QUIZ 2

ECE 816: Spring quarter 2009

A 5 GHz radar system has an antenna of gain 40 dBi; its half power beamwidths are 0.03 radians in both the elevation and azimuth planes. This radar observes a random medium that occupies all space between ranges 2000 and 2010 m. Since the radar has a narrow antenna pattern, the random medium can be modeled as a layer existing between two planes (range 2000 m and range 2010 m) along the radar's line of sight.

The random medium is known to have a backscattering cross section per unit volume of $\rho\langle\sigma_b\rangle = 7 \times 10^{-5}$ per meter.

a) It is known that the random medium moves toward the radar at a constant velocity. The backscattered fields show a Doppler shift of 100 Hz. Find the velocity of the layer.

> Basic Doppler shift equation: $\frac{\omega}{\omega_o} = -\left((\hat{i} - \hat{O}) \cdot \frac{\overline{U}}{c}\right) = -\hat{i} \cdot 2\frac{\overline{U}}{c}$ for backscattering. Writing $\overline{U} = (-\hat{i})U$, and solving for U, yields U = 3. $\mathbf{\overline{U} = -\hat{i} \cdot 3\ m/s}$

b) In a different measurement, the velocity of the individual particles ($\overline{V} = \hat{x}V_x + \hat{y}V_y + \hat{z}V_z$) in the random medium is known to have components (i.e. V_x, V_y, V_z) that are independent zero mean Gaussian random variables with a standard deviation 0.15 m/s. Write the temporal correlation function for voltages output from the receiver as a function of the time separation between measurements τ. Assume c (a calibration factor) = 1. (You may also assume $e^{-2\gamma} \approx 1$ and that the "narrow-beam" approximation is applicable.)

> We have $c = 1$, $G_t(\hat{i}) = G_r(\hat{O}) = 40$ dBi, $\theta_1 = \phi_1 = 0.03$ $R = 2$ km, $d = 10$ m
> Also $\sigma_v = 0.15$ m/s
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> Using the "narrow beam" approximation for backscattering and assuming that $1/R^2$ can be approximated as a constant over the layer for simplicity, we have
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> $$B_v(\tau) = (2.855 \times 10^{-4})\frac{\lambda^2 G_t^2(\hat{i})}{R^2}\theta_1\phi_1[\rho\langle\sigma_{bi}(-\hat{i}, \hat{i}, \tau)\rangle]d$$
>
> The velocity of particles is $\overline{V} = \overline{V}_f$, $|\overline{V}_f|$ is a Gaussian r.v. with the standard deviation $\sigma_v = 0.15$
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> $$\sigma_{bi}(-\hat{i}, \hat{i}, \tau) = \sigma_{bi}(-\hat{i}, \hat{i})e^{-(2k)^2\sigma_v^2\tau^2/2}$$
>
> Final answer: $\mathbf{B_v}(\tau) = (\mathbf{1.6188 \times 10^{-11}})\mathbf{e^{-50\pi^2\tau^2}} = (1.6188 \times 10^{-11})e^{-(\tau/0.045)^2}$ Correlation time is 45 msec.