Chapter 4

Single-Sideband Amplitude Modulation

4.1 Background

A shortcoming of double-sideband, suppressed-carrier amplitude modulation (DSBSC-AM) is the spectral inefficiency due to redundancy of the upper and lower sidebands. In this chapter, we explore the modulation and demodulation of single sideband signals.

4.1.1 Hartley modulator

A direct approach for creating a single sideband AM signal (SSB-AM) is to remove either the upper or lower sideband by filtering the DSBSC-AM signal. This approach is shown in Figure 4.1 and is known as the *frequency discriminator* method. The baseband, DSBSC-AM, and SSB-AM spectra are illustrated in Figure 4.2. For a baseband signal with one-sided bandwidth W, the *upper* sideband consists of frequencies $[f_c, f_c + W]$, and the *lower* sideband signal is the energy is the range $[f_c - W, f_c + W]$.

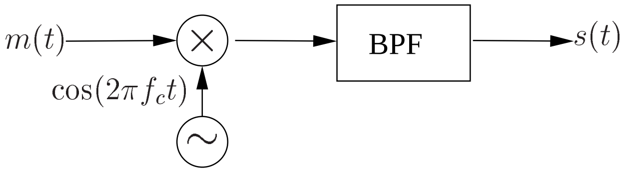

Figure 4.1: Single sideband modulator using the frequency discrimination approach.

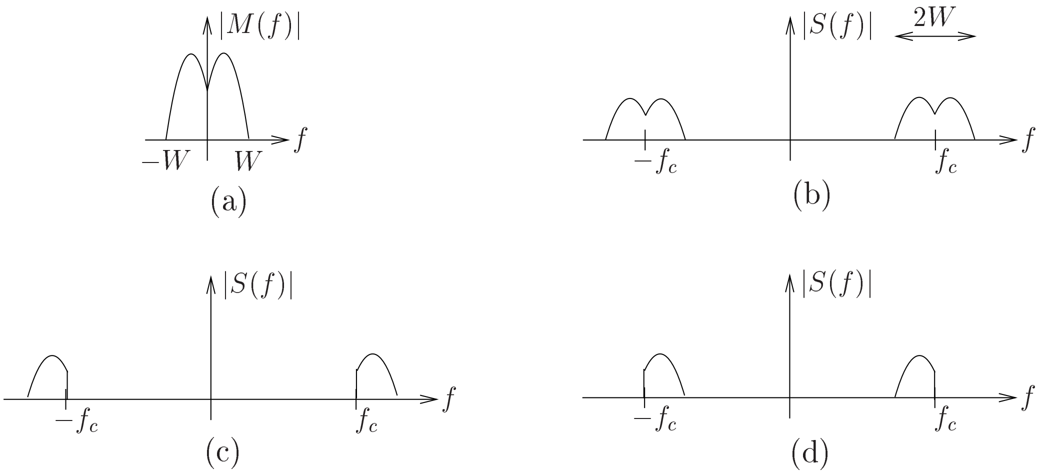

Figure 4.2: Magnitude spectra: (a) baseband; (b) DSBSC-AM; (c) upper SSB; (d) lower SSB.

Alternatively, a quadrature modulator can be used to create a SSB-AM signal by selecting the quadrature signal to coherently cancel either the upper or lower sideband from the inphase channel. This approach is known as the *Hartley modulator* or *phase discrimination* method and is shown in Figure 4.3. Specifically, the SSB-AM signal is given by

$$s(t) = m(t)\cos(2\pi f_c t) \mp \hat{m}(t)\sin(2\pi f_c t), \tag{4.1}$$

where $\hat{m}(t)$ is the *Hilbert transform* of the message $m(t)$. Using the minus sign in Equation 4.1 results in *upper SSB*, whereas selection of the plus sign yields *lower SSB*. The Hilbert transform has impulse response and frequency response

$$h(t) = \frac{1}{\pi t} \qquad \overset{\mathcal{F}}{\longleftrightarrow} \qquad H(f) = \begin{cases} -j, & f > 0 \\ j, & f < 0 \end{cases} \tag{4.2}$$

The Hilbert transform is a wideband −90° phase shifter.

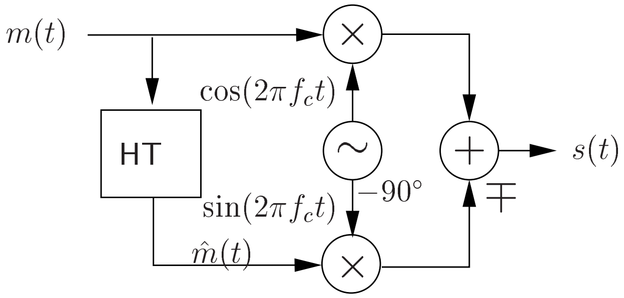

Figure 4.3: Hartley modulator for SSB; "−" gives upper SSB, and "+" gives lower SSB.

4.1.2 SSB demodulation

Single sideband demodulation can be accomplished using the coherent demodulator of Figure 3.2. Although for SDBSC-AM the Costas loop can be use to recover the carrier frequency and phase, SSB-AM requires an alternative approach due to the nonzero quadrature signal. Figure 4.4 illustrates a coherent demodulator for SSC-AM using a low-power pilot tone,

$A_p \cos(2\pi f_c t)$, added to the SSB-AM signal. In the figure, the signal $\tilde{v}[n]$ is the complex-valued IF signal produced by an RF-to-IF downconversion. The dashed lines represent complex-valued signals, and solid lines denote real-valued signals. For an upper SSB signal, the lowpass filter captures the pilot tone for use in the mixer. The highpass filter removes the pilot tone from the upper SSB signal. If groups delays are equivalent through each filter, then the real-valued output, $y_{\mathrm{I}}[n]$, is the inphase output of a coherent demodulator. Thus, the demodulator recovers the message $m(t)$ (up to a scale factor) in the absence of channel noise and distortion.

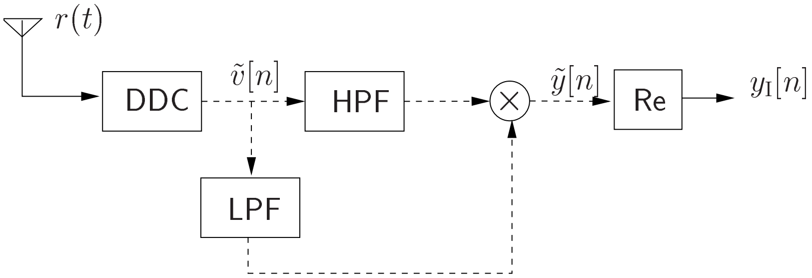

Figure 4.4: Coherent demodulation of SSC-AM using a pilot tone.

4.1.3 Vestigial sideband

under construction

4.2 Explorations

For preparing a lab report, eight requested responses are marked in the margin.

4.2.1 Exercise: Hartley modulator

In this exercise, you will explore single sideband signal generation using the Hartley modulator.

1. Recall the square wave $g(t)$ with period $1/f_m$ has Fourier series

$$g(t) = \frac{4}{\pi} \sum_{k=1,3,5,\dots} \frac{1}{k} \sin(2\pi f_m k t). \tag{4.3}$$

   With `TX Streaming.vi`, create a message signal, $m(t)$, using the first two odd harmonics ($k = 1, 3$) of a square wave with $f_m = 30\,\text{kHz}$. Be sure to set `IQrate` to match the `ActualIQrate` implemented by the digital upconverter. To learn about a useful command for the `Matlab Script Node`, use `help hilbert` in a MATLAB command window. View the waveform, $m(t)$, at the transmitter.

2. Use double sideband, suppressed-carrier amplitude modulation with carrier frequency 10 MHz. Transmit and receive the signal using the PCI-5640R IF transceiver; view the signal spectrum at the receiver.

3. Transmit $m(t)$ using single sideband modulation with the upper sideband; view the signal spectrum at the receiver.

4. Transmit $m(t)$ using single sideband modulation with the lower sideband; view the signal spectrum at the receiver.

   Compare and contrast the three spectra: DSBSC-AM, SSB-AM upper sideband, and SSB-AM lower sideband. Question 4.1

5. Create a single sideband modulation of the message $m(t)$ with the upper sideband and add a pilot tone at the carrier frequency. Select a pilot amplitude so that less than half of the transmitted signal energy is in the pilot. The pilot will aid in coherent demodulation; view the signal spectrum at the receiver.

4.2.2 Exercise: SSB coherent demodulation

In this exercise, you will implement the SSB demodulator shown in Figure 4.4. You will create an intermediate frequency (IF) signal, $\tilde{v}[n]$, by using the PCI-5640R digital downconverter with $f_{c:Rx} \neq f_{c:Tx}$. The demodulator you implement will then coherently convert the signal from IF to baseband, which requires generating a reference oscillator with the proper frequency and phase. The pilot tone is used to create this oscillator.

1. Set your receiver for $f_c = 9.7\,\mathrm{MHz}$, yielding IQ samples from the digital downconverter at an intermediate frequency of 300 kHz. Select the `IQrate` at the receiver to 1.5625 MHz.

2. Modify the `Rx Streaming.vi` template to implement the coherent demodulator shown in Figure 4.4.

   - Before implementing the design in LabVIEW, first make a sketch of the filters' required magnitude responses, noting the edge frequencies of passbands and stopbands. Question 4.2
   - To maintain coherence, the lowpass and highpass filters applied to the IF signal should have the same group delay. Equiripple FIR lowpass and highpass filters are recommended; the vi's are located at `Signal Processing >> Filters`. A recommended filter length is `151`; the highpass filter length must be odd.
   - At the receiver, display the IF magnitude spectrum, $|\tilde{V}(f)|$, and the spectra at the outputs of the lowpass and highpass filters. In addition, display the real-valued demodulated message, $y_{\mathrm{I}}[n]$. Note: to remove from the display of $y_{\mathrm{I}}[n]$ the edge effects due to the data block size, use the `array subset` vi to keep the first 3000 samples.
   - You may need to modify your filter designs in order to achieve proper demodulation; the displays can aid in iterating your design choices.

   Question 4.3 List the final parameters selected in your filter designs.

3. Once you have achieved proper demodulation of the message $m(t)$, explore the operation of the coherent demodulator.
   (a) Change the transmit carrier frequency in steps of ±1 kHz and observe the demodulated waveform, $y_{\mathrm{I}}[n]$. What range of transmit carrier frequencies is supported by your demodulator? Explain your observations in terms of the LPF frequency response, HPF frequency response, and f_m. Question 4.4
   (b) Lower the `ActualIQrate` at the receiver. At what value does the receiver fail? Why? Hint: `ActualIQRate` configures the decimation filter in the digital downconverter (DDC). Question 4.5
   (c) Change the filter order of the lowpass filter by two or more and observe the demodulator output, $y_{\mathrm{I}}[n]$. What do you observe? Explain the observed behavior. Question 4.6
   (d) Vary the amplitude of the pilot tone at the transmitter; experiment to find a low value of amplitude at which the demodulator operates properly. Question 4.7
   (e) For a filter length of 151 and a sampling rate of 1.5625 MHz, what is the delay, in milliseconds, of a linear phase FIR filter? Question 4.8
   (f) *(Optional theory question from digital signal processing)* Why must a highpass linear phase FIR filter have odd length? Question 4.9