Consider a field independently scattered from three particles at $\bar{r}_1, \bar{r}_2, \& \bar{r}_3$

Particle positions are random & independent.

$$\bar{E} = \frac{e^{ikR}}{R}\left[\bar{f}_1 e^{ik(\hat{i}-\hat{o})\cdot\bar{r}_1} + \bar{f}_2 e^{ik(\hat{i}-\hat{o})\cdot\bar{r}_2} + \bar{f}_3 e^{ik(\hat{i}-\hat{o})\cdot\bar{r}_3}\right]$$

$$\langle\bar{E}\rangle = \frac{e^{ikR}}{R}\left[\bar{f}_1 \langle e^{ik(\hat{i}-\hat{o})\cdot\bar{r}_1}\rangle + \bar{f}_2 \langle e^{ik(\hat{i}-\hat{o})\cdot\bar{r}_2}\rangle + \bar{f}_3 \langle e^{ik(\hat{i}-\hat{o})\cdot\bar{r}_3}\rangle\right]$$

$= 0$ for $\hat{i} \neq \hat{o}$ & for very random positions

coherent field

Now work on incoherent power...

$$|\bar{E}|^2 = \bar{E}\cdot\bar{E}^* = \frac{1}{R^2}\Big[|\bar{f}_1|^2 + |\bar{f}_2|^2 + |\bar{f}_3|^2$$
$$+ \bar{f}_1\cdot\bar{f}_2^* e^{ik(\hat{i}-\hat{o})\cdot(\bar{r}_1-\bar{r}_2)}$$
$$+ \bar{f}_1\cdot\bar{f}_3^* e^{ik(\hat{i}-\hat{o})\cdot(\bar{r}_1-\bar{r}_3)}$$
$$+ \ \ldots\Big]$$

$$= \frac{1}{R^2}\left[\sum_{n=1}^{3}|\bar{f}_n|^2 + \sum_{n=1}^{3}\sum_{m=n+1}^{3} 2\,\mathrm{Re}\left\{\bar{f}_n\cdot\bar{f}_m^* e^{ik(\hat{i}-\hat{o})\cdot(\bar{r}_n-\bar{r}_m)}\right\}\right]$$

Now $$\langle|\bar{E}|^2\rangle = \frac{1}{R^2}\left[\sum_{n=1}^{3}|\bar{f}_n|^2 + \sum_{n=1}^{3}\sum_{m=n+1}^{3} 2\,\mathrm{Re}\left\{\bar{f}_n\cdot\bar{f}_m^* \langle e^{ik(\hat{i}-\hat{o})\cdot(\bar{r}_n-\bar{r}_m)}\rangle\right\}\right]$$

Note $$\langle e^{ik(\hat{i}-\hat{o})\cdot(\bar{r}_n-\bar{r}_m)}\rangle = \underbrace{\langle e^{ik(\hat{i}-\hat{o})\cdot\bar{r}_n}\rangle\langle e^{-ik(\hat{i}-\hat{o})\cdot\bar{r}_m}\rangle}_{\text{independence}}$$

We saw previously that these averages are 0 for very random positions...

Thus $$\langle|\bar{E}|^2\rangle = \frac{1}{R^2}\left[\sum_{n=1}^{3}|\bar{f}_n|^2\right]$$ & powers from each particle add!