EE 816 - Lecture 21

1. Rough surface scattering
2. Rayleigh criterion
3. Power received above a rough surface

I. Rough surface scattering

- We now begin our study of scattering from rough surfaces
- Useful for many practical problems: propagation over Earth terrain, scattering from ocean or soil surfaces, surface tolerance of antennas, etc.
- Note the "roughness" of a given surface depends on frequency; size and scale of surface features relative to the electromagnetic wavelength is what matters
- For statistically described probles, we will have both "coherent" and "incoherent" scattering, usually in an ensemble average sense
- Coherent scattering essentially is the same as reflection from a flat surface, but attenuated depending on surface roughness; exists only in specular direction
- Incoherent scattering exists in all directions and is a result of surface roughness

II. Rayleigh criterion

- Lord Rayleigh first considered the effects on rough surfaces on scattering
- A simple model for scattering from a rough surface comes from examining specular reflection from each point on the surface
- Doing this, it is possible to work out that the phase difference between fields specularly reflected from different points on the surface separated by height h is

$$\delta\phi = 2kh\cos\theta_i \tag{1}$$

  where k is the electromagnetic wavenumber and θ_i is the incidence angle
- This phase difference needs to be appreciable for the surface roughness to have an appreciable effect on scattering

- By choosing $\pi/2$ as "appreciable" we can find that the surface peak-to-peak height should be bigger than

$$\lambda/(8\cos\theta_i) \tag{2}$$

  to have a large effect on scattering
- Note this is an approximate equation, we can throw in some constants if we want to be more conservative
- This equation is telling us that a surface must be at least slightly rough relative to λ in order to cause surface scattering
- In addition we find that as θ_i becomes larger, the surface must be rougher to achieve the same effect; this is because points at different heights become more in phase as θ_i approaches $\pi/2$
- Explains the "specular" reflections seen off of rough surfaces as we observe them nearer to grazing

- We can also discuss a few basic expectations regarding surface scattering...
- For a surface that is very flat relative to λ, we should expect the fields to resemble fields above a flat interface
- As surface roughness increases, we should expect a reduction in the specularly reflected field accompanied by an increase in the incoherent scattered fields in other directions
- As the surface becomes extremely rough, we should expect the specularly reflected component to disappear and all the power to be scattered throughout the upper and lower hemispheres
- These basic ideas are usually correct, but the goal of surface scattering theory is to quantify the amounts of power that are reflected or scattered for a particular surface description and radar parameters

III. Power received above a rough surface

- We'll proceed with studying surface scattering by considering a radar system operating above a rough surface and try to find an expression for the power received relative to the power transmitted
- If we consider a radar system operating above a flat surface and assume specular reflection, we can find

$$\frac{P_{r,coh}}{P_t} = \frac{\lambda^2}{(4\pi)^2}\frac{G_t G_r}{(R_1+R_2)^2} R_f^2 \left|\chi\right|^2 \qquad (3)$$

where R_f represents the flat surface reflection coefficient (for the appropriate polarization) and χ is a factor included to account for surface roughness effects

- This relationship applies for power received along the specular direction; in other directions there would be no coherent power received

- This results follows from the expectation that the coherent power should be the same as that of a flat surface regardless of surface properties except for an attenuation effect modeled by χ
- To discuss the scattered (or "diffuse" or "incoherent") fields, it is most useful to use a cross section per unit area
- This quantity is useful again because the amount of scattered power received from a rough surface depends on the amount of area illuminated; a cross section per unit area removes the dependence on illuminated area
- Define the cross section per unit area as

$$\sigma(\hat{O},\hat{i}) = \frac{4\pi R^2 < \left|\overline{E}_s\right|^2 >}{\Delta S} \qquad (4)$$

where $\overline{E}_s$ is the field scattered by the surface in direction $\hat{O}$ with a unit amplitude incident field in direction $\hat{i}$ which illuminates area ΔS

- Again neglecting polarization effects; we will include later

- Note ΔS needs to be small enough such that the incident field can be considered to be a plane wave, but large enough so that the surface statistical properties are captured in ΔS
- This will turn out to mean large dimensions compared to the surface correlation length later on...
- If this is true, then fields scattered from different pieces of surface area are uncorrelated (i.e. have randomized phases) and we can write the total power received from an entire illuminated surface as

$$\frac{P_{r,incoh}}{P_t} = \frac{\lambda^2}{(4\pi)^3}\int dS\, \frac{G_t(\hat{i})G_r(\hat{O})}{R_1^2 R_2^2}\sigma(\hat{O},\hat{i}) \qquad (5)$$

- Ishimaru also discusses scattering from a temporally varying surface for which a receiver voltage correlation function and temporal frequency spectrum can be formulated similar to our previous studies

- Thus in rough surface scattering theory, the goal is to compute the specular reflection coefficient modification χ and the bistatic radar cross section per unit area, σ, for given EM and surface parameters
- We will describe our surfaces as stochastic processes in space; it is clear that surface heights are separate random variables that should be correlated for small separations
- We will apply a second order moment characterization: surface described in terms of a correlation function or equivalently a spatial power spectral density
- Solving the rough surface scattering problem turns out to be a tough EM problem; lots of possible scattering effects
- We will resort to two approximate methods: small perturbation method (SPM) and physical optics (Kirchhoff approach)
- The former will be useful for surfaces whose heights are small compared to λ, while the latter will be useful for smoothly varying surfaces even with large heights

EE 816 - Lecture 22

1. Small perturbation method: PEC surface
2. Expressions for scattered fields
3. Perturbation series
4. 1st order solution

I. Small perturbation method

- We now begin our study of the small perturbation method for rough surface scattering: essentially a perturbation method (or iterative technique) for surfaces with small heights relative to lambda
- Denoting the surface profile function as $z = \zeta(x, y)$, this will require $|k\zeta \cos\theta_i| << 1$, and also $|\frac{\partial\zeta}{\partial x}|$ and $|\frac{\partial\zeta}{\partial y}| << 1$, i.e. not only small heights but also small slopes
- Consider a zero mean surface, so that $< \zeta(x, y) >= 0$. Note again for $\zeta(x, y)$ a random process: each point is a random variable but there are correlations among points that depend on the spatial separation
- We will consider actually scattering from a periodic surface with period L in the x and y directions; non periodic surface obtained in the limit L goes to infinity
- We're also assuming the surface is PEC to make things easier

- Let's begin by assuming we have a horizontally polarized incident plane wave; here horizontal polarization means a $\hat{y}$ directed $\overline{E}$ field:

$$\overline{E}_i = \hat{y}E_{yi} = \hat{y}\exp(i(\beta x - \gamma z)) \quad (6)$$

where $\beta = k\sin\theta_i$ and $\gamma = k\cos\theta_i$

- Since the surface is supposed to be near flat, let's go ahead and take an initial guess at the reflected field as that of a flat surface:

$$\overline{E}_r = \hat{y}E_{yr} = -\hat{y}\exp(i(\beta x + \gamma z)) \quad (7)$$

since the horizontal reflection coefficent on a PEC is -1

- Surface roughness will cause fields to propagate in directions other than the specularly reflected field above; we'll have to propose a form for the scattered field that includes waves propagating in many directions

II. Expression for scattered fields

- In general we should have both upgoing and downgoing scattered waves near the surface profile (due to possible multiple reflections), but we're going to neglect any downgoing scattered waves.
- It turns out this is reasonable as long as surface slopes aren't too large; this idea is called the "Rayleigh hypothesis"
- Since we're considering a periodic surface, it turns out it is a lot easier to write down a form for the scattered fields
- If we think about a periodic surface excited by a plane wave, it is clear that all the fields in the problem have to be periodic with period L in the x and y directions times the horizontal phase variations of the incident field, otherwise we couldn't match the boundary conditions

- This turns out to require that the scattered field be expressed as a sum of a discrete set of plane waves called "Floquet modes"; these discrete plane waves are chosen to have the correct periodicities
- Thus the expression we choose for scattered fields is

$$\overline{E}_s = \hat{x}E_{xs} + \hat{y}E_{ys} + \hat{z}E_{zs} \tag{8}$$

$$E_{xs} = \sum_{m=-\infty}^{\infty}\sum_{n=-\infty}^{\infty} e^{\frac{i2\pi mx}{L}} e^{\frac{i2\pi ny}{L}} \left(e^{i\beta x}e^{ib_{mn}z}A_{mn}\right) \tag{9}$$

$$E_{ys} = \sum_{m=-\infty}^{\infty}\sum_{n=-\infty}^{\infty} e^{\frac{i2\pi mx}{L}} e^{\frac{i2\pi ny}{L}} \left(e^{i\beta x}e^{ib_{mn}z}B_{mn}\right) \tag{10}$$

$$E_{zs} = \sum_{m=-\infty}^{\infty}\sum_{n=-\infty}^{\infty} e^{\frac{i2\pi mx}{L}} e^{\frac{i2\pi ny}{L}} \left(e^{i\beta x}e^{ib_{mn}z}C_{mn}\right) \tag{11}$$

with $b_{mn} = \sqrt{k^2 - (\beta + \frac{2\pi m}{L})^2 - (\frac{2\pi n}{L})^2}$ to satisfy the plane wave dispersion relation

- Note these fields are certain to be periodic after $e^{i\beta x}$ is factored out

- Now we write down the boundary condition equation on the surface: tangential total $\overline{E}$ must vanish:

$$\overline{E} - \hat{n}(\hat{n}\cdot\overline{E}) = 0 \tag{12}$$

on $z = \zeta$

- Here $\hat{n}$ is given by

$$\frac{1}{\sqrt{1 + (\frac{\partial\zeta}{\partial x})^2 + (\frac{\partial\zeta}{\partial y})^2}}\left(\hat{z} - \hat{x}\frac{\partial\zeta}{\partial x} - \hat{y}\frac{\partial\zeta}{\partial y}\right) \tag{13}$$

- Taking the $\hat{x}$ and $\hat{y}$ components of the above equation we find

$$E_x + \frac{\frac{\partial\zeta}{\partial x}}{1 + (\frac{\partial\zeta}{\partial x})^2 + (\frac{\partial\zeta}{\partial y})^2}(E_z - \frac{\partial\zeta}{\partial x}E_x - \frac{\partial\zeta}{\partial y}E_y) = 0 \tag{14}$$

$$E_y + \frac{\frac{\partial\zeta}{\partial y}}{1 + (\frac{\partial\zeta}{\partial x})^2 + (\frac{\partial\zeta}{\partial y})^2}(E_z - \frac{\partial\zeta}{\partial x}E_x - \frac{\partial\zeta}{\partial y}E_y) = 0 \tag{15}$$

- Now if we plug in the total field as the incident plus reflected plus scattered terms we have an equation to solve for the A_{mn}'s, etc.

III. Perturbation series

- If we could solve these equations, we'd have an exact answer except for use of the Rayleigh hypothesis
- The problem with getting an exact solution is the fact that we've got Fourier series expressions combined with $e^{ib_{mn}z} = e^{ib_{mn}\zeta(x,y)}$ terms. Thus orthogonality cannot be applied and we are stuck
- A perturbation solution gets rid of this problem by expanding $e^{ib_{mn}z} = 1 + ib_{mn}z + ...$ Keeping only the first term we can use orthogonality, and it turns out we can with higher order too. This series should work well if $b_{mn}z$ is small, i.e. surface height is small compared to λ
- Acutally to do the perturbation solution, we'll expand both the exponentials and the unknown coefficients A_{mn}, etc. in terms of a series in surface height. Write $A_{mn} = A_{mn}^{(0)} + A_{mn}^{(1)} + A_{mn}^{(2)} + ...$

- We'll define the zeroth order A_{mn} to be a constant, first order term to be linearly proportional to surface height, etc.
- Finally we'll consider slope and height terms to be equivalent, i.e. $\frac{\partial\zeta}{\partial x}$ is the same order as ζ
- Expanding our previous equations to zeroth and first order we get:

$$E_x^{(0)} = 0 \tag{16}$$

$$E_y^{(0)} = 0 \tag{17}$$

$$E_x^{(1)} + \frac{\partial\zeta}{\partial x} E_z^{(0)} = 0 \tag{18}$$

$$E_y^{(1)} + \frac{\partial\zeta}{\partial y} E_z^{(0)} = 0 \tag{19}$$

- Our total fields are

$$E_x^{(0)} = E_{xs}^{(0)} = \sum_{m=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} e^{\frac{i2\pi mx}{L}} e^{\frac{i2\pi ny}{L}} \left(e^{i\beta x} A_{mn}^{(0)} \right) = 0$$

so that $A_{mn}^{(0)} = 0$

- The zeroth order y component equation is

$$\begin{aligned} E_y^{(0)} &= E_{yi}^{(0)} + E_{yr}^{(0)} + E_{ys}^{(0)} = 0 \\ &= \exp(i\beta x) - \exp(i\beta x) + \sum_{m=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} e^{\frac{i2\pi mx}{L}} e^{\frac{i2\pi ny}{L}} \left(e^{i\beta x} B_{mn}^{(0)} \right) \end{aligned}$$

and we find $B_{mn}^{(0)}$ also is 0

- We can also find $E_z^{(0)} = C_{mn}^{(0)} = 0$
- First order is a little more tricky because of the $(A_{mn}^{(0)} + A_{mn}^{(1)} + ...)(1 + ib_{mn}z - ...)$ terms in the scattered fields. The first order term is thus $A_{mn}^{(0)} ib_{mn}z + A_{mn}^{(1)} = A_{mn}^{(1)}$
- Using these facts our first order equation for E_x is

$$\sum_{m=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} e^{\frac{i2\pi mx}{L}} e^{\frac{i2\pi ny}{L}} \left(e^{i\beta x} A_{mn}^{(1)} \right) = 0$$

- Thus $A_{mn}^{(1)} = 0$
- For the first order y component,

$$\begin{aligned} E_y^{(1)} &= E_{yi}^{(1)} + E_{yr}^{(1)} + E_{ys}^{(1)} = 0 \\ &= (-2i\gamma z)\exp(i\beta x) + \sum_{m=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} e^{\frac{i2\pi mx}{L}} e^{\frac{i2\pi ny}{L}} \left(e^{i\beta x} B_{mn}^{(1)} \right) \end{aligned}$$

and $B_{mn}^{(1)}$ will not be zero!

- Cancel $e^{i\beta x}$ and re-write equation as

$$\sum_{m=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} e^{\frac{i2\pi mx}{L}} e^{\frac{i2\pi ny}{L}} B_{mn}^{(1)} = 2i\gamma z$$

- Exactly a Fourier series expansion! Take inverse to find

$$\begin{aligned} B_{mn}^{(1)} &= 2i\gamma \frac{1}{L^2} \int_0^L \int_0^L dx\, dy\, e^{\frac{-i2\pi mx}{L}} e^{\frac{-i2\pi ny}{L}} \zeta(x,y) \\ &= 2i\gamma P(m,n) \end{aligned} \tag{20}$$

- This defines $P(m,n)$ through

$$\zeta(x,y) = \sum_{m=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} e^{\frac{i2\pi mx}{L}} e^{\frac{i2\pi ny}{L}} P(m,n)$$

- Repeating the procedure for E_z we find

$$C_{mn}^{(1)} = -2i\gamma \frac{2\pi n/L}{b_{mn}} P(m,n) \tag{21}$$

- Thus we have now found the scattered fields to first order in surface height; really to first order in surface Fourier coefficients
- The scattered magnetic field can also be found to first order by applying Maxwell's equations to the scattered electric field
- We can also repeat the whole process for a vertically polarized incident field; Ishimaru presents results
- Note Ishimaru drops a minus sign in the book; equivalent to changing amplitude of incident field to -1 instead of 1
- Solution applies for deterministic surface; we'll randomize next time

EE 816 - Lecture 23

1. Analysis of SPM result
2. Cross section per unit area
3. Stochastic process description of surface
4. Final ensemble average cross sections

I. Analysis of SPM result

- Last time we found the amplitude of y and z electric field components of scattered plane waves to go as $B_{mn}^{(1)} = 2i\gamma P(m,n)$ and $C_{mn}^{(1)} = -2i\gamma \frac{2\pi n/L}{b_{mn}} P(m,n)$
- Note here that the (mn) indices indicate the $e^{i2\pi mx/L}$ and $e^{i2\pi ny/L}$ factors which control the direction of scattered wave propagation
- It is interesting to observe that these first order results state that the amplitude of a scattered wave in a particular direction is directly linked to a surface Fourier coefficient that "corresponds" to that direction through the above equation
- This type of scattering is called "Bragg" scattering; scattering in a particular direction is due only to surface variations on a specific length scale; other variations do not affect (at first order)

- Notice that as (mn) increase, which means propagation closer to horizontal (grazing), the important surface Fourier coefficients move to higher and higher frequencies
- A little work can show that the important surface variations for backscattering at oblique incidence are in the range of λ
- This fact allows a lot of insight into scattering from the ocean surface: ocean surface is a "multi-scale" surface having roughness on many scales
- When backscattering is measured, it turns out primarily only a single length scale in the ocean "spectrum" is being measured
- It turns out even with ocean surfaces which do have large heights relative to λ that the first order SPM solution is not too terribly inaccurate for oblique backscattering

II. Cross section per unit area

- Given our SPM solution, we can try to find a cross section per unit area to describe the scattered fields
- At the moment, this is a little bit strange because we have so far considered a periodic surface for which the scattered fields are plane waves; it doesn't make sense to talk about the cross section of a plane wave
- We avoid this problem by letting the surface periods go to infinity. In this case, scattered plane waves get closer and closer together and we obtain a continuous scattered field
- Going through this process, we can find a bistatic cross section per unit area to be

$$\sigma(\hat{i}, \hat{O}) \quad = \quad 4\pi k_0^2 \cos^2\theta_s \frac{|E_s(\hat{O})|^2}{\delta k_x \delta k_y} \tag{22}$$

where $\delta k_x = \delta k_y = \frac{2\pi}{L}$ and θ_s refers to the polar angle of the scattered field in direction $\hat{O}$

- We can also separate the scattered field into horizontal and vertical polarizations to describe σ_{hh}, σ_{hv}, etc.
- Doing this we eventually find

$$\begin{aligned}\sigma_{hh}(\hat{i},\hat{O}) &= 4\pi k_0^2 \cos^2\theta_s 4\gamma^2 |P(m,n)|^2 \frac{\cos^2\phi_s}{\delta k_x \delta k_y} \\ &= 16\pi k_0^4 \cos^2\theta_s \cos^2\theta_i |P(m,n)|^2 \frac{\cos^2\phi_s}{\delta k_x \delta k_y}\end{aligned}$$

- The $\cos\phi_s$ factor results from taking $|\hat{h}_s \cdot \overline{E}_s|^2$ to get the polarized cross section. Ishimaru writes $\hat{h}_s$ as $\hat{\phi}_s$ and $\hat{v}_s$ as $\hat{\theta}_s$
- We can find similar expressions for the other polarized scattering cross sections per unit area: equations (21-61) through (21-64) in the book
- Note there are no cross polarized scattered fields in the plane of incidence at first order: proportional to $\sin(\phi_s)$. Need to go to second order to find cross polarized backscattering

- These results are still expressed in terms of the Fourier coefficients of a periodic surface $P(m,n)$, but as the periods approach infinity it is better to write in terms of a continuous surface spectrum $W(p,q)$
- By comparing a Fourier series to a Fourier transform we can define

$$W(2\pi m/L, 2\pi n/L) = 4\frac{|P(m,n)|^2}{\delta k_x \delta k_y} \quad (23)$$

- Note also that $2\pi m/L = k_{xs} - k_{xi}$, etc. so we can re-write this as $W(2\pi m/L, 2\pi n/L) = W(k_{xs} - k_{xi}, k_{ys} - k_{yi})$
- Under this definition it will turn out that W is proportional to the power spectral density of the continuous surface random process, i.e. the Fourier transform of the surface spatial correlation function
- For a non-random surface, W is related to the amplitude squared of the Fourier transform of the surface. Since the surface is a real function, W must be real and origin symmetric

III. Stochastic process description

- Thus far we've been talking about deterministic surfaces; since we can solve this problem (to first order in height anyway) we can solve the random surface problem to first order
- It is easiest to start with the periodic surface again, expressed in a Fourier series expansion:

$$\zeta(x,y) = \sum_{m=-\infty}^{\infty}\sum_{n=-\infty}^{\infty} e^{\frac{i2\pi m x}{L}} e^{\frac{i2\pi n y}{L}} P(m,n)$$

- We'll make things random here by assuming the $P(m,n)$'s to be random variables. Each has zero mean, and all are uncorrelated. Essentially a "sum of random sine and cosine" waves description
- However, since the surface is real, these coefficients must have a conjugate symmetry about the origin, and we'll define

$$< P(m,n)P(-m,-n) >=< |P(m,n)|^2 >= \frac{1}{4} W(2\pi m/L, 2\pi n/L)\delta k_x \delta k_y$$

- Notice the spectrum W specifies the amount of power on average in a particular Fourier coefficient = spatial length scales
- Ishimaru shows that W is related to the spatial correlation function of the surface through a Fourier transform
- Again W is only a second order characterization of the surface stochastic process, but the first order SPM only requires W so we don't need any more information than this
- In a higher order SPM, we would need higher order surface statistical properties
- Again the continuous surface result is obtained by letting the surface periods approach infinity; W remains the spatial psd of the random process
- Note W is a function of k_x and k_y so we can describe surfaces whose statistics are different in x and y; an ocean surface for example.

III. Final ensemble average cross sections

- Examining our SPM first order cross sections, taking an ensemble average only averages the surface spectrum, since this is on the only random quantity.
- So if we define W as related to $< |P(m,n)|^2 >$ we'll have taken the ensemble average already
- Final answers for a PEC surface:

$$\begin{aligned}
\sigma_{hh} &= 4\pi k^4 \cos^2\theta_i \cos^2\theta_s \cos^2\phi_s W(k_{xi} - k_{xs}, -k_{ys}) \\
\sigma_{vh} &= 4\pi k^4 \cos^2\theta_i \sin^2\phi_s W(k_{xi} - k_{xs}, -k_{ys}) \\
\sigma_{hv} &= 4\pi k^4 \cos^2\theta_s \sin^2\phi_s W(k_{xi} - k_{xs}, -k_{ys}) \\
\sigma_{vv} &= 4\pi k^4 (\sin\theta_i \sin\theta_s - \cos\phi_s)^2 W(k_{xi} - k_{xs}, -k_{ys})
\end{aligned}$$

  since we defined $k_{yi} = 0$, $\phi_i = 0$

- Note k^4 dependence on frequency if spectrum constant, also shift to higher spatial frequencies as frequency increases

- For backscattering, $\phi_s = 0$ and $\theta_s = -\theta_i$,

$$\begin{aligned}
\sigma_{hh} &= 4\pi k^4 \cos^4\theta_i W(2k\sin\theta_i, 0) \\
\sigma_{vv} &= 4\pi k^4 (\sin^2\theta_i + 1)^2 W(2k\sin\theta_i, 0) \\
\sigma_{vh} &= \sigma_{hv} = 0
\end{aligned}$$

- Note there is a polarization dependence here; as θ_i approaches 90 degrees, hh will fall off rapidly while vv will go to a constant
- Also again as θ_i approaches 90 we depend on higher spatial frequencies. Limit is $2k$ at 90 degrees, which corresponds to a length scale $\lambda/2$ in the surface spectrum
- As θ_i approaches 0 we depend on lower frequency components, and reach the DC component at $\theta_i = 0$
- Recall we've also got the reflected field as our zeroth order solution; same as a flat surface but exists only in specular direction
- It is possible also to derive the SPM for a penetrable surface; Ishimaru presents first order results