

## EE 816 - Lecture 13

1. RT integral equations
2. Received power
3. Polarization effects
4. Relationship with Poynting vector

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- Thus as far space variations are concerned we have a first order differential equation:

$$\frac{dI}{ds} + P(s) I = Q(s) \quad (1)$$

- Recall that a first order differential equation

$$\frac{dy}{dx} + P(x)y(x) = f(x) \quad (2)$$

is solved through the use of the “integrating factor”

$I(x) = \exp\left\{\int_{-\infty}^x P(x')dx'\right\}$  to obtain

$$y(x) = \frac{c_1}{I(x)} + \frac{1}{I(x)} \int_{-\infty}^x dx' f(x') I(x') \quad (3)$$

- The constant  $c_1$  and the lower limits on the integral are determined through the boundary conditions

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## I. RT integral equations

- The RT equation we have at the moment:

$$\frac{dI(\bar{r}, \hat{s})}{ds} = -\rho\sigma_t I(\bar{r}, \hat{s}) + \frac{\rho\sigma_t}{4\pi} \int_{4\pi} d\omega' p(\hat{s}, \hat{s}') I(\bar{r}, \hat{s}') + \epsilon(\bar{r}, \hat{s})$$

is an “integro-differential” equation since it contains both derivatives and integrals of the specific intensity  $I(\bar{r}, \hat{s})$

- This form can be solved for some simple geometries or through approximations, but sometimes integral forms can be more useful
- Note  $\frac{dI(\bar{r}, \hat{s})}{ds}$  describes the variation in  $I$  with position  $\bar{r}$  along direction  $\hat{s}$ : this is a space derivative
- The integral is over all other directions: it is not an integral over space

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- Applying this solution to our equation:

$$\frac{dI}{ds} + P(s) I = Q(s) \quad (4)$$

we obtain

$$I(\bar{r}, \hat{s}) = c e^{-\tau} + e^{-\tau} \int_{-\infty}^s ds_1 Q(s_1) e^{\tau_1} \quad (5)$$

where

$$Q(s_1) = \frac{\rho\sigma_t}{4\pi} \int_{4\pi} d\omega' p(\hat{s}, \hat{s}') I(\bar{r}_1, \hat{s}') + \epsilon(\bar{r}_1, \hat{s}) \quad (6)$$

and

$$\tau = \int_{-\infty}^s ds \rho\sigma_t \quad \tau_1 = \int_{-\infty}^{s_1} ds \rho\sigma_t \quad (7)$$

- Applying the boundary condition that at the point of incidence  $\bar{r}_0$ , we have only an incident intensity, we find

$$I_{ri}(\bar{r}, \hat{s}) = I_i(\bar{r}_0, \hat{s}) e^{-\tau} \quad (8)$$

$$I_d(\bar{r}, \hat{s}) = \int_0^s ds_1 \exp[-(\tau - \tau_1)] \left[ \frac{\rho\sigma_t}{4\pi} \int_{4\pi} d\omega' p(\hat{s}, \hat{s}') I(\bar{r}_1, \hat{s}') + \epsilon(\bar{r}_1, \hat{s}) \right]$$

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- This is not a complete description because we still couple intensities in all directions. We would need a series of these equations at all  $\hat{s}$  to complete the description
- We can also formulate an integral equation in terms of the “average” specific intensity  $U$  (intensity integrated over  $4\pi$  steradians)
- Integrating our previous equations over  $4\pi$  steradians we find

$$U_{ri}(\bar{r}) = \frac{1}{4\pi} \int_{4\pi} d\omega I_i(\bar{r}_0, \hat{s}) e^{-\tau} \quad (9)$$

while

$$U(\bar{r}) = U_{ri}(\bar{r}) + \int_V dV_1 \frac{\exp[-(\tau - \tau_1)]}{4\pi|\bar{r} - \bar{r}_1|^2} \left[ \frac{\rho\sigma_t}{4\pi} \int_{4\pi} d\omega' p(\hat{s}, \hat{s}') I(\bar{r}_1, \hat{s}') + \epsilon(\bar{r}_1, \hat{s}) \right]$$

- Total average intensity comes from the incident intensity plus the contributions from all other points inside the scattering volume but attenuated according to loss and distance

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## II. Received power

- Ishimaru now reviews again the relationship between specific intensity and received power for a specific antenna (not necessarily  $\cos\theta$  power pattern)
- Ishimaru describes receiving antenna performance in terms of a “receiving cross section”  $A_r(\hat{s}_r, \hat{s})$ .  $\hat{s}_r$  describes the orientation of the antenna while  $\hat{s}$  describes the direction incoming radiation
- When a wave with power density  $S_i(\hat{s})$  is incident with the receiver oriented in the  $-\hat{s}_r$  direction, the power received is

$$P_r(\hat{s}_r) = A_r(\hat{s}_r, \hat{s}) S_i(\hat{s}) \quad (10)$$

- For a specific intensity incident on the antenna, the received power is

$$P_r(\hat{s}_r) = \int_{\Omega} d\omega A_r(\hat{s}_r, \hat{s}) I(\bar{r}, \hat{s}) \quad (11)$$

where  $\Omega$  describes the solid angles the antenna can view

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- Note again that the separation into “reduced incident” and “diffuse” intensities is a separation of coherent and incoherent fields in a radar problem
- The diffuse intensity results due to scattering; if scattering is neglected there is no diffuse intensity and we obtain only the attenuated incident intensity
- RT theory will describe the diffuse intensity scattered from a random medium; once we find the outgoing diffuse intensity on the boundary, this diffuse intensity will propagate unchanged through homogeneous space
- Thus we can find monostatic and bistatic received powers in a radar problem once the diffuse intensity on the boundaries is known
- We will focus on layered media in particular and try to study the angular dependence of scattered fields that results...

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## III. Polarization effects

- A more complete RT includes polarization effects by replacing the scalar specific intensity  $I$  with a four component Stokes vector  $\bar{I}$  of specific intensity
- In this case, the phase function becomes a  $4 \times 4$  phase matrix  $\bar{\bar{P}}$  defined in terms of the earlier Stokes matrix discussed with regard to single particle scattering
- In Ishimaru’s coordinate system we also have to consider rotations of this Stokes matrix because we must couple Stokes vectors described in differing coordinate systems
- In the most general case the extinction term  $-\rho\sigma_t$  also becomes a  $4 \times 4$  matrix called the extinction matrix
- Rapidly becomes quite complex Ishimaru is satisfied with a scalar theory; we will follow Tsang, Kong, and Shin to discuss some simple problems including polarization

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## IV. Relationship with Poynting vector

- Ishimaru also discusses the relationship between a Poynting vector and specific intensity
- Recall that a Poynting vector has units of Watts per  $m^2$  but also has an associated direction
- For a randomly time varying Poynting vector this direction can vary in time, as well as the overall length of the vector
- Ishimaru relates the specific intensity to the expected value of all Poynting vectors (of differing overall magnitudes) which lie within a small solid angle
- Doesn't give much insight: better to rely on "brightness" interpretation of specific intensity and fundamental power relationship

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### I. Tenuous medium approximation

- Now that we have discussed the radiative transfer equation, we consider its solution
- In general this is not easy since it is an integro-differential equation; will require some sort of approximation
- We will consider two solution methods: iteration (Ch. 8) and discretization of angles (Ch. 11)
- The iterative method applies when the effects of scattering are dominated by absorption, i.e. low albedo and small optical depths
- Since effects of scattering are small, our first guess at  $I$  solves the equations without scattering, then "iterate" by adding in scattering of the no scattering intensity, etc.
- Provides analytical solutions, but becomes very difficult for higher than first order. Only up to 2nd order results have been achieved

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## EE 816 - Lecture 14

1. Tenuous medium approximation
2. Plane parallel medium
3. First order solution

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Basic idea of iterative solution: begin with

$$\frac{dI(\bar{r}, \hat{s})}{ds} = -\rho\sigma_t I(\bar{r}, \hat{s}) + \frac{\rho\sigma_t}{4\pi} \int_{4\pi} d\omega' p(\hat{s}, \hat{s}') I(\bar{r}, \hat{s}')$$

and write

$$I(\bar{r}, \hat{s}) = I^{(0)}(\bar{r}, \hat{s}) + I^{(1)}(\bar{r}, \hat{s}) + \dots \quad (12)$$

where the superscript indicates different "orders" of scattering. Substitute this in and neglect scattering at zeroth order:

$$\frac{dI^{(0)}(\bar{r}, \hat{s})}{ds} = -\rho\sigma_t I^{(0)}(\bar{r}, \hat{s})$$

which is an easy to solve first order DE. At higher order,

$$\frac{dI^{(n)}(\bar{r}, \hat{s})}{ds} = -\rho\sigma_t I^{(n)}(\bar{r}, \hat{s}) + \frac{\rho\sigma_t}{4\pi} \int_{4\pi} d\omega' p(\hat{s}, \hat{s}') I^{(n-1)}(\bar{r}, \hat{s}')$$

again easier to solve since now the scattering term is known.

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- Ishimaru's separation of  $I$  into the “reduced incident” and “diffuse” components has already started an iterative procedure
- $I_{ri}$  is in fact the zeroth order solution; however remember that Ishimaru derived this neglecting any boundary reflections
- Ishimaru obtains the first order solution from his integral form of the RT equation, but an identical result is obtained from the procedure described in the last slide
- Ishimaru's “general” result:

$$I_d(\bar{r}, \hat{s}) = \int_0^s ds_1 \exp[-(\tau - \tau_1)] \left[ \frac{\rho\sigma_t}{4\pi} \int_{4\pi} d\omega' p(\hat{s}, \hat{s}') I_{ri}(\bar{r}_1, \hat{s}') + \epsilon(\bar{r}_1, \hat{s}) \right]$$

which is identical to the first order multiple scattering solution

- Now however we have a consistent procedure to obtain higher order predictions
- Again remember Ishimaru has neglected boundary reflections in his equations; probably better to follow our procedure and apply b.c.'s

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Begin with the RT equation:

$$\frac{dI(\bar{r}, \hat{s})}{ds} = -\rho\sigma_t I(\bar{r}, \hat{s}) + \frac{\rho\sigma_t}{4\pi} \int_{4\pi} d\omega' p(\hat{s}, \hat{s}') I(\bar{r}, \hat{s}')$$

and define

$$d\tau = \rho \sigma_t dz = \cos\theta \rho \sigma_t ds \quad (13)$$

and assume that  $I(\bar{r}, \hat{s}) = I(z, \hat{s}) = I(\tau, \theta, \phi)$  (constant in  $x$  and  $y$ ):

$$\cos\theta \frac{dI(\tau, \theta, \phi)}{d\tau} = -I(\tau, \theta, \phi) + \frac{1}{4\pi} \int_{4\pi} d\omega' p(\hat{s}, \hat{s}') I(\tau, \theta', \phi')$$

Define  $I$  as  $I_+$  for  $0 < \theta < \pi/2$  and  $I_-$  for  $\pi/2 < \theta < \pi$ ; both satisfy above equation but have different boundary conditions:

$$I_+(\tau = 0, \theta, \phi) = I_{inc}(\tau = 0, \theta, \phi) \quad I_-(\tau = \tau_0, \theta, \phi) = 0 \quad (14)$$

neglecting any boundary reflections; with reflecting boundaries  $I_+$  and  $I_-$  become coupled at boundaries through a reflection coefficient

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## II. Plane parallel medium

- Now consider the problem of scattering from a “plane parallel” medium. This is a common model for atmospheres or other objects
- The  $z$  direction is defined to be normal to the planes;  $\hat{s}$  can then be described in terms of the angles  $\theta$  and  $\phi$
- Under this definition an intensity propagating with a  $+z$  component has  $0 < \theta < \pi/2$  while the  $-z$  direction has  $\pi/2 < \theta < \pi$
- It is typical to define these two as the forward going  $I_{d+}$  and backward going  $I_{d-}$  intensities; different boundary conditions on each
- Also typically distances are measured along  $z$  rather than along ray paths using  $dz = ds \cos\theta = ds \mu$
- Sometimes  $I_{d+}$  is written as  $I(\bar{r}, \theta, \phi)$  while  $I_{d-}$  is written as  $I(\bar{r}, \pi - \theta, \phi)$  both with  $0 < \theta < \pi/2$

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Now solve iteratively: at zeroth order both  $I_+$  and  $I_-$  satisfy

$$\cos\theta \frac{dI^{(0)}(\tau, \theta, \phi)}{d\tau} = -I^{(0)}(\tau, \theta, \phi) \quad (15)$$

whose solution is

$$I_+^{(0)}(\tau, \theta, \phi) = c_+(\theta, \phi) e^{-\int_0^\tau \sec\theta d\tau} = c_+(\theta, \phi) e^{-\tau \sec\theta} \quad (16)$$

$$I_-^{(0)}(\tau, \theta, \phi) = c_-(\theta, \phi) e^{-\int_0^\tau \sec\theta d\tau} = c_-(\theta, \phi) e^{-\tau \sec\theta} \quad (17)$$

Applying the boundary conditions determines  $c_+$  and  $c_-$  and shows

$$I_+^{(0)}(\tau, \theta, \phi) = I_{inc}(\tau = 0, \theta, \phi) e^{-\tau \sec\theta} \quad (18)$$

$$I_-^{(0)}(\tau, \theta, \phi) = 0 \quad (19)$$

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At first order both  $I_+$  and  $I_-$  satisfy

$$\cos \theta \frac{dI^{(1)}(\tau, \theta, \phi)}{d\tau} = -I^{(1)}(\tau, \theta, \phi) + \frac{1}{4\pi} \int_{4\pi} d\omega' p(\hat{s}, \hat{s}') I^{(0)}(\tau, \theta', \phi')$$

where  $I^{(0)}$  in the integral contains both  $I_+^{(0)}$  and  $I_-^{(0)}$  since the integration is over  $4\pi$  steradians. Again there are different boundary conditions:

$$I_+(\tau = 0, \theta, \phi) = 0 \quad I_-(\tau = \tau_0, \theta, \phi) = 0 \quad (20)$$

since we have already taken care of the incident intensity at zeroth order.

The solution to the above equation is

$$I_{\pm}^{(1)}(\tau, \theta, \phi) = c_{\pm}(\theta, \phi) e^{-\tau \sec \theta} + e^{-\tau \sec \theta} \int_0^{\tau} d\tau_1 e^{\tau_1 \sec \theta} \frac{\sec \theta}{4\pi} \left[ \int_{4\pi} d\omega' p(\theta, \phi, \theta', \phi') I^{(0)}(\tau_1, \theta', \phi') \right]$$

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## EE 816 - Lecture 15

1. Plane parallel medium: plane wave incidence
2. Iterative solution including polarization effects

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Applying the b.c.'s we find

$$c_+ = 0$$

$$c_- = - \int_0^{\tau_0} d\tau_1 e^{-\tau_1 \sec \theta} \frac{\sec \theta}{4\pi} \left[ \int_{4\pi} d\omega' p(\theta, \phi, \theta', \phi') I^{(0)}(\tau_1, \theta', \phi') \right]$$

which can be combined into the original solutions to obtain

$$I_+^{(1)}(\tau, \theta, \phi) = \int_0^{\tau} d\tau_1 e^{(\tau_1 - \tau) \sec \theta} \frac{\sec \theta}{4\pi} \left[ \int_{4\pi} d\omega' p(\theta, \phi, \theta', \phi') I^{(0)}(\tau_1, \theta', \phi') \right]$$

$$I_-^{(1)}(\tau, \theta, \phi) = - \int_{\tau}^{\tau_0} d\tau_1 e^{(\tau_1 - \tau) \sec \theta} \frac{\sec \theta}{4\pi} \left[ \int_{4\pi} d\omega' p(\theta, \phi, \theta', \phi') I^{(0)}(\tau_1, \theta', \phi') \right]$$

Ishimaru presents these results as if they are obvious! In fact they were derived through solving the first order DE's and applying the appropriate boundary conditions.

Again remember that we use  $I_+$  for  $0 < \theta < \pi/2$  and  $I_-$  for  $\pi/2 < \theta < \pi$

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## I. Plane parallel medium: plane wave incidence

The first order solution for a plane parallel medium is

$$I_+^{(1)}(\tau, \theta, \phi) = \int_0^{\tau} d\tau_1 e^{(\tau_1 - \tau) \sec \theta} \frac{\sec \theta}{4\pi} \left[ \int_{4\pi} d\omega' p(\theta, \phi, \theta', \phi') I^{(0)}(\tau_1, \theta', \phi') \right]$$

$$I_-^{(1)}(\tau, \theta, \phi) = - \int_{\tau}^{\tau_0} d\tau_1 e^{(\tau_1 - \tau) \sec \theta} \frac{\sec \theta}{4\pi} \left[ \int_{4\pi} d\omega' p(\theta, \phi, \theta', \phi') I^{(0)}(\tau_1, \theta', \phi') \right]$$

which applies for an arbitrary incident source as long as boundary reflections are neglected. For plane wave incidence,

$$I_{inc}(\vec{r}, \hat{s}) = F_0 \delta(\hat{\omega} - \hat{\omega}_0) \quad (21)$$

and

$$I^{(0)}(\tau, \theta, \phi) = F_0 \delta(\hat{\omega} - \hat{\omega}_0) e^{-\tau \sec \theta_0} \quad (22)$$

so the delta function will remove the integral over  $d\omega'$  and make things a lot easier.

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For plane wave incidence,

$$\begin{aligned}
 I_+^{(1)}(\tau, \theta, \phi) &= \int_0^\tau d\tau_1 e^{(\tau_1 - \tau) \sec \theta} \frac{\sec \theta}{4\pi} p(\theta, \phi, \theta_0, \phi_0) F_0 e^{-\tau_1 \sec \theta_0} \\
 &= F_0 \frac{\sec \theta}{4\pi} p(\theta, \phi, \theta_0, \phi_0) e^{-\tau \sec \theta} \int_0^\tau d\tau_1 e^{\tau_1 (\sec \theta - \sec \theta_0)} \\
 &= F_0 \frac{\sec \theta}{4\pi} p(\theta, \phi, \theta_0, \phi_0) e^{-\tau \sec \theta} \frac{e^{\tau (\sec \theta - \sec \theta_0)} - 1}{(\sec \theta - \sec \theta_0)} \\
 &= F_0 \frac{\cos \theta_0}{(\cos \theta_0 - \cos \theta)} \frac{p(\theta, \phi, \theta_0, \phi_0)}{4\pi} (e^{-\tau \sec \theta_0} - e^{-\tau \sec \theta})
 \end{aligned}$$

By a similar analysis

$$I_-^{(1)} = F_0 \frac{\cos \theta_0}{(\cos \theta_0 - \cos \theta)} \frac{p(\theta, \phi, \theta_0, \phi_0)}{4\pi} (e^{-\tau \sec \theta_0} - e^{-\tau_0 \sec \theta_0 + (\tau_0 - \tau) \sec \theta})$$

Note the phase function here does not influence the variation in  $z$ , only scales overall magnitude

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- Ishimaru also discusses some terms to add if boundary reflections are included; you will derive these on your homework
- He also discusses a non-plane wave source; just plug in source terms into our general first order equations. More complicated sources will not get rid of the  $d\omega'$  integral so things are a little more complicated
- Remember this is only a first order iterative solution; we could now take our first order solution and try to calculate second order but this would be tough!
- We'll be satisfied with first order solutions
- The neglect of higher order scattering makes the first order solution tend to underestimate backscattered fields
- Once specific intensities are known we can use Ishimaru's equations to find the power received by a receiver

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- Remember we must include the zeroth order contribution when calculating total intensity; zeroth order is the reduced incident plane wave
- Ishimaru determines the location of maximum forward propagating intensity also
- We have solved for intensities inside the plane parallel layer; Ishimaru obtains intensities outside the layer just using transmission coefficients  $T_{21}$  or  $T_{23}$
- This is a little inconsistent since we neglected boundary reflections; unless  $T_{21} \approx T_{23} \approx 1$  the equations should be re-derived including reflections off boundaries
- If we consider scattered intensities from a half space ( $e^{-\tau_0 \sec \theta} = 0$ ) we obtain the "law of diffuse reflection":

$$I_d(\theta_r = \pi - \theta) = \frac{p(\theta, \phi, \theta_0, \phi_0)}{4\pi} \left[ \frac{\cos \theta_0}{(\cos \theta_0 + \cos \theta_r)} \right] T_{12} T_{21} F_i$$

which describes the angular dependence of diffusely scattered radiation under plane wave incidence

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## II. Including polarization effects

- Ishimaru's RT equations neglect polarization effects; however it is clear there are polarization effects due to the rotation matrices required in the scattering term as discussed in Ch. 7
- To include polarization effects,  $I$  becomes a four component Stokes vector,  $P$  becomes a  $4 \times 4$  matrix and in general  $\rho\sigma_t$  also becomes a  $4 \times 4$  matrix
- An iterative solution can still work for the general problem; requires some eigenvalue techniques
- If we consider only Rayleigh scattering from small spheres, things get a little easier
- It turns out that the extinction matrix is diagonal so we can still just use  $\rho\sigma_t$ . Also the Rayleigh phase matrix is fairly simple

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Let's study scattering from a layer of Rayleigh scatterers neglecting interface reflections but including polarization effects.

The vector RT equation for this case is

$$\frac{d\bar{I}(\bar{r}, \hat{s})}{ds} = -\rho\sigma_t \bar{I}(\bar{r}, \hat{s}) + \frac{\rho\sigma_t}{4\pi} \int_{4\pi} d\omega' \bar{P}(\hat{s}, \hat{s}') \cdot \bar{I}(\bar{r}, \hat{s}')$$

using the diagonal extinction matrix. We can still define  $\tau$  as before and re-write this as

$$\cos\theta \frac{d\bar{I}(\tau, \theta, \phi)}{d\tau} = -\bar{I}(\tau, \theta, \phi) + \frac{1}{4\pi} \int_{4\pi} d\omega' \bar{P}(\hat{s}, \hat{s}') \bar{I}(\tau, \theta', \phi')$$

and the forward and backward going intensities can also be defined as before and satisfy the same boundary conditions as before.

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At first order both  $\bar{I}_+$  and  $\bar{I}_-$  satisfy

$$\cos\theta \frac{d\bar{I}^{(1)}(\tau, \theta, \phi)}{d\tau} = -\bar{I}^{(1)}(\tau, \theta, \phi) + \frac{1}{4\pi} \int_{4\pi} d\omega' \bar{P}(\hat{s}, \hat{s}') \cdot \bar{I}^{(0)}(\tau, \theta', \phi')$$

which for plane wave incidence reduces to

$$\cos\theta \frac{d\bar{I}^{(1)}(\tau, \theta, \phi)}{d\tau} = -\bar{I}^{(1)}(\tau, \theta, \phi) + \frac{1}{4\pi} \bar{P}(\hat{s}, \hat{s}') \cdot \bar{I}_0 e^{-\tau \sec\theta_0}$$

which again decouples into four scalar equations identical to our previous equation - we already have the solution!

$$\bar{I}_+^{(1)}(\tau, \theta, \phi) = \frac{\cos\theta_0}{4\pi(\cos\theta_0 - \cos\theta)} \left[ \bar{P}(\hat{s}, \hat{s}') \cdot \bar{I}_0 \right] \left( e^{-\tau \sec\theta_0} - e^{-\tau \sec\theta} \right)$$

$$\bar{I}_-^{(1)}(\tau, \theta, \phi) = \frac{\cos\theta_0}{4\pi(\cos\theta_0 - \cos\theta)} \left[ \bar{P}(\hat{s}, \hat{s}') \cdot \bar{I}_0 \right] \left( e^{-\tau \sec\theta_0} - e^{-\tau_0 \sec\theta_0 + (\tau_0 - \tau) \sec\theta} \right)$$

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Using the iterative solution, the zeroth order intensities satisfy

$$\cos\theta \frac{d\bar{I}_{\pm}^{(0)}(\tau, \theta, \phi)}{d\tau} = -\bar{I}_{\pm}^{(0)}(\tau, \theta, \phi)$$

which reduces to four scalar equations identical to our previous scalar equation. The zeroth order solution is thus

$$\bar{I}_+^{(0)}(\tau, \theta, \phi) = \bar{I}_{inc}(\tau=0, \theta, \phi) e^{-\tau \sec\theta} \quad (23)$$

$$\bar{I}_-^{(0)}(\tau, \theta, \phi) = 0 \quad (24)$$

Again very similar to the scalar case! For plane wave incidence we have

$$\bar{I}_+^{(0)}(\tau, \theta, \phi) = \bar{I}_0 \delta(\hat{\omega} - \hat{\omega}_0) e^{-\tau \sec\theta_0} \quad (25)$$

$$\bar{I}_-^{(0)}(\tau, \theta, \phi) = 0 \quad (26)$$

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- Thus the vector solution is similar to the scalar solution except we now involve the Rayleigh phase matrix  $\bar{P}$
- A common way of describing polarized scattered intensities is in terms of the scattered polarization and incident polarization; for example  $HH$ ,  $HV$ ,  $VH$ ,  $VV$  as we defined in our phase matrix notes
- It is also most common to describe this type of scattering in terms of a “bistatic scattering coefficient” and not in terms of specific intensities
- A bistatic scattering coefficient is a cross section per unit area; makes sense because as we illuminate larger areas we get more scattering from a volume scattering medium
- See handed out notes for exact definitions of the bistatic scattering coefficient  $\gamma_{\beta\alpha}$  and the backscattering coefficient  $\sigma_{\beta\alpha}$
- These quantities are proportional to the scattered intensities  $I$  so our first order solution provides a prediction of these scattering coefficients

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