EE 752 Homework #4
Solutions

Problem 1

The feedback system is given as $C(s) = \frac{0.5}{s+0.001}$ $\qquad P_0(s) = \frac{1}{s^2+s+1}$

with possible delay in the plant $P(s) = e^{-hs} P_0(s)$

a-) To calculate h_{max} find the feed-forward transfer function

$$G(s) = e^{-hs} G_0(s) \qquad G_0(s) = \frac{0.5}{(s^2+s+1)(s+0.001)}$$

The Bode plot and the phase and gain margins are given in the following Figure 1.

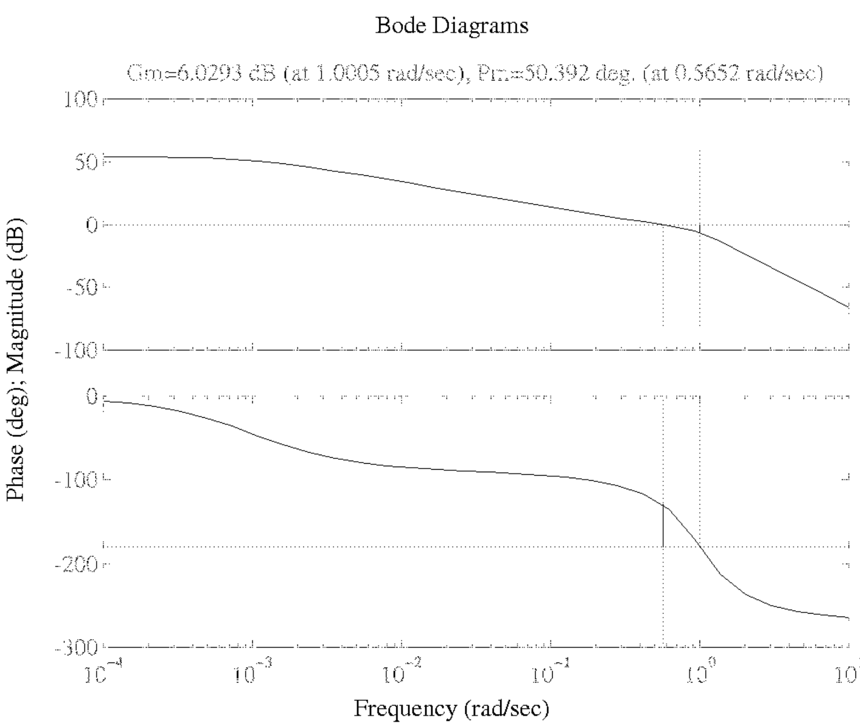

Figure 1

From the Bode plot

Phase margin (*PM*) is found to be 0.8795 rad.
Corresponding w_c is 0.5652 rad/sec.

$$h_{\max} = \frac{PM}{w_c} \qquad h_{max}=1.5561 \text{ sec.}$$

b-) h=1, Using the algorithm given in Section 7.2, minimal order Pade approximation is found to be 4.

c-) To obtain the locus of the dominant roots of $1+G_0(s)e^{-hs}=0$, Pade approximation with order 10 is used. The poles with greatest real parts are kept and plotted. The locus is given in Figure 2.

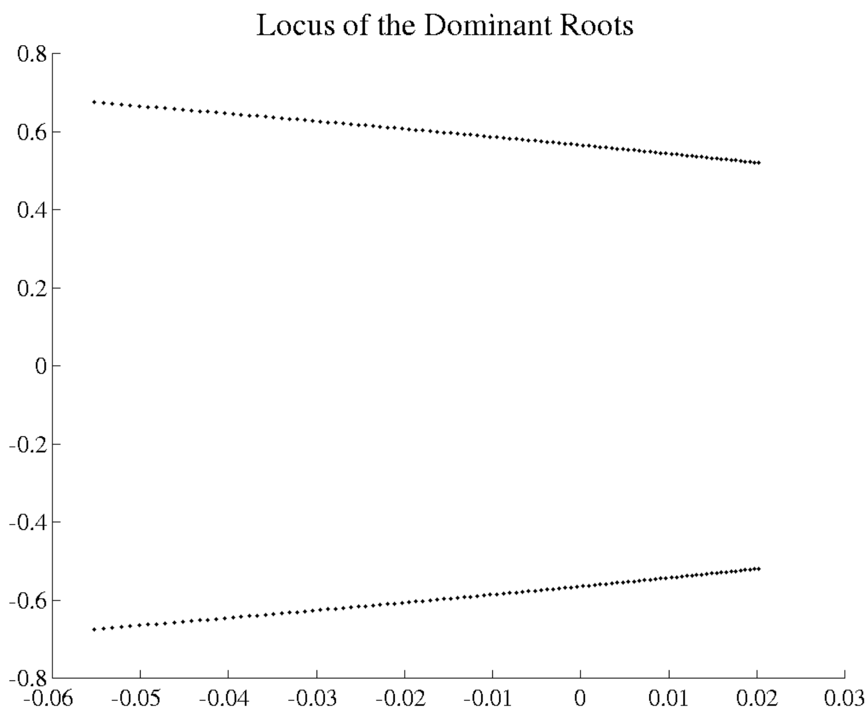

Figure 2

d-) The Nyquist plot for three different *h* values is given in Figure 3.

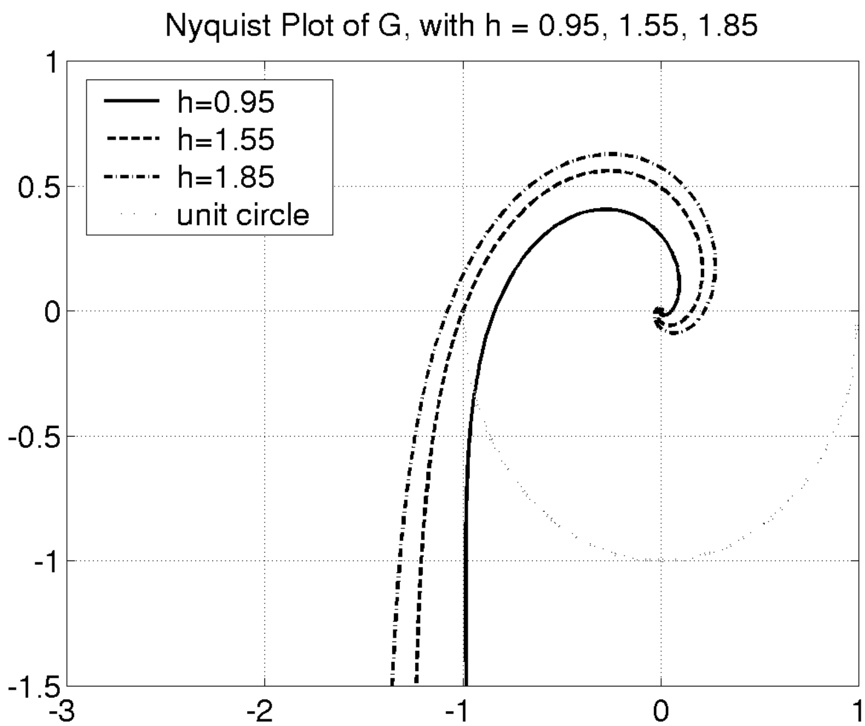

Figure 3

From Figure 3 it is observed that the system stays stable when *h=0.95 sec.* ($h< h_{max}$) and becomes unstable when *h=1.85 sec.* ($h< h_{max}$). The limit value of *h* for stability is *h=1.55 sec.* ($h= h_{max}$).

The delay margin when h=1 is found to be $h_{max} -1= 0.55$ *sec.*

Problem 2

The plant is given as $P(s) = \dfrac{e^{-hs}}{s^2 + s + 1}$ *h* is the amount of time delay in the process.

The first order controller is $C(s) = K_c \dfrac{s + z_0}{s + 5}$

Requirement: When *h=0* the closed loop system poles are equal.

a-) Characteristic polynomial of the system is:

$$\Delta(s) = (s^2 + s + 1)(s + 5) + K_c (s + z_0) = 0$$

Desired characteristic polynomial is of the form:

$$\Delta_d(s) = (s - r)^3 = 0$$

Equating these two characteristic polynomials:

$r = -2$ $\quad$ $K_c = 6$ $\quad$ $z_0 = 0.5$

The controller is of the form: $C(s) = 6\dfrac{s+0.5}{s+5}$

To find h_{max} Bode plot of the feed-forward system is plotted in Figure 4.

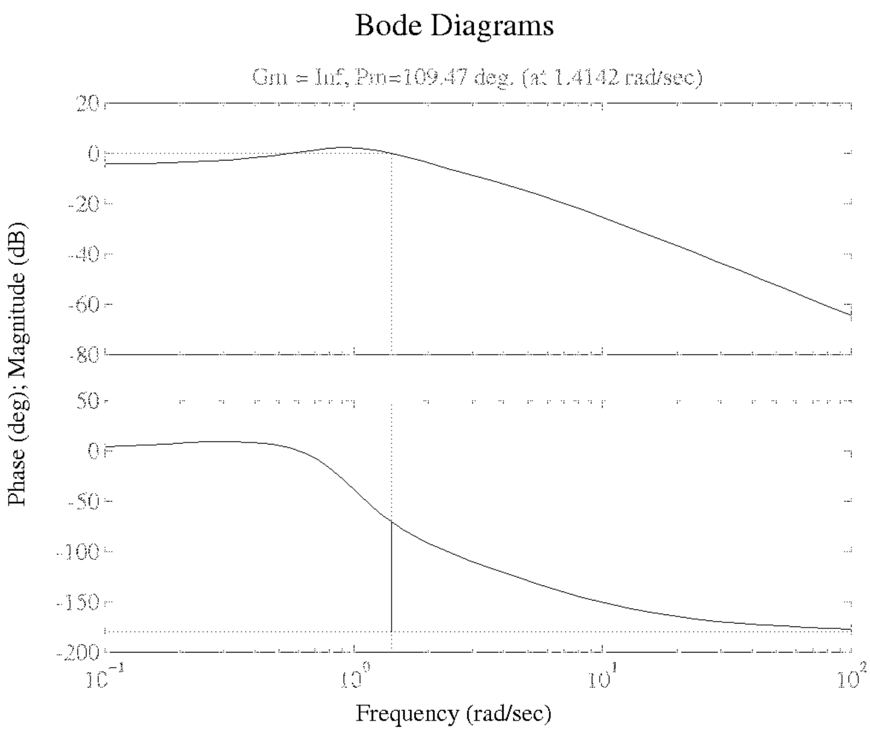

Figure 4

The Bode plot and the possible h_{max} are found to be h_{max1} =1.3510 sec. and h_{max2} =5.236 sec.
$h_{max} = min(h_{max1}, h_{max2})$ h_{max} = 1.3510 sec.
b-) Pade approximation is used to calculate the location of the dominant closed loop poles. The dominant closed loop poles are found to be $r = -0.0359 \pm i1.504$ with rlocfind Kc can be multiplied by at most 1.0723 for a stable system. Nyquits plot for the system is given in Figure 5.

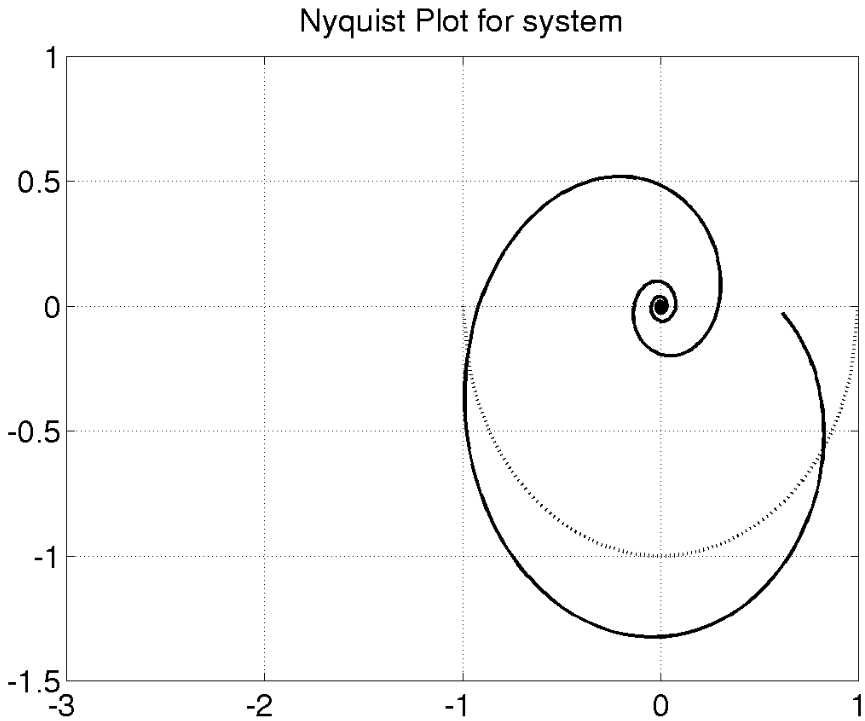

Figure 5