EE 816 - Lecture 10

1. Microwave scattering from rain
2. Independent scattering for pulses
3. Independent scattering: line of sight propagation
4. Higher fractional volumes: multiple scattering theory

Microwave scattering from rain

- A typical problem to which independent scattering theory can be applied is microwave scattering from rain
- Our previous equations essentially described the received power in terms of a bistatic scattering coefficient per unit volume given by $\rho\sigma_{bi}(\hat{O}, \hat{\imath})$
- Assuming Rayleigh scattering, this reduces to $\sigma_{bi} = \frac{64\pi^5}{\lambda^4} a^6 |K|^2$, where K is defined in terms of ϵ
- Due to the particle size distribution we have to average this over particle size
- Required integrations can be performed analytically for Rayleigh scattering
- Measurements of backscattering cross sections per unit volume therefore determine rain rate: useful for weather monitoring!

- You are performing this type of calculation on problem 2 of your homework
- Narrow beam approximation can be applied; still need to integrate out range dependence however
- Ishimaru defines some factors to relate antenna effective aperture and gain to its geometrical area
- You should consider whether rain is a strongly scattering (high albedo) or absorbing (low albedo) medium
- You should also consider the applicability of Rayleigh scattering as rain rates or frequency increase

Independent scattering for pulses

- Chapter 5 of the book considers scattering of time domain pulses from a random medium
- We so far have been considering CW transmit and receive signals
- However, it is not too difficult to handle a pulse through use of the Fourier transform
- The theory shows the important quantity to be the two frequency correlation function: time domain correlation function between CW fields at two different frequencies
- The main result is in equation 5-31 on page 100; this expression defines the correlation function between complex fields with a time pulse incident
- There are a few interesting examples illustrating how pulse shapes can change when scattered from a random medium

- The concepts of "coherence time" and "coherence bandwidth" are also introduced in Chapter 5
- Consider a CW radar measuring a received field as a function of time; "coherence time" is the time delay at which the correlation function becomes small (usually $1/e$)
- A good measure of an average time delay over which fields will be correlated; note not as useful as the correlation function itself however
- Consider two CW radars using the same antenna but operating at tuneable and different carrier frequencies; "coherence bandwidth" is the frequency separation at which the correlation between two frequency measurements becomes small
- A good measure of the expected frequency band over which measurements will be related; again not as good as entire two frequency correlation function
- Both of these quantities are captured by the two frequency correlation function; also commonly discussed in many areas

Line of sight propagation

- Chapter 6 of the book considers independent scattering theory along the line of sight
- The introduction to Chapter 6 gives a simple explanation: coherent intensity attenuates at extinction rate, but only absorption cross section results in power loss
- From this argument:

$$I_c \propto \exp(-\rho\sigma_t z) \quad (1)$$
$$I_t \propto \exp(-\rho\sigma_a z) \quad (2)$$
$$I_i \propto \exp(-\rho\sigma_a z) - \exp(-\rho\sigma_t z) \quad (3)$$

- Eventually all power even along line of sight is incoherent
- The first order multiple scattering approximation is studied in chapter 6 for field spatial and temporal correlation functions, both for CW and pulse signals

- Theories are also developed both for a plane wave and including transmit and receive antenna patterns
- Field spatial and temporal correlation functions are found to be very sensitive to antenna patterns
- A new independent scattering theory is also discussed for line of sight propagation: the Rytov approximation
- This assumes the line of sight field to be a deterministic phasor with random exponential amplitude and phase modulations
- The Rytov approximation is more commonly used than FOMS for line of sight propagation

Higher fractional volumes: multiple scattering theory

- The independent scattering theory we have studied so far applies only to low fractional volumes
- Clearly as fractional volumes increase there will be several problems: correlated particles positions, muliple scattering effects, etc...
- Multiple scattering theories attempt to take some of these effects into account, but formulations are very complex
- Some theories essentially try to take averages using equations much like our DDA method
- To account for correlated particles positions, a "pair distribution" function is introduced
- This quantity is based on a two point pdf for particle positions: $f_{\overline{r}_1,\overline{r}_2}(\overline{r}_1,\overline{r}_2) = f_{\overline{r}_1|\overline{r}_2}(\overline{r}_1|\overline{r}_2) f_{\overline{r}_2}(\overline{r}_2)$

- The conditional probability is related to the pair distribution function; differing models for particles interactions lead to differing pair functions
- Some of the moments resulting from multiple scattering theories involve the pair function
- Ishimaru considers multiple scattering theories in chapters 14 and 15; some sections are a little outdated
- Many of these efforts are similar to work done in quantum mechanics
- Also as particles fractional volumes become large, the coherent field no longer appears to propagate in the background medium, but rather in an "effective" medium with a different dielectric constant due to scattering and absorption effects
- Determining this effective permittivity comprises a large part of multiple scattering theory

EE 816 - Lecture 11

1. Introduction to radiative transfer theory
2. Specific intensity
3. Basic properties of specific intensity
4. Reflection and transmission of specific intensity

I. Radiative transfer theory

- We now start to address a distinct theory for propagation and scattering in random media: radiative transfer (or RT) theory
- RT is a heuristic description of wave propagation, and is not derived from Maxwell's equations
- Neglects all phase effects in multiple particle scattering and interface reflections
- Formulated in terms of the "specific intensity", a new description of power flux
- Originated in radio astronomy, has also been applied to radiative heat transfer problems
- Extensive literature and used in many areas!
- Can approximately include multiple scattering effects; can also handle polarization phenomena but Ishimaru neglects

II. Specific intensity

- Since RT theory started in radio astronomy, let's start from this point of view also
- Consider an antenna of area dA located at position $\overline{r}$ observing the sky. Assume the antenna receives incident power with a $\cos\theta$ weighting function.
- The specific intensity $I(\overline{r}, \hat{s})$ is defined in terms of the amount of power per unit frequency received by this antenna from a source occupying a small solid angle centered on $\hat{s}$:

$$dP = I(\overline{r}, \hat{s}) \cos\theta \; dA \; d\omega \; d\nu \qquad (4)$$

- Here the symbols are a little funny: $d\omega$ now is a small solid angle while $d\nu$ is a small frequency bandwidth
- Recall that an object of area a at distance R from $\overline{r}$ subtends a solid angle $\frac{a\cos\beta}{R^2}$ steradians when viewed from $\overline{r}$; here β is the angle between the normal to a and $\hat{s}$

- The units of I are thus watts per m^2 per sr per Hz
- These units are a little funny: Poynting power spread out in both frequency and solid angle; note at any point in space we have power going in all directions
- In radio astronomy, it is thermal emission that is received; this is almost always noise power spread out in frequency so it makes sense to talk about power per unit bandwidth
- Also in radio astronomy, there are sources of thermal emission everywhere in the sky so it makes sense to talk about power per unit solid angle
- Specific intensity is also called "brightness"; you can also think about observing objects with your eyes
- Using power flux per unit solid angle turns out to make I independent of distance from a source. Further objects subtend less solid angle so total power falls off as $1/R^2$ but I remains constant

- We have here considered an "antenna" with area dA which received power from a source in the sky
- However this equation actually defines the specific intensity at any point in space, whether an antenna is there or not. If we put an antenna at a position with a narrow power pattern we could determine I
- Ishimaru discusses specific intensities flowing into or out of an area; the equation also applies to power *emitted* from an area
- Another quantity important in RT theory is the "total flux" F flowing through an area dA with normal $\hat{s}_0$, defined by

$$\hat{s}_0 \cdot \overline{F}(\overline{r}) = \hat{s}_0 \cdot \int_{4\pi} I(\overline{r}, \hat{s})\hat{s} \, d\omega \tag{5}$$

  in watts per m^2 per Hz
- We can also consider the "energy density" $u(\overline{r})$ at a point in space; this is given in units of Joule per m^3 per Hz as

$$u(\overline{r}) = \frac{1}{c} \int_{4\pi} I(\overline{r}, \hat{s}) \, d\omega \tag{6}$$

III. Basic properties of specific intensity

- Does not change with distance in an ideal medium: consider two small areas da_1 and da_2 oriented normal to each other and at distance r
- Define the specific intensity at da_1 as I_1, that at da_2 as I_2
- If we think in terms of the amount of power emitted from da_1 to da_2 we find

$$\begin{aligned} dP_2 &= I_1 \, da_1 \, d\omega_1 d\nu && (7) \\ &= I_1 \, da_1 \, \frac{da_2}{r^2} d\nu && (8) \end{aligned}$$

  where $d\omega_1$ is the solid angle subtended by da_2 when viewed from da_1

- If we think in terms of the amount of power received at da_2 from da_1 we find

$$\begin{aligned} dP_2 &= I_2 \, da_2 \, d\omega_2 d\nu && (9) \\ &= I_2 \, da_2 \, \frac{da_1}{r^2} d\nu && (10) \end{aligned}$$

- For this to be true, I_1 must equal I_2, i.e. the specific intensity does not change along a path in an ideal medium
- This fact will make keeping track of changes in specific intensities in non-ideal media a lot easier
- Note again even though I is remaining constant with distance, the power total flux between da_1 and da_2 is changing with distance since the solid angles subtended are getting smaller
- Ishimaru also considers specific intensity coming from a spherical surface and shows that the flux falls off as $1/R^2$ as expected

IV. Reflection and transmission of specific intensity

- Since specific intensity is related to a power flux, some of its properties can be related to properties of Poynting vectors
- A particular case is reflection and transmission at a dielectric interface
- It is fairly easy to show that the relationship between reflected and incident specific intensities is

$$I_r = |R|^2 I_i \tag{11}$$

where R is the appropriate Fresnel reflection coefficient

- This is simple because the propagation angles of the incident and reflected intensities are identical, so no solid angle changes occur

- Note R above depends on polarization: a complete RT theory must include a Stokes vector like representation of I, but this becomes more complicated
- The relationship between transmitted and incident intensities is more complicated since they propagate at different angles; Snell's law used to relate $d\omega_i$ to $d\omega_t$
- Ishimaru shows that

$$I_t = \frac{n_2^2}{n_1^2} |1 - R|^2 I_i \tag{12}$$

where n_2 and n_1 are the indices of refraction ($\sqrt{\epsilon}$) of the transmitted and incident media

- These equations will be useful later when we consider RT theory for a medium with planar boundaries

EE 816 - Lecture 12

1. Radiative transfer equation
2. Scattering contributions
3. Reduced incident and diffuse components

I. Radiative transfer equation

- We found previously that a specific intensity propagating in an ideal medium remains constant
- We also know how to handle problems involving planar interfaces between ideal media
- The radiative transfer equation is a heuristic description of changes in specific intensity as it propagates through an absorbing and scattering medium
- Think in terms of our radio astronomy antenna observing a source but then a cloud comes between. Power from source may be absorbed or scattered as it propagates; RT equation describes this process

- Consider a specific intensity $I(\overline{r}, \hat{s})$ incident upon a cylindrical elementary volume with unit area and length dS
- The volume of this cylinder contains ρ particles per unit volume
- We can solve the scattering problem for a single particles to determine the scattering and absorption cross sections, σ_s and σ_a
- Each particles now absorbs a "power" $I\ \sigma_a$ and scatters the "power" $I\ \sigma_s$ so the total power removed is $I\ \sigma_t\ \rho\ ds$
- The first term in the radiative transfer equation describes this extinction process:

$$dI(\overline{r}, \hat{s}) \quad = \quad -\rho\ \sigma_t\ I(\overline{r}, \hat{s})\ ds \tag{13}$$

- However, the scattered power is not lost but merely changes direction; it should be possible to have some power scattered into direction $\hat{s}$ from other directions $\hat{s}'$

II. Scattering contributions

- Let's try to figure out the scattering contribution...
- Consider a specific intensity incident on the volume from direction $\hat{s}'$ in a small solid angle $d\omega'$
- The flux (Watts per m^2 per Hz) incident is $I(\overline{r}, \hat{s}')d\omega'$ which can be interpreted as a Poynting flux S_i
- Particles inside the volume will scatter some of this flux in direction $\hat{s}$ according to

$$S_r = \frac{|f(\hat{s}, \hat{s}')|^2}{R^2} S_i \tag{14}$$

where f is the scattering amplitude of a single particle and R^2 is the distance from the particle to the observation point

- Re-writing this in terms of a solid angle gets rid of the R^2 factor:

$$S_r R^2 \quad = \quad |f(\hat{s}, \hat{s}')|^2\, S_i = |f(\hat{s}, \hat{s}')|^2\, I(\overline{r}, \hat{s}')d\omega' \tag{15}$$

- Now adding up the specific intensity scattered into direction $\hat{s}$ from all other directions and including all the particles in the small volume yields

$$\int_{4\pi} d\omega'\ \rho ds\, |f(\hat{s}, \hat{s}')|^2\, I(\overline{r}, \hat{s}') \tag{16}$$

which increases the power traveling in direction $\hat{s}$

- This can also be rewritten in terms of the "phase" function $p(\hat{s}, \hat{s}')$ using

$$p(\hat{s}, \hat{s}') = \frac{4\pi}{\sigma_t}\, |f(\hat{s}, \hat{s}')|^2 \tag{17}$$

as

$$\frac{\sigma_t}{4\pi} \int_{4\pi} d\omega'\ \rho ds\ p(\hat{s}, \hat{s}')\ I(\overline{r}, \hat{s}') \tag{18}$$

- Finally there is one last contribution to the RT equation: thermal emission
- Define this contribution as $ds\ \epsilon(\overline{r}, \hat{s})$ which increases intensity

- We have now obtained the radiative transfer equation:

$$\frac{dI(\overline{r}, \hat{s})}{ds} \quad = \quad -\rho\sigma_t\ I(\overline{r}, \hat{s}) + \frac{\rho\sigma_t}{4\pi} \int_{4\pi} d\omega'\ p(\hat{s}, \hat{s}')I(\overline{r}, \hat{s}') + \epsilon(\overline{r}, \hat{s})$$

- The left hand side can be re-written as $\frac{dI(\overline{r}, \hat{s})}{ds} = \hat{s} \cdot \nabla I(\overline{r}, \hat{s}) = \nabla \cdot (\hat{s} I(\overline{r}, \hat{s}))$
- Note also that ρ and σ_t can vary with location which makes this equation a little more complicated. Eliminate these variations by defining the unitless "optical distance" τ as

$$d\tau \quad = \quad \rho\sigma_t\ ds \tag{19}$$

$$\tau \quad = \quad \int \rho\sigma_t\ ds \tag{20}$$

- In terms of τ the RT equation is now

$$\frac{dI(\tau, \hat{s})}{d\tau} \quad = \quad -I(\tau, \hat{s}) + \frac{1}{4\pi} \int_{4\pi} d\omega' p(\hat{s}, \hat{s}')I(\tau, \hat{s}') + J(\tau, \hat{s})$$

where $J(\tau, \hat{s}) = \epsilon(\overline{r}, \hat{s})/\rho\sigma_t$

- Ishimaru shows that the RT equation conserves total flux

III. Reduced incident and diffuse components

- RT theory is frequently applied to thermal radiation problems where all power is noise power
- However in microwave remote sensing it is also applied in radar problems where there is a coherent incident field
- For this reason it can be useful to separate I into a "reduced incident intensity" and the "diffuse intensity"; the former is likely to be coherent, the latter incoherent
- Let's write $I = I_{ri} + I_d$ and define I_{ri} to satisfy

$$\frac{dI_{ri}(\overline{r},\hat{s})}{ds} = -\rho\sigma_t \, I_{ri}(\overline{r},\hat{s}) \tag{21}$$

that is, the attenuated incident intensity

- This choice means that I_d must satisfy

$$\frac{dI_d(\overline{r},\hat{s})}{ds} = -\rho\sigma_t \, I_d(\overline{r},\hat{s}) + \frac{\rho\sigma_t}{4\pi}\int_{4\pi} d\omega' \; p(\hat{s},\hat{s}')I_d(\overline{r},\hat{s}') + \epsilon(\overline{r},\hat{s}) + \epsilon_{ri}(\overline{r},\hat{s})$$

where

$$\epsilon_{ri}(\overline{r},\hat{s}) = \frac{\rho\sigma_t}{4\pi}\int_{4\pi} d\omega' \; p(\hat{s},\hat{s}')I_{ri}(\overline{r},\hat{s}')$$

is an "equivalent source function" due to scattering of the reduced incident intensity

- We'll also need boundary conditions when solving the RT equation: diffuse intensity does not enter a volume scattering medium (generated all within):

$$I_d(\overline{r},\hat{s}) = 0 \tag{22}$$

on the boundary when $\hat{s}$ is directed inward

- We can also have different types of incident fields in RT problems: "collimated" or "diffuse"
- A "collimated" source is focused in a particular direction, as a plane wave or laser beam. Define as the "collimated" incident intensity

$$I_{ci}(\overline{r},\hat{s}) = F_0\delta(\hat{\omega}-\hat{\omega_0}) = \frac{F_0}{\sin\theta}\delta(\theta-\theta_0)\delta(\phi-\phi_0) \tag{23}$$

- A "diffuse" incident intensity is not located in a particular direction; again like our radio astronomy example or problems involving thermal emission. Need to specify angular dependence
- We will usually consider collimated sources
- Finally examine the source function $J(\tau,\hat{s}) = \epsilon(\overline{r},\hat{s})/\rho\sigma_t$. If a medium contains sources we can model their radiation with J.
- However, all media at non-absolute zero temperature produce thermal emission according to Kirchhoff's law. This is a very small power though and can be neglected in radar problems.