PROBLEM SET 5
ECE 816

Assigned: May 19th
Quiz: May 28th (problems 1 and 2) in class Instructor: Joel Johnson

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Problem 1

In this problem we will derive the first order SPM solution for scattering of a vertically polarized incident field from a PEC surface. The vertically polarized incident field is written as $\bar{E}_i = (\hat{x}\cos\theta_i + \hat{z}\sin\theta_i)e^{i\beta x}e^{-i\gamma z}$, with $\beta = k\sin\theta_i$ and $\gamma = k\cos\theta_i$.

(a) Postulate a form for the reflected field and show that the sum of the incident and reflected fields reduces to the first two equations on page 465 of Ishimaru.

(b) Using the expansions given in the third through 5th equations on page 465 of Ishimaru, derive $A_{mn}^{(0)}$, $B_{mn}^{(0)}$, and $C_{mn}^{(0)}$ (zeroth order).

(c) Using the expansions given in the third through 5th equations on page 465 of Ishimaru, derive $A_{mn}^{(1)}$, $B_{mn}^{(1)}$, and $C_{mn}^{(1)}$ (first order). Do you obtain Ishimaru's results?

(d) Find the cross sections per unit area σ_{hv} and σ_{vv} following the derivation in the notes. Do you obtain Ishimaru's results?

Problem 2

In this problem we will consider 100 MHz scattering from the ocean surface (model as a perfect conductor) using the first order SPM and the empirical ocean surface spectrum on page 475 of Ishimaru.

(a) Plot the ocean surface spectrum versus $\sqrt{p^2+q^2}$ for wind speed 5 m/s. Use a log-log axis. Interpret any features in your plot. How would this plot change if the wind speed is increased or decreased? Note $g = 9.81$ m/sec^2.

(b) For an incidence angle of $50°$, plot the bistatic cross section per unit area for $\phi_s = 0°$ (in-plane scattering) and for $\theta_s = -90°$ to $90°$. Consider all four polarized cross sections, i.e. hh, hv, vh, and vv. Interpret any features in this plot. What value of $\sqrt{p^2+q^2}$ is important for $\theta_s = 49°$? Discuss.

(c) Plot the backscattering cross section per unit area for incidence angles from $1°$ to $89°$, again for hh, hv, vh, and vv polarizations. What values of $\sqrt{p^2+q^2}$ are important for $\theta_i = 1°$, $\theta_i = 45°$, and $\theta_i = 80°$? Discuss.

(d) Discuss the applicability of first order SPM to the ocean surface at 100 MHz. An empirical formula for the height variance of the ocean is $h^2 = (0.0054U^2)^2$ where U is the windspeed in m/s. For what wind speeds does first order SPM become questionable?

Problem 3

In this problem we will consider the physical optics approximation for scattering of a vertically polarized incident field from a PEC surface. The vertically polarized incident field is written as $\bar{E}_i = -(\hat{x}\cos\theta_i + \hat{z}\sin\theta_i)e^{ik_{ix}x}e^{-ik_{iz}z}$ with $k_{ix} = k\sin\theta_i$ and $k_{iz} = k\cos\theta_i$.

(a) Find the physical optics induced currents on the surface $z = \zeta(x, y)$.

(b) Find the vector potential radiated by these currents in the far field. Use an integration by parts to get rid of any surface derivative terms.

(c) Find the horizontally and vertically polarized scattered electric fields in the far field. How do these fields relate to those found for a horizontally polarized incident field?

(d) Find the bistatic cross sections per unit area σ_{hv} and σ_{vv}. How do these cross sections relate to those found for a horizontally polarized incident field? If backscattering is considered?

Problem 4

In this problem we will consider 1 GHz scattering under the physical optics approximation from a PEC rough surface with a Gaussian correlation function.

(a) For a surface with $h = 20$ cm and $l = 2$ m and for an incidence angle of $50°$, plot the bistatic cross section per unit area $\sigma_{\alpha\beta}$ for $\phi_s = 0°$ (in-plane scattering) and for $\theta_s = -90°$ to $90°$. Consider all four polarized cross sections, i.e. hh, hv, vh, and vv. Interpret any features in this plot.

(b) Repeat part (a) but for $h = 1$ cm and $l = 10$ cm. Compare your cross sections with those from first order perturbation theory. Discuss any differences. Note for a Gaussian surface,

$$W(p, q) = \frac{h^2 l^2}{\pi} e^{-(p^2 + q^2)\frac{l^2}{4}}.$$

(c) Repeat part (a) for $h = 3$ m and $l = 30$ m. In this case extreme care must be exercised in adding up the PO series and thousands or tens of thousands of terms may be required. It may help computationally to evaluate each term in the series as $\exp(A)$, where A is the logarithm of the term. Compare your cross sections with those from geometrical optics. Discuss your results. Note for a Gaussian surface, the slope variance $s^2 = \frac{2h^2}{l^2}$.

(d) Plot geometrical optics results as in part (a) but for h values of 2, 4, and 6 m with $l = 30$ m. Interpret your results. Discuss the use of physical and geometrical optics for computing scattering from PEC rough surfaces based on your part (a) through (d) results.