

EE 816 - Extra Lecture

1. Midterm questions
2. Thermal noise
3. Brightness temperature
4. Uniform atmosphere
5. Layered atmosphere

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III. Brightness temperature

- Because intensity is directly related to the physical temperature of a blackbody, it is more common to talk about the “brightness temperature” of a source (units of Kelvin), rather than the specific intensity radiated
 - Objects that are not blackbodies do not satisfy the Planck law. However, it is still used as a reference: the brightness temperature of an object is the temperature of a blackbody that would produce the same specific intensity as the real object.
 - Emissivity is defined as the ratio of an object’s brightness temperature to its physical temperature:
- $$T_B = e T_{phys} \quad (3)$$
- Energy conservation arguments can be used to relate the emissivity of an object to its scattering properties: “Kirchhoff’s Law”

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II. Thermal noise

- All objects at non-zero absolute temperature emit radiation over a wide range of frequencies: thermal noise
- The standard for this emission is a “blackbody”: an object that perfectly absorbs all incident radiation
- If it remains in thermal equilibrium, a blackbody must also emit radiation; this is thermal noise however, not reflected incident radiation
- The Planck blackbody law describes the specific intensity radiated by a blackbody at Kelvin temperature T :

$$I = \frac{1}{c^2} \frac{h\nu^3}{e^{h\nu/kT} - 1} \quad (1)$$

where h is Planck’s constant, ν is the frequency, and k is Boltzmann’s constant 1.38×10^{-23} J/K.

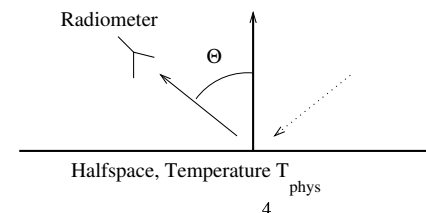
- In the microwave region, the exponent can be expanded to yield

$$I = \frac{kT}{\lambda^2} \quad (2)$$

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Brightness temperature of a halfspace

- A simple argument can be used to find the brightness temperature of a halfspace medium at constant physical temperature T_{phys} :
- $$T_{B,\beta}(\theta) = T_{phys} \left(1 - |\Gamma_\beta(\theta)|^2\right) \quad (4)$$
- β is the polarization observed, while Γ is the halfspace reflection coefficient. The brightness temperature function of β and θ comes from the reflection coefficient.
 - The more reflective a boundary is, the “colder” it appears.
 - Properties of a halfspace can be determined from thermal noise measurement: microwave radiometry.

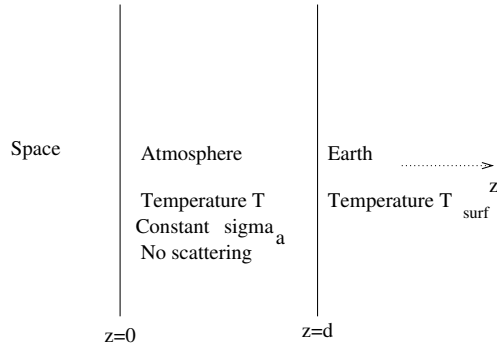


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IV. T_B of a uniform atmosphere

- We can use radiative transfer theory to study T_B 's of absorbing and scattering media; I is simply related to T_B .
- We have to add the emission source term to our RT equation:

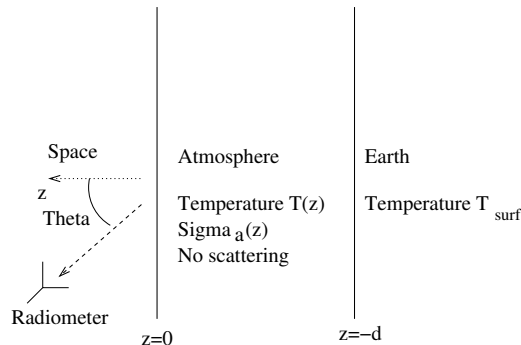
$$\frac{dI(\bar{r}, \hat{s})}{ds} = -\rho\sigma_t I(\bar{r}, \hat{s}) + \frac{\rho\sigma_t}{4\pi} \int_{4\pi} d\omega' p(\hat{s}, \hat{s}') I(\bar{r}, \hat{s}') + \rho\sigma_a \frac{K T}{\lambda^2}$$



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V. T_B of a “layered” atmosphere

- It is more realistic to have a temperature profile $T(z)$ in the atmosphere, as well as an absorption profile. Here define $\kappa_a(z) = (\rho\sigma_a)(z)$
- Solution of RT equation neglecting scattering remains easy. Note different coordinate system below.



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T_B of a uniform atmosphere

- We can still divide into forward and reverse going intensities as before. However the Earth boundary is reflective, so we need:

$$I_-(z=d) = rI_+(z=d) + \frac{KT_{surf}}{\lambda^2}(1-r) \quad (5)$$

where T_{surf} is the physical temperature of the Earth and r is the reflection coefficient at the boundary.

- In many cases, scattering in the atmosphere can be neglected. The RT solution is a straightforward 1st order DE solution. Write solution in terms of T_B in space region:

$$T_B(\theta) = T(1 - e^{-\tau \sec \theta})(1 + re^{-\tau \sec \theta}) + T_{B,surf}e^{-\tau \sec \theta}$$

- Here $\tau = \rho \sigma_a d$. There are three terms:
 - Direct upward emission by layer
 - Downward emission of layer reflected off boundary and attenuated
 - Direct upward surface emission attenuated by layer

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T_B of a “layered” atmosphere

- RT equation solution is:

$$\begin{aligned} T_B = & \sec \theta \int_{-d}^0 dz \kappa_a(z) T(z) \exp\left(-\int_z^0 dz' \kappa_a(z') \sec \theta\right) \\ & + \sec \theta r e^{-\int_{-d}^0 dz \kappa_a(z) \sec \theta} \int_{-d}^0 dz \kappa_a(z) T(z) \exp\left(-\int_{-d}^z dz' \kappa_a(z') \sec \theta\right) \\ & + T_{surf}(1-r)e^{-\int_{-d}^0 dz \kappa_a(z) \sec \theta} \end{aligned}$$

- Terms are similar to before but include atmospheric profiles
- We could think of the first term only as:

$$T_B = \int_{-d}^0 dz T(z) w(z) \quad (6)$$

where $w(z)$ is a “weighting function” connecting the atmospheric temperature profile to the observed brightness

- With a proper sensor design, it is possible to use multi-frequency thermal noise measurements to sense atmospheric temperature profiles.

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