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ECE 327: *Electronic Devices and Circuits Laboratory I*

Class Pre-Quiz (Ungraded)

Description. *This* **ungraded** *quiz provides the instructor with information on your present comfort with analog circuits. Answer these questions as best as you can. Try to reason through problems that are unfamiliar to you. If you cannot find the desired solution, give as much information as you can about the circuit that you think would assist you in finding the correct solution.* ***Be sure to state all of your assumptions****; important information has been deliberately omitted from some of these problems. Complete this quiz with* ***closed book*** *and* ***closed notes.***

Problem Q0-1: Rubber Diode/Rubber Zener

Find the potential v_{out} from Figure Q0-1.1 as a numeric voltage. Assume that the current i_b is large enough to avoid the trivial case. Be sure to state all other assumptions (or approximations) that you make.

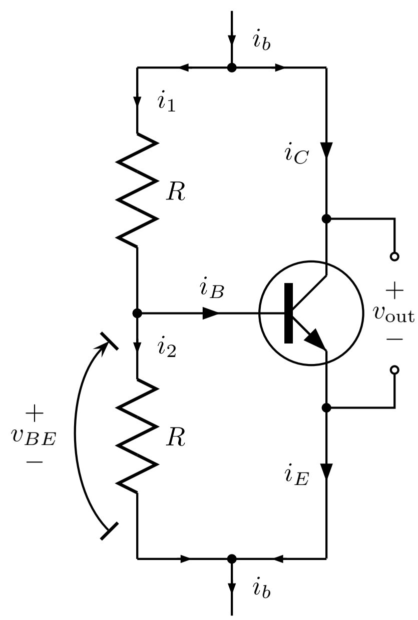
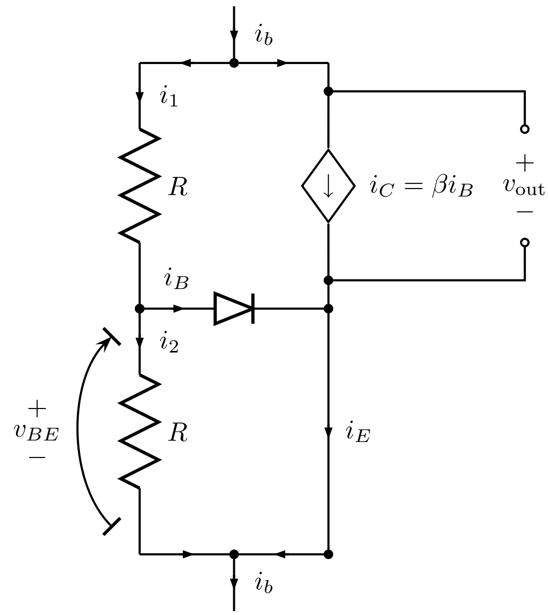


Figure Q0-1.1: Rubber Zener.

Solution

See next page.

Assume that the transistor is biased so that it is in active mode (i.e., assume that biasing current i_b is sufficiently large; we will be more precise momentarily). The circuit can then be redrawn with a diode and a current-controlled current source (CCCS) like



where β is a gain property of the transistor. Assume that β is very large (i.e., $\beta \gg 0$). So the larger i_B grows, the much larger i_C grows, which then absorbs a larger proportion of the i_b biasing current. As more of i_b is directed to i_C, the current supply to i_B is choked off. So, as long as β is very large, i_B will be nearly insignificant and $i_1 \approx i_2$ (and $i_C \approx i_E$). Thus, the voltage drop across the upper resistor must be the same as the voltage drop across the lower resistor (i.e., v_{BE}). However, because we assumed that the transistor is active, the diode is turned on, and will maintain a voltage drop of approximately $v_{BE} \approx 0.7\,\text{V}$. Therefore, the voltage drop across both resistors is maintained at $\boxed{v_{\text{out}} \approx 1.4\,\text{V}.}$

Biasing Current? We assumed that i_b was high enough to bias the transistor into its active region. We really need i_b to be high enough to direct enough current through the diode to turn it on. So we need $i_B > v_{BE}/R$; that is, $i_B > (0.7\,\text{V})/R$.

Voltage Regulator: The Rubber Zener is a kind of voltage regulator. At every time instant, the transistor adjusts i_C to "shunt" a proportion of i_b around the resistors so that $i_2 R \approx 0.7\,\text{V}$.

Recall the transfer characteristics of a transistor. If the current in i_2 is too low, v_{BE} will be low and i_C will be decreased. When i_C is decreased, a larger proportion of current will flow through i_1 and i_2, which will push v_{BE} closer to $0.7, \text{V}$. Similarly, if i_2 is too high, v_{BE} will be high and i_C will be increased. However, increasing i_C causes less current to flow through i_1 and i_2, and so v_{BE} will decrease.

It should be clear that this circuit is a negative feedback system. There is a natural *stable* equilibrium at $v_{BE} = 0.7\,\text{V}$. Whenever the system is perturbed off of that equilibrium, the system moves back toward it.

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Problem Q0-2: Operational Amplifier Analysis

Determine the v_{out}/v_{in} gain of the circuit shown in Figure Q0-2.1. Be sure to specify the sign of the gain.

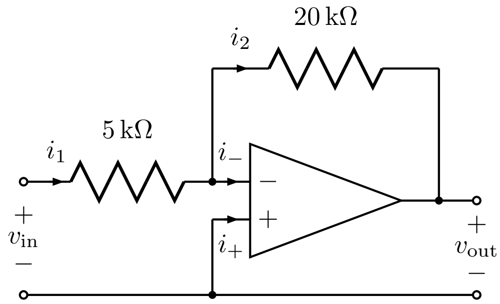

Figure Q0-2.1: An operational amplifier circuit.

Solution

Assume that the operational amplifier (OpAmp) is ideal. Therefore,

$$i_- = i_+ = 0\,\text{A},$$

and so

$$i_1 = i_2. \tag{Q0-2.1}$$

Because there is a direct current (DC) path from the output of the OpAmp to its inverting input (i.e., the input labeled "−"), we can assume that there is no difference in electric potential between its inverting and non-inverting inputs. Thus,

$$i_1 = \frac{v_{in}}{5\,\text{k}\Omega} \qquad \text{and} \qquad v_{out} = -i_2\,(20\,\text{k}\Omega)\,. \tag{Q0-2.2}$$

The sign of v_{out} is negative because the current has been drawn flowing into the OpAmp output. From combining Equation Q0-2.1 with Equation Q0-2.2,

$$v_{out} = -v_{in}\left(\frac{20\,\text{k}\Omega}{5\,\text{k}\Omega}\right) = -4\,v_{in},$$

and so $\boxed{v_{out}/v_{in} = -4.}$ That is, this circuit is $\boxed{\text{an } \textit{inverting voltage amplifier} \text{ with a magnitude 4 gain.}}$ □

Extension: What is the input impedance of this circuit?

When this circuit is operating, the inverting input (i.e., input "−") will have the same electric potential as the common reference for the circuit. Therefore, from the perspective of the input, the circuit is a 5 kΩ resistor. For $v_{in} = 5\,\text{V}$, the current required from the input will be $i_1 = 1\,\text{mA}$. That is, even though no current will go into the inputs of the ideal operational amplifier, possibly substantial current can go into the circuit. For example, imagine if 20 Ω and 5 Ω resistors were used instead. While the gain would be the same, the required current (from input and OpAmp) would be much higher (for 5 V input, it would be 1 A!).

Can resistor sizes be increased? Resistor packs are manufactured that provide paired resistors with precise resistance ratios, and so it's easy to increase resistor sizes while maintaining the same ratio. However, leakage current into a *real* operational amplifier input can cause *huge offsets* to appear at the output. Additionally, use of large resistors can activate parasitic capacitances[1] that can affect bandwidth or drive the circuit to instability.[2] So keeping the resistances small makes the design robust to leakage and some AC effects, but it greatly increases current drive.

[1] One such *parasitic capacitance* comes from the fact that adjacent pins separated by air form a natural capacitor.

[2] A related problem occurs with alternating-current (AC) coupled circuits that need a low-impedance circuit for DC biasing. *Bootstraping* can be used to increase AC input impedance. Current that would come from the input is actually sourced by the circuit's own output. See Horowitz and Hill's 1989 *The Art of Electronics* (2nd ed.) for more information on bootstrapping.

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Problem Q0-3: Name the Circuit

The circuit in Figure Q0-3.1 shows the prototypical implementation of a common element in systems theory (i.e., one of the blocks you use frequently in classes like ECE 351 and ECE 352). Name that element.

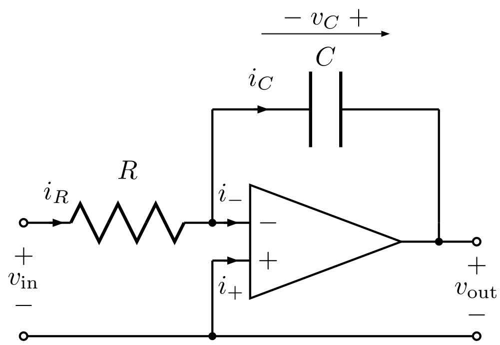


Figure Q0-3.1: Electronic systems building block.

Solution

There are two ways to solve this problem.

Method 1 (Frequency Domain)

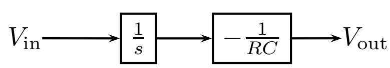
Take $V_{\text{in}}(s)$ to be the Laplace transform of $v_{\text{in}}(t)$. Recall that the complex impedance of the resistor Z_R and the complex impedance of the capacitor Z_C are

$$Z_R = R \qquad \text{and} \qquad Z_C = \frac{1}{sC},$$

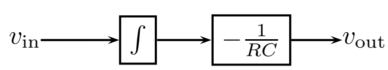
respectively. Strictly speaking, there is no DC path from the OpAmp output to its inverting input. However, if we assume that the capacitor is charging, there will be displacement current traveling from the OpAmp output to the inverting input, and so the analysis from Problem Q0-2 can still be used[3]. Thus, the Laplace transform $V_{\text{out}}(s)$ of the output $v_{\text{out}}(t)$ is

$$V_{\text{out}}(s) = -V_{\text{in}}(s)\frac{\frac{1}{sC}}{R} = -V_{\text{in}}(s)\frac{1}{sRC} = -V_{\text{in}}(s)\frac{1}{s}\frac{1}{RC}$$

which can be depicted with the block diagram



However, division by s in the Laplace domain is equivalent to integration in the time domain. So an equivalent time-domain block diagram is



That is,

$$v_{\text{out}}(t) = -\frac{1}{RC}\int_0^t v_{\text{in}}(\tau)\,\mathrm{d}\tau.$$

Therefore, the circuit is an $\boxed{\textit{integrator}}$ with gain $-1/(RC)$. Recall that displacement current must be flowing through the capacitor for this analysis to be valid. At some point, the OpAmp may saturate and integration will be clipped.

[3] In a *real* OpAmp, there will be a slight imbalance between the two inputs that has the effect of a small DC source attached to one of the inputs. We will discuss the impact of this effect (and how to compensate for it) briefly in a later class.

Method 2 (Time Domain)

Using the displacement current argument from the previous method[4],

$$i_C(t) = i_R(t) = \frac{v_{\text{in}}(t)}{R} \qquad \text{and} \qquad v_{\text{out}}(t) = -v_C(t) \tag{Q0-3.1}$$

However, the transfer characteristics of a capacitor dictate that

$$i_C(t) = C\frac{\mathrm{d}v_C(t)}{\mathrm{d}t}, \tag{Q0-3.2}$$

and so

$$v_C(t) = \frac{1}{C}\int_0^t i_C(\tau)\,\mathrm{d}\tau. \tag{Q0-3.3}$$

Combining Equation Q0-3.1 and Equation Q0-3.3 yields

$$v_{\text{out}}(t) = -\frac{1}{RC}\int_0^t v_{\text{in}}(\tau)\,\mathrm{d}\tau,$$

as expected. This time-domain analysis provides insight into how the OpAmp integrator works. The transfer characteristics (i.e., the physics) of the capacitor actually performs the integration. By consistently lowering the output by the appropriate amount, the OpAmp pulls current into the capacitor that is directly proportional to the input voltage to the circuit. The resulting accumulation of charge is the source of the integration.

Extension: Ideal OpAmp

So far, we have only concerned ourselves with the use of an OpAmp. That is, we have taken for granted the implementation of the OpAmp. Here, we illustrate how an OpAmp works with a motivating example.

Limiting Circuit Elements: Consider the resistor with value R.

- The *limiting circuit element (LCE)* of this resistor as $R \to 0$ is a short circuit.
- The LCE of this resistor as $R \to \infty$ is an open circuit.

Recall the transfer characteristics of a capacitor, shown in Equation Q0-3.2 and Equation Q0-3.3.

- The LCE of the capacitor as $C \to 0$ has $i_c(t) \equiv 0$, which is an open circuit.
- The LCE of the capacitor as $C \to \infty$ has $v_c(t) \equiv 0$, which is a short circuit (i.e., a short circuit to everything but DC).

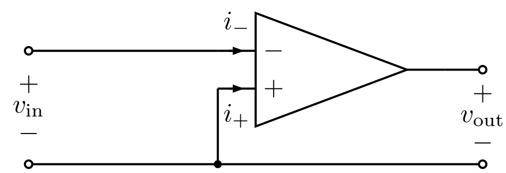
Capacitors with infinite capacitance (i.e., $C \to \infty$) are often used in circuit diagrams when an arbitrarily large capacitor is required that passes all frequencies of interest but do not pass DC.

Limit of an Integrator: Consider Figure Q0-3.1; however, take $R \to 0$ and $C \to 0$ so that $RC \to 0$. The limiting results are (with some abuse of notation)

$$v_{\text{out}}(t) = -\infty\int_0^t v_{\text{in}}(\tau)\,\mathrm{d}\tau, \qquad \text{and} \qquad V_{\text{out}}(s) = -\frac{\infty}{s}V_{\text{in}}(s).$$

That is, the limiting circuit is an integrator with infinite gain. However, substituting the LCE capacitor and LCE resistor into Figure Q0-3.1 yields

[4] Again, there are serious problems with this method due to the imperfections in real OpAmps.



which is an OpAmp without any additional circuit elements added. Thus, an OpAmp may be viewed as an integrator with infinite gain.

Control Interpretation: Recall the characteristics of a system that feeds back the difference between its output and some reference.

- To increase the speed at which the output tracks changes in the reference input, the gain on the difference from the reference should be made as high as possible.
- To guarantee that the error between the output and a constant reference approaches zero as time goes to infinity, the difference from the reference should be integrated.

Now consider an OpAmp connected for negative feedback (i.e., output tied to inverting input). The inverting input may be viewed as the system's output, and the non-inverting input may be viewed as the reference to the system. The OpAmp calculates the difference between the two, multiplies them by a high gain, and integrates the result. The result is that the OpAmp quickly brings the inverting input as close to the non-inverting input as possible. Therefore, when an OpAmp is connected properly, the inverting input may be taken to be at the same level as the non-inverting input[5]

[5] For brevity, significant details have been omitted about OpAmp dynamic response and stability. These details cannot be taken for granted in general.

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