ECE 816 - Spring Quarter 2009

Midterm Exam

May 5th, 2009

Write your name below and sign the honor pledge "No aid given, received, or observed" if it applies.

There are 3 problems on this exam. Exam is open book and notes.

Please box or underline your final answers, and remember to include units.

Be sure to show all work clearly if you wish to obtain any partial credit.

Use the pages at the back of the exam for your work. Be sure to clearly indicate which problem is being worked on each page.

Name: Solutions

"The Pledge": No aid given, received, or observed.

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Problem 1 (5 parts, 75 points)

A 300 MHz plane wave with electric field $\bar{E}^{inc} = \hat{x}e^{ikz}$ (V/m) is incident upon a small sphere located at position $\bar{r}'$; assume that $\bar{r}'$ is zero unless otherwise instructed. The sphere has radius 1 mm and dielectric constant 3. Include numerical values for the following, and use of the Rayleigh approximation is acceptable.

in free space

a) Find the backscattering cross section for the sphere. (10%)

b) Find the scattering cross section for this sphere (10%).

c) Show why the forward scattering theorem is not applicable for predicting the extinction cross section of this sphere (20%).

d) In a medium containing ~~one hundred~~ 10^{15} of these particles per meter cubed, how many dB would a propagating wave be attenuated after propagating 1 m in this medium? (15%)

e) Find the expected value of the backscattering scattering amplitude and backscatter cross section if the particle's position is $\bar{r}' = \hat{x}x' + \hat{z}z'$ where x' and z' are independent random variables. Assume that x' is uniformly distributed between 0 and 1 m, and z' is ~~a Rayleigh~~ an exponential random variable having mean 1 m. (20%)

(a) Rayleigh scatterer $\sigma_{bi}(\hat{o},\hat{i}) = \frac{k^4}{4\pi}\left|\frac{3(\epsilon-1)}{\epsilon+2}\right|^2 V^2 \sin^2\chi$

Here $\hat{o} = -\hat{i}$ and $\chi = \pi/2$ since the angle between $\hat{e}_i = \hat{x}$ & $-\hat{i} = -\hat{z}$ is $\pi/2$.

$$\sigma_{back} = \frac{(2\pi)^4}{4\pi}\left|\frac{3(2)}{5}\right|^2\left(\frac{4}{3}\pi\, 10^{-9}\right)^2 = \frac{16\pi^4\, 6^2 (4\pi)}{9\cdot 5^2}10^{-18} = (64)(16)\pi^5 \times 10^{-20}$$
$$= \boxed{3.13\times 10^{-15}\ m^2}$$

(b) $$\sigma_s = \frac{3k^4V^2}{2\pi}\left|\frac{\epsilon-1}{\epsilon+2}\right|^2 = \frac{3(2\pi)^4 4}{2\pi\cdot 5^2}\left(\frac{4}{3}\pi\,10^{-9}\right)^2 = \frac{4\cdot 24\pi^3}{25}\left(\frac{16\pi^2}{9}10^{-18}\right)$$
$$= \boxed{2.09\times 10^{-15}\ m^2}$$

(c) Forward scattering theorem says $\sigma_t = \frac{4\pi}{k}\operatorname{Im}\{\bar{f}(\hat{i},\hat{i})\cdot\hat{e}_i\}$

Here $$\bar{f} = \frac{k^2}{4\pi}\frac{3(\epsilon-1)}{\epsilon+2}V(-\hat{o}\times\hat{o}\times\hat{e}_i) = \frac{(2\pi)^2}{4\pi}\frac{3\cdot 2}{5}\left(\frac{4}{3}\pi\,10^{-9}\right)\underbrace{(-\hat{z}\times\hat{z}\times\hat{x})}_{\hat{x}}$$

This is entirely real, so $\operatorname{Im}\{\bar{f}(\hat{i},\hat{i})\cdot\hat{e}_i\} = 0$! Rayleigh approx not accurate enough to satisfy forward scattering theorem.

(d) Power attenuates as $e^{-\rho\sigma_t z} = e^{-10^{15}\sigma_s} \Rightarrow 10\log_{10} e^{-(10^{15})\sigma_s} = -9.1$ dB

Problem 2 (5 parts, 25 points): circle the best answer

1) A plane wave has a Stokes' vector (not a modified Stokes vector): $\begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \end{bmatrix}$ (Watts per sq. meter)

Which of the following describes the polarization state of the plane wave?

(a) linear (b) right hand circular $\boxed{\text{(c) left hand circular}}$ (d) elliptical (e) none of the above

2) A radar system observes backscattering from a random medium whose particles are moving. The random medium has a low fractional volume of scatterers so that independent scattering theory is applicable. Which of the following is true?

(a) the received power will be larger when particles are moving away from the radar
(b) there will be a non-zero mean Doppler shift only if the particle velocities are zero mean
(c) the temporal correlation function of the received fields will always be an increasing function of time
(d) all of the above
(e) none of the above

3) Which of the following pdf's is typically expected to describe the real or imaginary parts of the fields backscattered from a random medium?

(a) Exponential (b) Rayleigh (c) Gaussian (d) Uniform (e) none of the above

4) Which of the following statements about first-order multiple scattering theory (i.e. independent scattering theory including attenuation in the random medium) is true?

(a) the theory includes the effects of correlated particle positions for higher fractional volumes
(b) the theory cannot be applied if particles in the random medium are of varying sizes
(c) It can be shown that a coherent field exists in the forward scattered direction
(d) The theory is most applicable to random media with high fractional volumes
(e) none of the above

5) Which of the following statements is true?

(a) the radiative transfer theory is derived from Maxwell's equations
(b) the specific intensity decreases as 1/R^2 when it propagates in an ideal medium
(c) the radiative transfer theory includes interference effects
(d) all of the above
(e) none of the above

1(e): For a particle not at the origin, $\bar{f} = \frac{k^2}{4\pi}\frac{3(\epsilon-1)}{\epsilon+2} V \underbrace{(\hat{z} \times -\hat{z} \times \hat{x})}_{\hat{x}} e^{ik(\hat{i}-\hat{o})\cdot\bar{r}'}$

$= \hat{x}(1.579 \times 10^{-8}) e^{i2\pi\, 2(\hat{z}\cdot\bar{r}')}$. Only the z' position matters!

$\langle \bar{f} \rangle = \hat{x}(1.579\times10^{-8}) \langle e^{i4\pi z'} \rangle = \hat{x}(1.579\times10^{-8}) \int_0^\infty dz'\, e^{-z'}\, e^{i4\pi z'} = \frac{\hat{x}(1.579\times10^{-8})}{(1-i4\pi)}$

Backscattering cross section is not affected by position $= 3.13\times10^{-15}\ m^2$