Quiz #3
ECE 816 Spring Quarter 2007
April 30th, 2007

This problem concerns scalar scattering, i.e. polarization effects can be neglected throughout.

A halfspace random medium exists for $z > 0$. The background permittivity for $z < 0$ is that of free space, while for $z > 0$ the background medium has $\varepsilon > \varepsilon_0$ so that interface reflections are important.

An incident specific intensity $\tilde{I}_{inc}(z, \theta, \phi)$ impinges upon the halfspace from the free space region; the incident intensity transmitted into the halfspace at the boundary is written as $I_{inc}(z=0+, \theta, \phi)$, where 0+ implies a point at the boundary but within the halfspace. Note that $I_{inc}(z=0+, \theta, \phi)$ already includes any necessary transmission coefficients.

For specific intensities within the random medium, interface reflections are described in terms of a reflection coefficient $r(\theta)$ that couples a specific intensity incident at angle θ into a reflected specific intensity propagating at angle $\pi - \theta$.

Other properties of the medium include the particle number density ρ and the particle extinction cross section σ_t.

(a) Write appropriate boundary conditions for the plus and minus going specific intensities within the halfspace. This process might be simplified by thinking of the halfspace as a very thick layer.

Our previous boundary conditions neglecting interface reflections were $I^+(z=0,\theta,\phi) = I_{inc}(z=0+, \theta, \phi)$ and $I^-(z=d,\theta,\phi) = 0$, note with no interface reflections I_{inc} and $\tilde{I}_{inc}$ are identical, but with interface reflections we need to use I_{inc}. First, for the minus going intensity nothing has changed except we have a thick layer, so appropriate equation is $I^-(z=\infty,\theta,\phi) = 0$. At interface, we have to include reflections now, so we have: $I^+(z=0,\theta,\phi) = I_{inc}(z=0+, \theta, \phi) + r(\pi-\theta)I^-(z=0,\pi-\theta,\phi)$.

(b) Determine the zeroth order iterative solution of the radiative transfer equation for specific intensities within the halfspace.

Both plus and minus going intensities have the form $c(\theta,\phi)e^{-\tau\sec\theta}$ at zeroth order, where $\tau(z) = \int_0^z \rho\sigma_t dz$. Applying the boundary conditions, we find that the minus going intensity is zero at zeroth order, therefore there is no change in the plus going intensity either: $I^-(z,\theta,\phi) = 0$, $I^+(z,\theta,\phi) = I_{inc}(z=0+, \theta, \phi)e^{-\tau\sec\theta}$.