EE752 Homework#1 Solution

2-1

With h_0 is an equilibrium point for the system with input μ_0, ($\dot{h}(t) = f(h_0, u_0) = 0$). Around this equilibrium point, linearize the system with respect to the first order taylor's series.

$$\delta \dot{h}(t) = \dot{h}(t) \approx f(h_0, u_0) + \frac{\partial f}{\partial u}|_{u_0}\ \delta u(t-\tau) + \frac{\partial f}{\partial h}|_{h_0}\ \delta h(t)$$

where $\frac{\partial f}{\partial u}|_{u_0} = \frac{1}{A_0}$ and $\frac{\partial f}{\partial h}|_{h_0} = \frac{V_0}{2A_0\sqrt{h_0}}$, substitute into equation above and apply Laplace Transform, we will obtain,

$$G(s) = \frac{\delta h(s)}{\delta u(s)} = \frac{e^{-s\tau}}{A_0\left(s + \frac{V_0}{2A_0\sqrt{h_0}}\right)}.$$

2-2

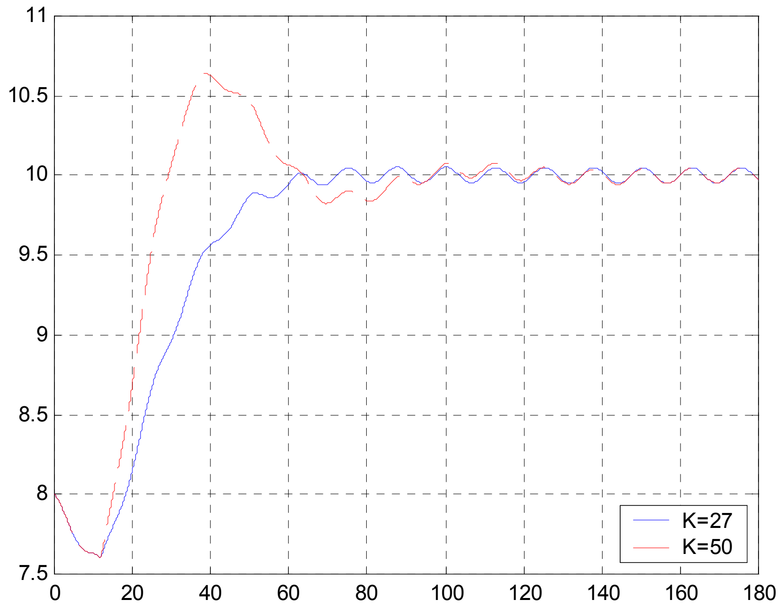
(i) $\tau = 0$

The system dynamics $\dot{h} = \frac{u(t) - v(t)}{A(h(t))}$

$v(t)$ is known, define $u'(t) = u(t) - v(t)$. Since we need $h(t) \to h_d(t)$ as $t \to \infty$, the designed dynamics could be $\mu \dot{h}(t) + h(t) = h_d(t)$, $\mu > 0$ ($h(t) \to h_d(t)$ with $\mu \to 0$, what if $\mu < 0$?).

To achieve the designed dynamics, choose $u'(t) = \frac{A(h(t))}{\mu}(-h(t) + h_d(t))$ with small μ (why?).

(ii)



Matlab code:

```
clear all;

% Given constants
A0=300;
A1=12;
A2=120;
Tao=12; % the time delay
v0=24;
h_d=10;
h_0=8;
% Feedback gain to be designed
K_min=27; % lower bound for K
K_max=50; % upper bound for K
% Initialize the simulation
Ts=0.04;
t=0:Ts:180;
N=length(t); % number of simulation steps
step_of_delay=Tao/Ts;
% Using Euler's first order method: X(k+1)=X(k)+Ts*f(k)
h_1=zeros(size(t)); % Initialize h vector for K_min case
h_1(1)=h_0;
h_2=zeros(size(t)); % Initialize h vector for K_max case
h_2(1)=h_0;
for i=2:N,
if i<=step_of_delay
u1=0; % control for K_min case
u2=0; % control for K_max case
else
u1=K_min*(h_d-h_1(i-step_of_delay))+v0; % control for K_min case
u2=K_max*(h_d-h_2(i-step_of_delay))+v0; % control for K_max case
end
h_1(i)=h_1(i-1)+Ts*(u1-(v0+20*sin(0.5*t(i-1))))/(A0+A1*h_1(i-...
1)+A2*sqrt(h_1(i-1)));
h_2(i)=h_2(i-1)+Ts*(u2-(v0+20*sin(0.5*t(i-1))))/(A0+A1*h_2(i-...
1)+A2*sqrt(h_2(i-1)));
end
% Plot the results:
figure(1)
plot(t, h_1, 'b-', t, h_2, 'r--');grid on;zoom on;
legend('K=27','K=50', 4);
```

You can compare the performance of different Ks by using the code above.

2-5

The transfer function between disturbance $v(t)$ and output $y(t)$ is $\dfrac{P(s)}{1+P(s)C(s)}$.

Because this is a linear system, the output is the superposition from the input and the disturbance. Now consider the part of the output from the disturbance.

$$y_v(s) = \frac{P(s)}{1+P(s)C(s)}v(s) = \frac{P(s)}{1+P(s)C(s)}\frac{3}{s^2+9} \qquad (*) \qquad [\text{because } v(t)=\sin(3t)].$$

From $y(t) \to 0$ as $t \to \infty$, we have $y_v(t) \to 0$ as $t \to \infty$.

Thus, in (*), s^2+9 must be eliminated from the denominator. Otherwise, after the partial expansion, we know there will always be some sinusoidal components in the output. To cancel s^2+9, it is clear that the controller $C(s)$ must have s^2+9 term in the denominator.