Chapter 5

Frequency Modulation

5.1 Background

5.1.1 Modulation

In amplitude modulation, the message signal modulates the carrier amplitude; in contrast, for phase modulation the message signal modulates the instantaneous carrier phase, $\theta(m(t))$, to yield

$$s(t) = A\cos(\theta(m(t))). \tag{5.1}$$

Frequency modulation (FM) is the special case in which the message signal modulates the instantaneous frequency of the carrier waveform. Recall that the instantaneous frequency is given by

$$2\pi f_i(t) = \frac{d}{dt}\theta(m(t)) \tag{5.2}$$

For FM, let

$$f_i(t) = f_c + k_f m(t) \tag{5.3}$$

where k_f is the *frequency sensitivity* and f_c is the unmodulated *carrier frequency*. Thus, we have

$$s(t) = \cos(\theta(m(t))) = \cos\Big(2\pi f_c t + 2\pi k_f \int_0^t m(\tau)d\tau\Big). \tag{5.4}$$

The signal $s(t)$ can be written in terms of in-phase and quadrature components (see Fig. 3.1 using the trigonometric identity

$$\cos(A + B) = \cos(A)\cos(B) - \sin(A)\sin(B) \tag{5.5}$$

where

$$A = 2\pi f_c t \qquad \text{and} \qquad B = 2\pi k_f \int_0^T m(\tau)d\tau. \tag{5.6}$$

AM and FM modulation of a sinusoidal message signal are depicted in Figure 5.1.

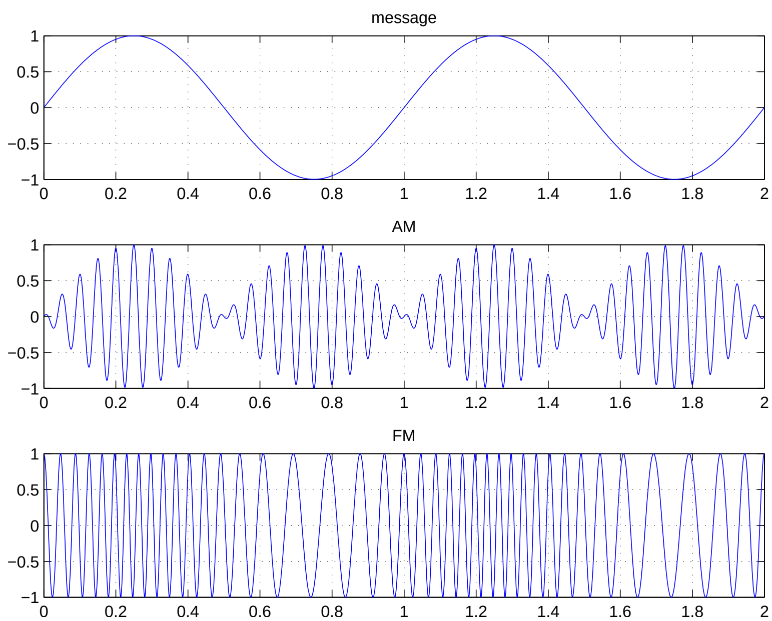

Figure 5.1: Comparison of AM and FM modulation of a sinusoidal message signal.

The *modulation index* β is the ratio of the *peak deviation frequency*, Δf, to the modulation frequency,

$$\beta = \frac{\Delta f}{f_c} = \frac{k_f \max|m(t)|}{f_c}. \tag{5.7}$$

FM is a nonlinear modulation, and in contrast to AM the spectrum of the modulated signal is not easily characterized. For the simple case of $m(t) = \sin(2\pi f_m t)$ it follows that

$$s(t) = \sum_{n=-\infty}^{\infty} J_n(\beta)\cos(2\pi(f_c + nf_m)t) \tag{5.8}$$

where J_n is the n-th order Bessel function of the first kind,

$$J_n(x) = \sum_{m=0}^{\infty} (-1)^m \frac{(\frac{1}{2}x)^{n+2m}}{m!(n+m)!}. \tag{5.9}$$

An approximation known as *Carson's rule* provides an estimate of the bandwidth of an FM signal,

$$B = 2(\Delta f + W) \tag{5.10}$$

where W is the one-sided bandwidth of the message signal. Increasing the frequency sensitivity, k_f, decreases spectral efficiency but increases robustness to noise and interference.

5.1.2 Demodulation

The discriminator is the earliest demodulation technique for FM signals. The chain rule of differentiation gives

$$\frac{d}{dt}\cos\Big(\varphi(t)\Big) = -\frac{d\varphi(t)}{dt}\sin\Big(\varphi(t)\Big). \tag{5.11}$$

Consequently, the derivative of the modulated waveform is

$$\begin{aligned} \frac{d}{dt}s(t) &= \frac{d}{dt}\cos\Big(2\pi f_c t + 2\pi k_\mathrm{f} \textstyle\int_0^t m(\tau)d\tau\Big) \\ &= -\Big[2\pi f_c + 2\pi k_\mathrm{f}\, m(t)\Big]\sin\Big(2\pi f_c t + 2\pi \textstyle\int_0^t m(\tau)d\tau\Big). \end{aligned} \tag{5.12}$$

Observe in Eqnation 5.12 that the derivative of $s(t)$ is a large-carrier AM signal (assuming $f_c > k_\mathrm{f}\, m(t)$), which can be demodulated using an envelope detector, as shown in Figure 5.2.

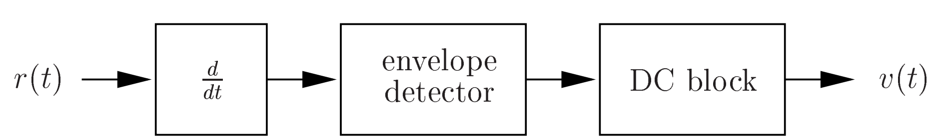

Figure 5.2: Discriminator for FM demodulation.

For digital implementation, consider processing the inphase and quadrature components, $\tilde{v}(t) = v_{\mathrm{I}}(t) + jv_{\mathrm{Q}}(t)$, provided by an RF-to-IF downconverter. The downconverter shifts the carrier frequency f_c to an IF frequency f_{IF} and introduces a phase offset, ϕ. Accordingly, the IF signal can be written

$$\tilde{v}(t) = A\exp\{2\pi f_{IF}t + \phi + \theta_m(t)\} \tag{5.13}$$

where $\theta_m(t) = 2\pi k_f \int_0^t m(\tau)d\tau$. The phase of $\tilde{v}(t)$ is given by

$$\psi(t) = \mathrm{atan}\frac{v_{\mathrm{Q}}(t)}{v_{\mathrm{I}}(t)} = 2\pi f_{IF}t + \phi + \theta_m(t). \tag{5.14}$$

Observe that the derivative of this phase yields

$$\begin{aligned} \frac{d}{dt}\psi(t) &= 2\pi f_{IF} + \frac{d}{dt}\theta_m(t) \\ &= \frac{d}{dt}\mathrm{atan}\frac{v_{\mathrm{Q}}(t)}{v_{\mathrm{I}}(t)} \\ &= \frac{v_{\mathrm{I}}(t)\frac{d}{dt}v_{\mathrm{Q}}(t) - v_{\mathrm{Q}}(t)\frac{d}{dt}v_{\mathrm{I}}(t)}{v_{\mathrm{I}}^2(t) + v_{\mathrm{Q}}^2(t)}. \end{aligned} \tag{5.15}$$

Equation 5.15 suggests a digital implementation of the discriminator, as shown in Figure 5.3.

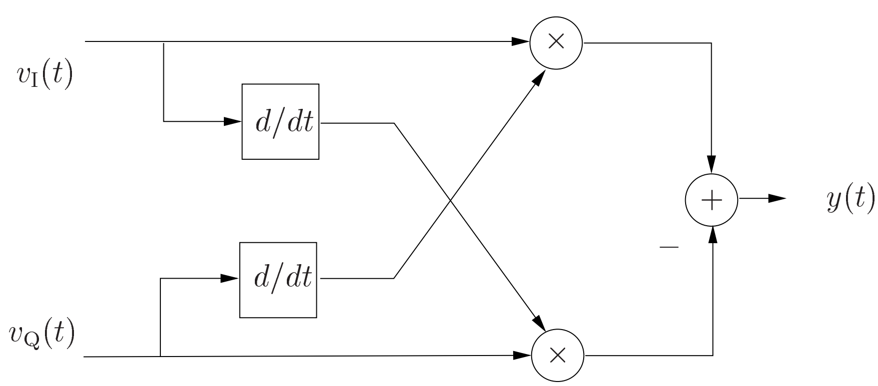

Figure 5.3: Digital implementation of an FM discriminator.

5.2 Exercise: Analog discriminator

In this exercise, you will use the simple analog components from Chapter 2 to implement a discriminator circuit for noncoherent demodulation of an FM signal. From the Fourier transform, recall that a derivative corresponds to a linear frequency response,

$$\frac{ds(t)}{dt} \stackrel{\mathcal{F}}{\longleftrightarrow} j2\pi f S(f).$$

Thus, the derivative of a bandpass signal can be approximated (up to a scaling factor) by a filter $H(f)$ that has linear magnitude response and constant group delay across the passband of the signal.

1. To begin, find the transfer function $H(f)$ of a parallel LC circuit in Figure 2.7. Recall from Chapter 2 that the variable capacitor has capacitance $30 - 160\,\mu$F and the coil has nominal inductance to give resonance at 880 kHz. Question 5.1: $H(f)$

   Use MATLAB to create an approximate magnitude response plot, and write the transfer function in terms of arbitrary L and C. Question 5.2: plot

2. Build the circuit shown in Figure 5.4. The italicized notes in the figure identify the colors of leads to the various components. The variable capacitor has three leads, which are labeled left-to-right with tuning knob facing upwards. (See Chapter 2 for accessing $s(t)$ from the SMA connector using alligator clips.)

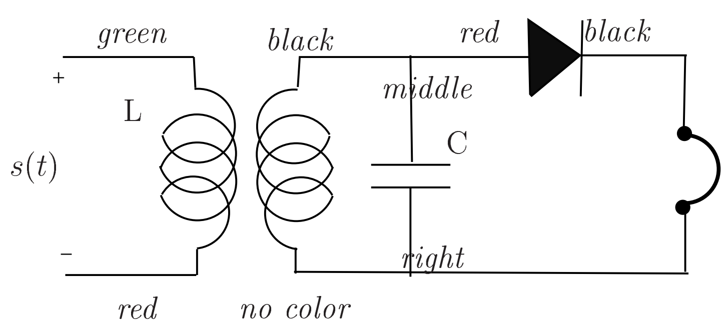

Figure 5.4: Simple circuit to implement an FM discriminator.

3. Copy `Tx Streaming.vi` to your workspace and modify the MATLAB SCRIPT NODE to implement FM modulation of an audio or sinusoidal message signal. You will need to determine the inphase and quadrature components to implement Equation 5.4. (In MATLAB, you may find the command `cumsum` useful for integration of the message signal.)

   Question 5.3: $m_{\text{I}}(t)$ and $m_{\text{Q}}(t)$

   Select carrier frequency f_c and peak deviation frequency Δf for proper operation, based on your results from step (1).

   Question 5.4: Select f_c and Δf.

4. Transmit an audio waveform and listen to the demodulated output. Repeat using a 440 Hz tone.

5. Using an audio signal as the message, increase the peak deviation frequency Δf in small increments until you can hear significant distortion of the demodulated waveform. Record your results and explain your observation in terms of $H(f)$.

   Question 5.5: Observe & explain.

6. Using an audio signal as the message, *increase* f_c on the `Tx Streaming.vi` Front Panel. Listen to the demodulated waveform. Record your results and explain your observation in terms of $H(f)$.

   Question 5.6: Observe & explain.

5.3 Exercise: Digital discriminator

In this exercise, you will create and test a digital implementation of an FM discriminator.

1. Copy `Rx Streaming.vi` to your workspace. Modify the VI to implement the receiver structure shown in Figure 5.3. (Search LabVIEW for `derivative`.) Set `IQRate` to 1 MHz and match the carrier frequency to the transmitter. Display the time signals $v_{\text{I}}(t)$ and $y(t)$.

2. Use `Tx Streaming.vi` to transmit an audio file and observe the results. Repeat using a 100 kHz tone as the message signal.

3. Using a 100 kHz tone as the message signal, vary the receiver carrier frequency so that there exists a frequency offset between the transmitter and receiver. Observe $y(t)$ and $v_I(t)$. Record your results (amplitude, frequency, DC shift, etc.) and explain your observation in terms of Equation 5.15.

   Question 5.7: Observe & explain.

4. Lower the `IQRate` at your receiver until the message is distorted. Record your results and explain your observations.

Question 5.8: Observe & explain.

5. In Figure 5.3, signals are split into two paths and multiplied. In your LabVIEW implementation, do the signals remain phase coherent? Why or why not?

Question 5.9: Coherence?

5.4 Exercise: FM chirp waveform

under construction

5.5 Exercise: Range/Doppler radar

under construction