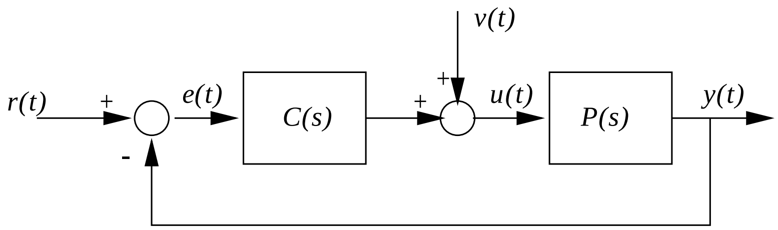
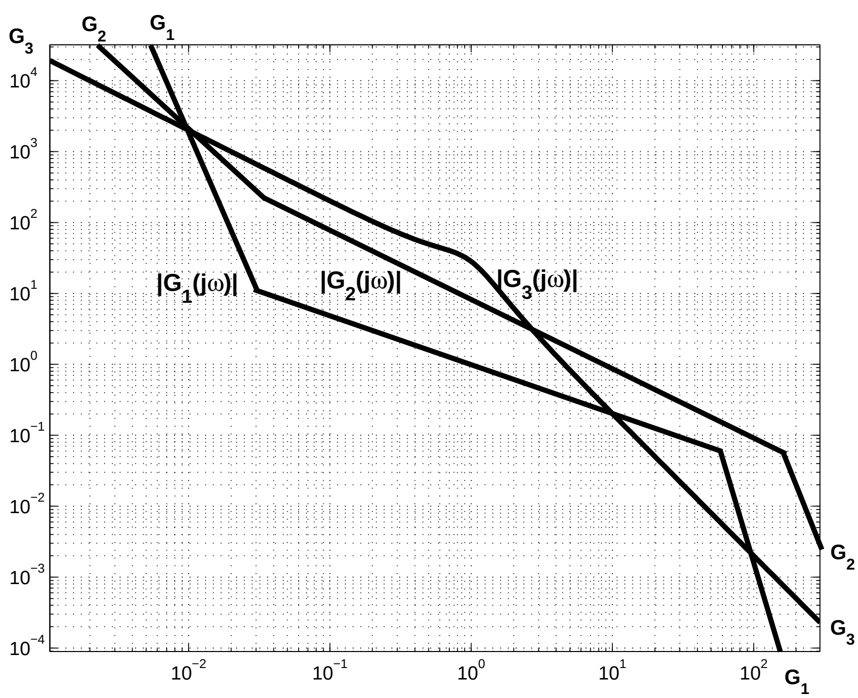
EE752 — Winter 2004 — Final Exam Part 2

This is the "In-Class" part of the final examination. It is worth 45% of the total Final Exam grade. You are asked to work alone and sign the pledge.

In this examination we deal with the following standard feedback control system, with linear time invariant controller C and plant P. When we define $G = PC$ we assume that there are no unstable pole zero cancellations in this product.



Problem 1. (10 points)
The Bode magnitude plots of three minimum phase systems, $G_i(s)$ $i = 1, 2, 3$, are as shown in the figure. We require to have the feedback system to have good phase margin, good tracking of "low frequency" ($\omega \leq 10^{-2}$rad/sec) signals and good rejection of "high frequency" ($\omega \geq 10^2 rad/sec$) noise. Which one of these G_i's represent the best design? Explain your answer in one sentence.



G_1 is better because it has the largest magnitude at low frequency (best tracking), slowest decay rate around the crossover frequency (best phase margin) , and smallest magnitude at high frequency (best noise attenuation).

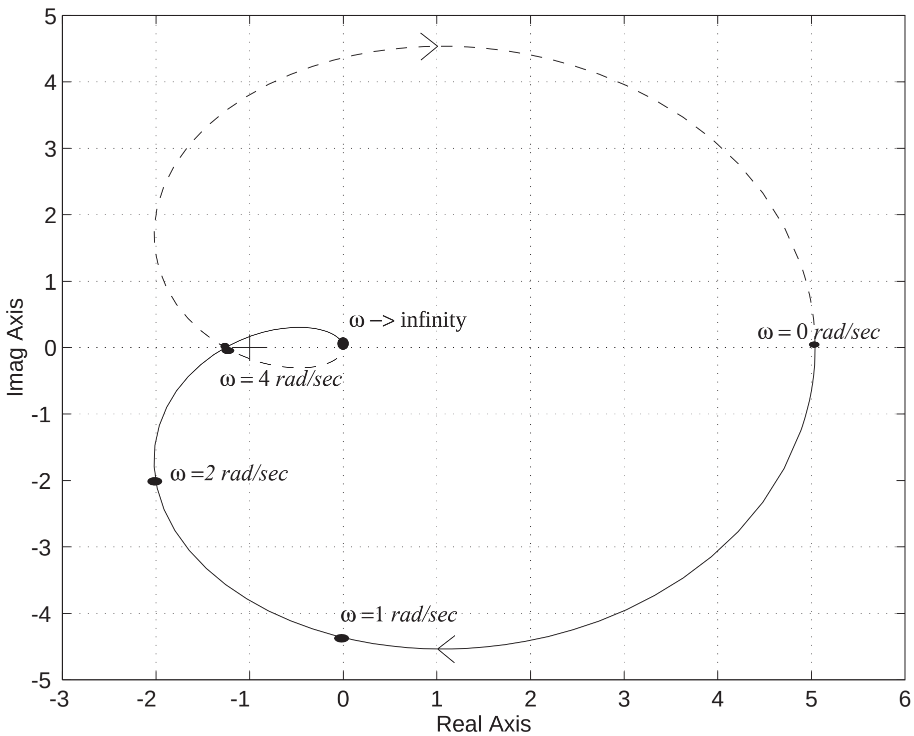
Problem 2. (15 points)

The Nyquist plot of $G(s) = P(s)C(s)$ is as given in the figure. We know that $P(s)$ and $C(s)$ do not have any right half plane poles.

(a) Is the feedback system stable? If no explain why, if yes, what is the gain margin?

(b) Consider the same plant, $P_1(s) = P(s)$, with the new controller $C_1(s) = \frac{\sqrt{2}}{4}C(s)$.
Is the feedback system stable? If no explain why, if yes, what is the phase margin?

(c) Consider the new plant $P_2(s) = e^{-0.3s}P(s)$ and the controller of part (b), $C_2(s) = C_1(s) = \frac{\sqrt{2}}{4}C(s)$
Is the feedback system stable? If no explain why, if yes, what is the delay margin?



(a) Unstable because -1 is encircled by $G(j\omega)$.
(b) Stable because we scale down $G(j\omega)$ by a factor $2\sqrt{2} > 2$, so negative real axis crossing move to the right of -1, hence no encirclement.
(c) a careful look at the data shows that $G(j2)$ passes through $(-2, -2j)$, when scaled down this point is brought to the unit circle, and then the phase margin of the non-delayed system is $\pi/4$. This means that non delayed system can tolerate a delay up to $h_{\max} = \pi/8 = 0.3927$sec. Since $h_{\max} > 0.3$ the system is stable with a $DM = 0.0927$sec.

Problem 3. (20 points)
For the plant

$$P_o(s) = \frac{s-1}{s^2+2s+2}$$

we are free to use any controller. We require that the steady state error for the unit step reference input is zero, and the feedback system is robustly stable for the set of all possible plants

$$\mathcal{P} = \{P = P_o + \Delta \quad : \quad \Delta \text{ is stable and } \ |\Delta(j\omega)| < \alpha \text{ for all } \omega \ \}$$

(a) Determine the largest possible α for which this problem is solvable.

(b) For the value of α determined in part (a), find a controller that meets all the design specifications.

Solution: All controllers stabilizing the nominal feedback system are in the form

$$C = \frac{Q}{1 - P_o Q} \qquad Q \text{ is stable.}$$

The requirement that $e_{\text{ss}} =$ for $R(s) = 1/s$ implies $Q(0) = 1/P_o(0) = -2$. On the other hand robustness condition is equivalent to

$$|\alpha \ Q(j\omega)| \leq 1 \quad \forall \ \omega$$

Thus, a necessary condition for the solution is $\alpha < \frac{1}{2}$. Actually, by taking $\alpha = \frac{1}{2}$, and $Q(s) = -2$ we can solve this problem. So maximum allowable value of $\alpha = 0.5$.

For $Q(s) = -2$ the controller becomes

$$C(s) = \frac{(-2)(s^2+2s+2)}{(s^2+2s+2) - (-2)(s-1)} = \frac{-2(s^2+2s+2)}{s(s+4)}$$