EE 894J Final Exam Solns, Spring '99

Problem 1

(a) Continuous medium theory, since under clear atmosphere conditions the Born approx should be satisfied & the atmosphere would be best described as a continuous medium. We'll need to know also the PSD of the permittivity random process $\Phi_n(k_s)$ to get an answer. This is a limitation since getting this can be tough realistically. Other limitations would occur if the Born approx was no longer valid.

(b) At 5 GHz, $\lambda = 6$ cm, so $h > \lambda$ & we should use the physical optics surface scattering method. Also since $l >> h$ the surface should be smooth on the scale of λ, making PO valid. We wouldn't need any more info on the surface to solve the problem. PO has problems however away from near specular angles. Also Xpol cross sections in the plane of incidence are predicted to be 0.

(c) Low fractional volumes & small particles → independent scattering theory, due to its simplicity. We would need more info on particle sizes, etc. to describe the medium. For low fractional volumes ind. scat. theory should probably give a reasonable answer unless for some reason particles are located extremely close together (not likely at small f). Note RT theory could also be used but would be somewhat more complex.

(d) At 10 GHz, $\lambda = 3$ cm so $h << \lambda$ & we can use first order SPM. Also l is large enough to keep surface slopes small on average. We wouldn't need any more info on the surface to do this problem. SPM should give a good answer for this problem although Xpol cross sections in the plane of incidence would be predicted to be zero. Also the reflection coefficient would still be predicted as 1. A 2nd order theory would correct these.

(e) For larger fractional volumes, RT theory should be used. We'd need to know details on particle sizes, permittivities, etc. to complete the derivation. RT theory (especially the discrete angles solution) should be fairly accurate, although problems would occur if any coherent position effects or reflecting boundaries were present.

Problem 2

(a) From the handout on "Iterative soln of the RT eqn", we find for a layer of Rayleigh scatterers that $\sigma_{vv}(\theta_i) = \frac{3}{4}\frac{\sigma_s}{\sigma_t}\cos\theta_i\left(1-e^{-2\tau_0\sec\theta_i}\right)$, where $\tau_0 = \rho\sigma_t d$.

Since particle radii (0.5 mm) are much less than λ (30 cm) here, Rayleigh scattering should apply and the above equation should hold. Thus we just need to find σ_s, σ_t, & τ_0 to compute the solution.

$$\sigma_s = \frac{k^4V^2}{6\pi}\left|\frac{3(\epsilon-1)}{\epsilon+2}\right|^2 = \frac{(20.943)^4(5.236\times10^{-10})^2}{6\pi}\underbrace{\left|\frac{3(2+i0.1)}{5+i0.1}\right|^2}_{1.443}$$

$$\sigma_s = 4.038\times10^{-15}\ (m^2)$$

$$\sigma_a = k\epsilon''\left|\frac{3}{\epsilon+2}\right|^2 V = (20.94)(0.1)\underbrace{\left|\frac{3}{5+i0.1}\right|^2}_{0.3599}(5.236\times10^{-10}) = 3.946\times10^{-10}\ (m^2)$$

$$\sigma_t = \sigma_s + \sigma_a = 3.946\times10^{-10}\ (m^2) \qquad \rho = \frac{\#\text{ particles}}{\text{unit volume}} = \frac{\text{vol of particles}}{\text{unit volume}}\cdot\frac{1}{\text{vol of a particle}} = (.01)\frac{1}{5.236\times10^{-10}}$$

$$\rho = 1.9099\times10^{7}\ \text{spheres/unit vol}$$

$$\tau_0 = \rho\sigma_t d = (1.9099\times10^{7})(3.946\times10^{-10})(100) = 0.7537$$

Finally $\sigma_{vv} = \frac{3}{4}\left(\frac{4.038\times10^{-15}}{3.946\times10^{-10}}\right)(\cos\theta_i)\left(1-e^{-1.507\sec\theta_i}\right) = \boxed{(7.674\times10^{6})\cos\theta_i\left(1-e^{-1.507\sec\theta_i}\right)}$

(b) From 1st order SPM, $\sigma_{vv}(\theta_i) = 4\pi k^4(\sin^2\theta_i+1)^2 W(2k\sin\theta_i, 0)$

$$\boxed{=(2.416\times10^{6})(\sin^2\theta_i+1)^2 W(41.88\sin\theta_i, 0)}$$

(c) At 45°, part (a) is 4.782×10^{-6} or -53.23 dB. Part (b) is $(5.436\times10^{6})W(29.614, 0)$.

So we need $W(29.614, 0) = \frac{4.782\times10^{-6}}{5.436\times10^{6}} = 8.769\times10^{-13} = \frac{h^2l^2}{\pi}e^{-219.2l^2}$

Noting $8.769\times10^{-13} \cong e^{-28}$ we need to choose l on the order of 0.36 m.

Using $l = 0.3$ gives $h = 0.1069$ m! This is too large for 1st order SPM.

Try $l = 0.25$ m, then $h = 0.0063$ m. Closer & prob. ok! Also $\boxed{l = 0.2 \text{ m gives } h = 0.000665 \text{ m}}$