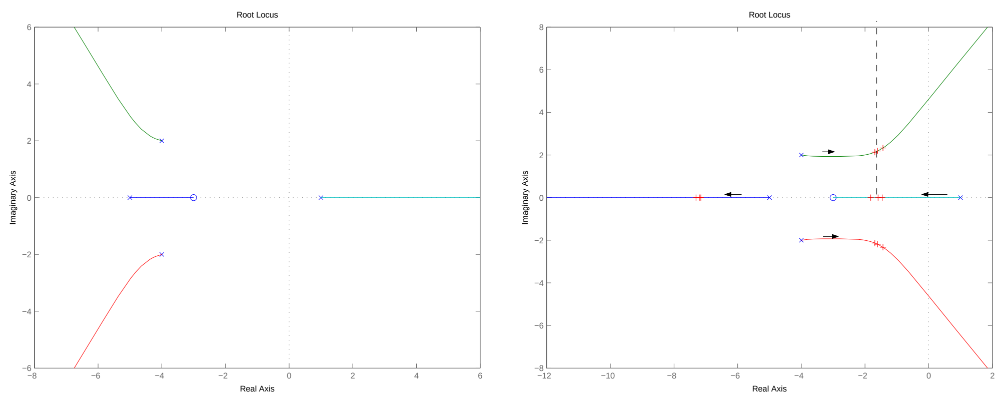
EE752 Wi2004, HW3 Solution

Ch5.Problem 3 The root locus and complementary root locus are shown in the figure. By using `rlocfind` command of Matlab with a trial an error we can find the point where the real parts of the roots are the smallest (as far to the left of Im-axis as possible). The result is: $K = 61.5$ leads to the roots

$$-1.6 \pm j2.2 \qquad -1.6 \qquad -7.2$$

If we further increase K complex conjugate roots move to the right of the line $Re(s) = -1.6$. On the other hand if we reduce K one of the real roots is greater than -1.6.



Ch5.Problem 4 The characteristic equation is

$$s^3 + (4 - p_c)s^2 + (K - 4p_c)s - Kz_c = 0$$

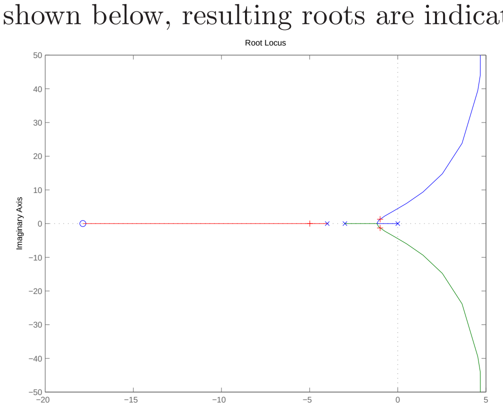
We want this to be equal to

$$s^3 + (2\zeta\omega_n + r)s^2 + (\omega_n^2 + 2\zeta\omega_n r)s + \omega_n^2 r = 0$$

with $\zeta\omega_n = 1$ and $\zeta \in [0.6\ ,\ 1]$. By equating the coefficients of the characteristic polynomial to the desired polynomial, we get $p_c = 2 - r$, $K = \omega_n^2 + 8 - 2r$, and $\omega_n^2 r = -Kz_c$. Therefore the quantity to be maximized is

$$\frac{Kz_c}{p_c} = \frac{\omega_n^2 r}{r - 2} \qquad r \geq 5, \quad \omega_n \in [1\ ,\ 10/6]$$

Obviously, the maximum is achieved when $\omega_n = 10/6$ and $r = 5$. That leads to $p_c = 3$, $z_c = -18$, $K = 0.77$. The root locus is as shown below, resulting roots are indicated for this value of K.



Ch5.Problem 6 The characteristic equation can be written as

$$1 + \frac{1}{\tau} \frac{s^2 + 5s + 10}{s(s^2 - 5s - 10)} = 0$$

drawing the root locus for $K = 1/\tau$ we get the figure shown below with grid lines for $\zeta = 0.85$ and $\zeta = 0.87$ (the largest ζ we can obtain). We see that for $\tau \in [1/27\ ,\ 1/21]$ we have $\zeta \geq 0.85$. With the largest $\zeta = 0.87$ obtained for $\tau = 1/22$.