defined in the handout).

(c) Plot the backscattering coefficients as a function of incidence angle from 0 to 89 degrees. Discuss the angular dependence of backscattering from a "volume scattering layer" under the first order iterative solution.

Problem 4

Derive equation (8-13) p. 172 to include the effects of a specularly reflecting interface in the first order iterative solution assuming plane wave incidence. Note the boundary condition to be

applied for $\frac{\pi}{2} < \theta < \pi$ is $I(\tau_0, \theta, \phi) = |R(\pi - \theta)|^2 I(\tau_0, \pi - \theta, \phi)$ for vertically or horizontally

polarized intensities, where R is the appropriate field reflection coefficient. Do you obtain terms not presented by Ishimaru? If so discuss the physical meaning of these terms.