

PROBLEM SET 4  
ECE 816

Assigned: May 7th  
Quiz: May 19th in class

Instructor: Joel Johnson

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**Computer project initial result (turn in in class, Thursday, May 21st)**

Turn in a document containing plots of an initial result from your course project numerical code. Briefly describe the problem considered, the code used, and the meaning of the results. Also describe your current plans for completing the project and comparing with any approximate theories. Your course computer project report is due on the Friday two weeks after this assignment.

**Problem 1**

(a) Derive an integral equation equivalent to the RT equation (11-20a) with boundary conditions (11-20b). This can be accomplished through use of the general linear first order differential equation solution. Compare your answer to equation (11-37).

(b) Derive equations (11-38) and (11-39) from this integral equation by substituting in the discretized angle solution of the RT equation.

(c) Explain the meanings of the terms (for example, the  $\alpha$ 's,  $\beta$ 's,  $\lambda$ 's, and  $C$ 's) in your equation, and describe how they are obtained in the discretized angle solution.

**Problem 2**

Consider the discretized angle solution for isotropic scattering using  $N=1$ , described in equations (11-43) through (11-52). Polarization effects can be neglected throughout the problem. Note there are typos in equation (11-52) as can be verified through comparison with (11-39).

(a) Plot the bistatic scattering coefficient versus scattering angle for a medium with  $W = 0.1$  and  $\tau_0 = 0.1$ . Compare with the first order iterative solution.

(b) Plot the bistatic scattering coefficient versus scattering angle for a medium with  $W = 0.6$  and  $\tau_0 = 0.1$ . Compare with the first order iterative solution.

(c) Repeat parts (a) and (b) using  $\tau_0 = 4$ .

(d) Interpret your part (a)-(c) results. When is a significant difference between the discretized angle and first order solutions observed?

Problem 3

Consider bistatic scattering from atmospheric inhomogeneities at 5 GHz (the basis for “tropospheric scatter” communications systems). The transmitter and receiver are separated by 100 km, and both antennas are directed at 5 degrees above the horizon. The transmit and receive antennas are identical, and are both 6 m diameter parabolic antennas with aperture efficiencies of 50% and  $\alpha$  illumination factors of 1.5. Earth curvature, polarization and extinction effects can be neglected, and the narrow beam approximation can be used.

(a) Find the ensemble average power received relative to the power transmitted if the atmosphere is modeled using the Booker-Gordon formula with correlation length 50 m and index of refraction variance  $10^{-6}$ . Compare this power received to the power received if the antennas were in the line of sight in a non-random medium and neglecting any Earth reflections.

(b) Repeat part (a) using a Gaussian correlation function model with the same parameters.

(c) Repeat part (a) using the von Karman spectrum with  $C_n = 10^{-7} \text{ m}^{-1/3}$ ,  $L_0 = 50 \text{ m}$  and  $l_0 = 1 \text{ cm}$ .

(d) Interpret the differences obtained using these spectra. What properties of the medium are different with these spectra? Also discuss issues associated with a tropospheric scatter communications system based on your results.

Problem 4

In this problem we will compare the continuous medium and discrete scatterer theories for radar cross section per unit volume. We will apply the Born approximation in both cases, so any differences obtained should primarily be due to the way the random medium is described.

(a) Using the Born approximation for scattering from a single lossless sphere (equation 2-40), find the bistatic cross section per unit volume for a medium containing  $\rho$  spheres per unit volume using independent scattering theory.

(b) Find the variance of the index of refraction corresponding to a medium containing  $\rho$  spheres per unit volume of radius  $a$ , all of which have a near unity relative permittivity. Equations (16-3) through (16-6) may clarify the meaning of this quantity and the average value of the relative permittivity can be taken to be approximately unity.

(c) Using continuous random medium theory, find the bistatic cross section per unit volume using the Booker-Gordon formula with the index of refraction variance found in part (b) and using  $a$  as the correlation length of the medium. How do your results compare to part (a)? Interpret any differences. A few plots may help clarify the differences.

(d) By comparing your part (a) answer with continuous random medium theory, determine the spectral density of a medium containing  $\rho$  spheres per unit volume of radius  $a$  and relative permittivity  $\epsilon$ . Note since both theories are found using the Born approximation, this should be the correct answer for a medium composed of a low fractional volume of discrete spheres. Discuss your results. An inverse Fourier transform of this spectral density should provide the correlation function of the medium.