Quiz #1
ECE 816 Spring Quarter 2007
April 9th, 2007

A plane wave propagating in the $\hat{y}$ direction in free space with a $\hat{z}$ directed electric field and of frequency 1 GHz is incident upon a small sphere of radius 1 mm and relative permittivity $\varepsilon = 2 + i0$.

(a) Find the extinction cross section of this particle (final answer should be a number of square meters).

We know $\sigma_t = \sigma_a + \sigma_s$, here $\sigma_a = 0$ since the particle is lossless. First note that at 1 GHz in free space, $\lambda = 0.3$ m and $k = 20\frac{\pi}{3}$ rads/m. From the notes, we have

$$\sigma_t = \sigma_s = \frac{128\pi^5 a^6}{3\lambda^4}\left|\frac{\varepsilon - 1}{\varepsilon + 2}\right|^2 = \frac{128\pi^5 10^{-18}}{3(0.3)^4}\left|\frac{1}{4}\right|^2 = 1.0075 \times 10^{-13} \text{ square meters.}$$

(b) The sphere is located at the origin of the coordinate system, and the scattered field is observed in direction $\hat{O} = \hat{x}$. Write the (vector) scattering amplitude in this direction.

From the notes on Rayleigh scattering,

$$\bar{f}(\hat{O}, \hat{i}) = \frac{k^2}{4\pi}\frac{3(\varepsilon - 1)}{(\varepsilon + 2)}\left(\frac{4}{3}\pi a^3\right)(-\hat{O} \times \hat{O} \times \hat{e}_i) = \left(\frac{20\pi}{3}\right)^2 \frac{1}{4\pi}\left(\frac{3}{4}\right)\left(\frac{4}{3}\right)\pi(10^{-9})\hat{z} = \hat{z}1.0966 \times 10^{-7}$$

m.

(c) Find the modified Stokes vector corresponding to the electric field $\bar{E} = (\hat{x} + (1 + i)\hat{y})e^{ikz}$ (V/m) that propagates in free space; note you will need to specify a coordinate system.

Here we have exactly Ishimaru's coordinate system, so we can choose $\hat{a}_1 = \hat{x}$ and $\hat{a}_2 = \hat{y}$, giving $a_1 = 1$, $\delta_1 = 0$, $a_2 = \sqrt{2}$, and $\delta_2 = -\frac{\pi}{4}$. Then we have $\delta = \delta_2 - \delta_1 = -\frac{\pi}{4}$. Ishimaru's equations for the modified Stokes' vector then give

$$\bar{I} = \begin{bmatrix} 1^2 \\ (\sqrt{2})^2 \\ 2(1)(\sqrt{2})\cos\left(-\frac{\pi}{4}\right) \\ 2(1)(\sqrt{2})\sin\left(-\frac{\pi}{4}\right) \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 2 \\ -2 \end{bmatrix} \text{ (in volts-squared per meter squared.)}$$