Chapter 2

Amplitude Modulation: Noncoherent Detection

Amplitude modulation (AM) was the first widespread modulation technique in commerical radio broadcasting in the early twentieth century and remains present in most wireless digital services. This introductory chapter has three goals: (1) to learn noncoherent detection of large-carrier, double-sideband (LC-DSB) AM signals; (2) to continue introduction of the hardware and software tools used throughout the laboratory lessons; and, (3) to build and use a simple passive circuit for envelope detection.

2.1 Background

2.1.1 Amplitude Modulation

Modulation is the process whereby some characteristic of a signal (carrier) is varied in accordance with another signal (message). Often, this process translates the frequency content of a baseband message. Modulation serves to match a signal to a channel; for wireless communication, the channel includes a radiating antenna with length on the order of the wavelength of the modulated signal. Modulation also enables multiple simultaneous users of a channel. For example, commercial AM radio broadcast in the United States uses frequency division multiple access (FDMA), with carriers separated by 10 kHz and ranging from 530 to 1700 kHz.

An AM signal is mathematically represented in the form

$$\begin{aligned} s(t) &= A_c\Big[1 + k_a m(t)\Big]\cos(2\pi f_c t) && (2.1)\\ &= \underbrace{A_c\cos(2\pi f_c t)}_{\text{carrier (pilot) tone}} + A_c k_a m(t)\cos(2\pi f_c t). && (2.2)\end{aligned}$$

The spectrum of the AM signal is given by

$$S(f) = \tfrac{A_c}{2}\Big[k_a M(f-f_c) + k_a M(f+f_c) + \delta(f-f_c) + \delta(f+f_c)\Big]. \qquad (2.3)$$

The frequency translation affected by amplitude modulation is shown in Figure 2.1.

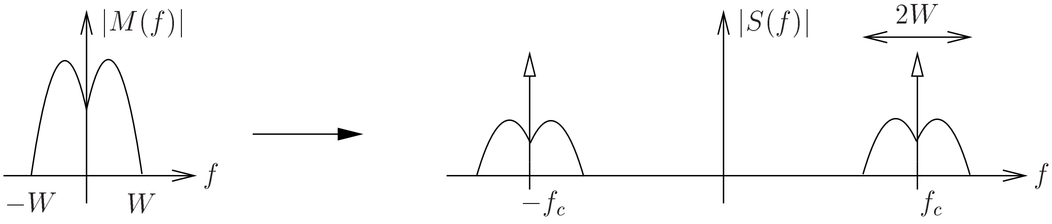

Figure 2.1: Amplitude modulation performs a translation in frequency. Left: notional baseband message spectrum. Right: LC-DSB AM spectrum.

The carrier frequency, f_c Hz, should be larger than the highest frequency in $m(t)$. The parameter k_a is called the *amplitude sensitivity* of the modulator. The *envelope* of $s(t)$ is given by

$$e(t) = A_c\Big[1 + k_a m(t)\Big]. \qquad (2.4)$$

For large-carrier, double-sideband (LC-DSB) AM, the amplitude sensitivity is set so that

$$1 + k_a m(t) \geq 0 \quad \text{for all } t. \qquad (2.5)$$

In this case, the message signal may be easily recovered by an *envelope detector*; however, signal power is consumed by the pilot signal.

2.1.2 Example: Single tone modulation

In the special case of a sinusoidal message, $m(t) = A_m \cos(2\pi f_m t)$, the AM signal is given by

$$s(t) = A_c \Big[1 + \mu \cos(2\pi f_m t) \Big] \cos(2\pi f_c t) \tag{2.6}$$

where $\mu = k_a A_m$ is called the *modulation index*.

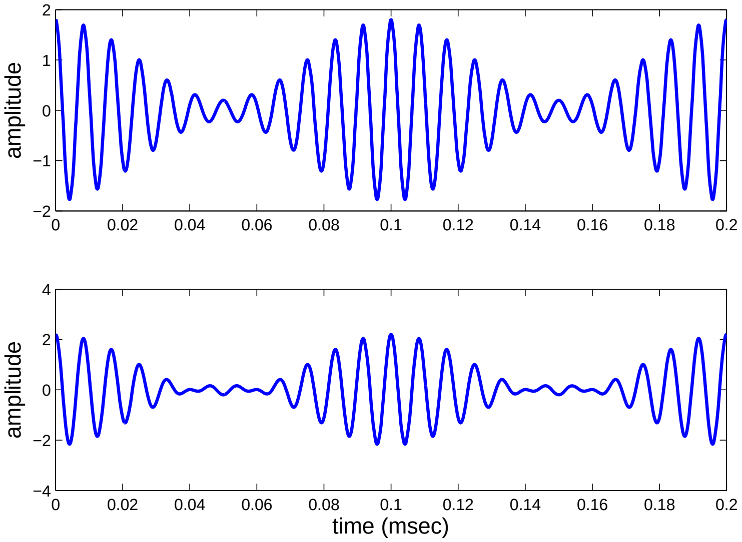

Figure 2.2: Illustration of AM modulation of a tone, $A_c = 1$, $f_m = 10\,\text{kHz}$, $f_c/f_m = 12$. Top: undermodulation, $\mu = 0.8$. Bottom: overmodulation, $\mu = 1.2$.

For $0 \leq \mu \leq 1$, the modulation index can be measured by observing that the envelope has minimum and maximum values

$$e_{min} = A_c(1 - \mu) \qquad \text{and} \qquad e_{max} = A_c(1 + \mu). \tag{2.7}$$

Then, from the ratio of the extremal values, μ is given by

$$\mu = \frac{1 - (e_{min}/e_{max})}{1 + (e_{min}/e_{max})}. \tag{2.8}$$

For $\mu = 1$ the AM signal is said to be 100% modulated and the envelope reaches 0. The signal is said to be *critically modulated* for $\mu = 1$. The signal is said to be *undermodulated* for $\mu < 1$ and *overmodulated* for $\mu > 1$. Thus, with overmodulation $1 + k_a m(t)$ goes negative on some time intervals and the message $m(t)$ cannot be recovered solely from the envelope.

From Equation 2.6, the total power in $s(t)$ is

$$P_s = 0.5A_c^2 + 0.25A_c^2\mu^2 \tag{2.9}$$

The power in the sidebands due to the message is

$$P_m = 0.25A_a^2\mu^2, \tag{2.10}$$

so the ratio is

$$\eta = \frac{P_m}{P_s} = \frac{\mu^2}{2 + \mu^2} \tag{2.11}$$

and is monotone increasing for $\mu \geq 0$. Thus, to avoid overmodulation, we constrain $0 < \mu \leq 1$ and learn that the modulation is most efficient for 100% modulation.

Question 2.1
1. Use time-domain reasoning to argue that 100% modulation is the most power efficient for LC-DSB AM using a noncoherent receiver.

Question 2.2
2. Derive Equation 2.8.

2.1.3 Envelope detector

Mathematically, an envelope detector is described by

$$\begin{aligned} v(t) &= \frac{\pi}{2}\mathrm{LPF}\{\,|r(t)|\,\} - A \qquad (2.12)\\ &\approx m(t). \qquad (2.13) \end{aligned}$$

The envelope detector is a *noncoherent* receiver in that it does not use knowledge of the phase of the carrier signal. The gain $\frac{\pi}{2}$ compensates for the loss incurred when lowpass filtering the rectified signal, as seen in Figure 2.3

$$\frac{\int_0^{\frac{1}{4f_c}} \cos(2\pi f_c t)dt}{\frac{1}{4f_c}} = \frac{2}{\pi}. \tag{2.14}$$

A circuit model of Equation 2.12 using passive devices is shown in Figure 2.4. The absolute value, $|\cdot|$ is easily approximated using a diode; an RC filter provides the lowpass filtering; and, an additional capacitor provides the DC block to achieve the level shift of $-A$. Further, a resonant LC circuit typically provides tuning of the antenna to the carrier frequency, f_c.

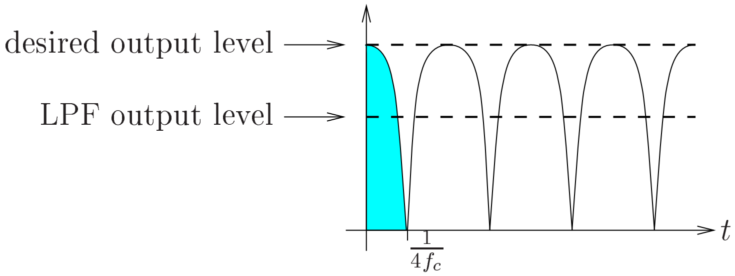

Figure 2.3: Derivation of gain factor in Equation 2.12.

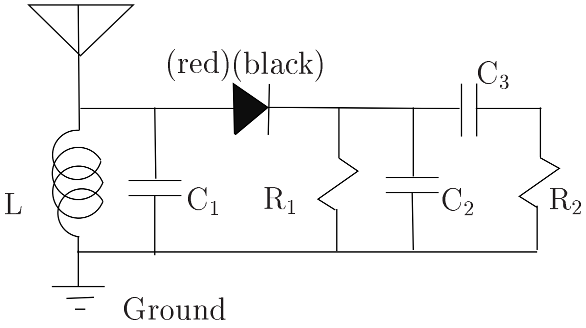

Figure 2.4: Circuit model for an envelope detector.

2.1.4 Square-law detector

A square-law detector is given in block diagram form in Figure 2.5. The detector, like the envelope detector, works without requiring generation of the carrier signal at the receiver.

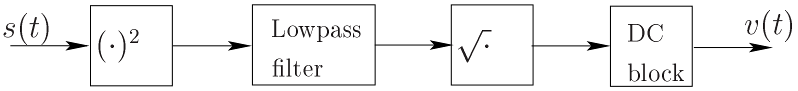

Figure 2.5: Block diagram for square-law detection of LC-DSB AM signals.

A simple trigonometric identity explains the operation of the square-law detector:

$$\begin{aligned} s^2(t) &= A_c^2\Big[1+k_a m(t)\Big]^2 \underbrace{\cos^2(2\pi f_c t)}_{0.5+0.5\cos(2\pi 2 f_c t)} \\ &= \frac{A_c^2}{2}\Big[1+k_a m(t)\Big]^2 + \underbrace{\frac{A_c^2}{2}\Big[1+k_a m(t)\Big]^2 \cos(2\pi 2 f_c t)}_{\text{remove } 2f_c \text{ term with LPF}} . \end{aligned} \quad (2.15)$$

Thus, by placing a square root operation after the lowpass filter, a scaled version of the message signal is recovered with a DC offset. For messsage signals without signal energy near zero frequency, the DC offset can be removed by a simple highpass filter.

2.2 Experiment: Construct a Crystal Radio

A basic AM radio receiver can be constructed using only passive components. By converting approximately one trillionth Watt of power from the transmitted waveform into acoustic energy, the circuit is able to produce sound pressures perceptible by the human ear.

2.2.1 Construction

Use the following parts[1] to construct a basic AM radio receiver.

1. A tunable resonant LC circuit for selecting a frequency channel.

---

[1] `http://www.scitoys.com`

   (a) Stranded insulated wire for an antenna (or an AM/FM/TV telescoping rod antenna)
   (b) A variable inductor is provided by a ferrite loop. The ferrite rod allows for a small coil and can be moved for coarse tuning.
   (c) A variable capacitor (30 to 150 pf)

2. A germanium diode (1N34A) for rectification

3. A piezoelectric earphone

4. Convenient alligator jumper wires

For a passive receiver, it is essential that the earphone transducer have very high impedance, so that the very low current induced in the antenna can be converted, with high sensitivity, to acoustic energy. The piezoelectric earphone (sometimes called "crystal earphone") used in the lab has over 1 M Ohms resistance, in contrast to the 8 Ohms of a typical speaker. The earphone is made of a disk of brass that is coated with barium titanate ceramic, which changes shape in response to electric current, causing acoustic waves. Quartz is an alternative piezoelectric material and was used with early AM radios.

To assemble your AM recevier, follow five simple steps. Refer to Figure 2.6

1. Connect the black wire from the ferrite loop to the center lead of the variable capacitor. Connect the unpainted wire to the rightmost lead of the variable capacitor.

2. Connect the red germanium diode to the rightmost lead of the variable capacitor.

3. Connect the black piezoelectric earphone wire to the red end of the germanium diode; connect the other earphone wire to the center lead of the variable capacitor.

4. Attach the red wire of the coil to the long wire antenna with an alligator clip lead.

5. Attach the green wire of the coil to a good electric ground using another alligator clip lead; black wire is available in the lab for connecting to the ground posts on the wall-mounted panels.

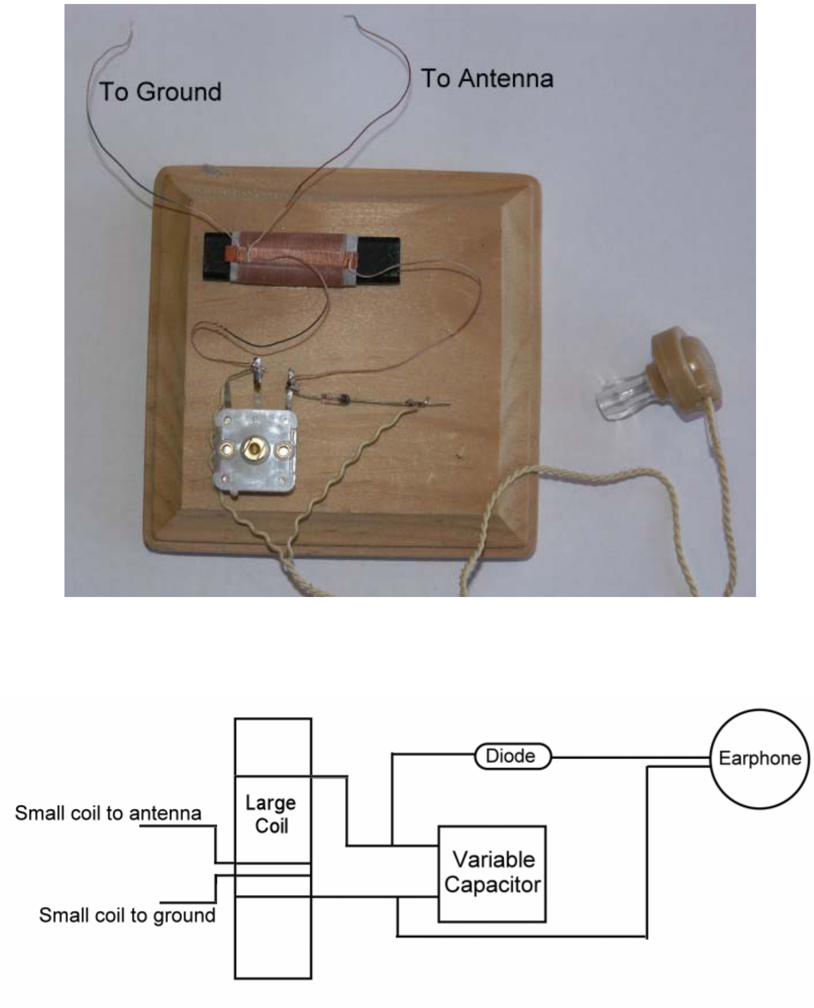

Figure 2.6: Photograph and diagram for crystal radio (http://www.sci-toys.com).

The LC circuit provides for tuning. There are two coils of wire wound around the ferrite rod: the larger coil is connected to the variable capacitor, and the smaller coil is connected to the antenna and ground. As the ferrite rod is inserted into the coils, more of each coil is affected by the ferrite, and so the inductance increases. Increasing the inductance reduces the resonant frequency.

If there are no strong broadcasts to potentially interfere with a desired receive signal, then the signal amplitude may be increased by connecting the antenna and ground directly to the large coil. (Connect the antenna to the center lead of the variable capacitor, and connect the ground to the rightmost lead of the variable capacitor.) With this connection, the resonant circuit will be less effective in suppressing adjacent stations, but the signal attenuation will be less.

The piezoelectric earphone provides the lowpass filtering and DC block represented in Figure 2.4 by capacitors C_2, C_3 and resistors R_1, R_2.

Simple envelope detectors have been made from everyday materials since the earliest days of wireless commuincations. For example, during World War I, soldiers in the field made their own radios from scrap wire, telephone receivers, and diodes constructed from a discarded razor blade and a pencil lead. By lightly contacting the pencil lead to an oxidized spot on the blade, a point contact diode is formed, whereby current flows better in one direction than the other.

Try to receive a commercial AM broadcast signal using your simple crystal radio. The signal strength will likely be very small, and the acoustic signal from the earphone may be weak. Use as long an antenna wire as possible to capture signal energy. For a list of local stations licensed by the US Federal Communications Commission, visit

`http://www.fcc.gov/mb/audio/amq.html`

and use the “Search within a radius” option at the bottom of the page. For example, the Ohio State University, Columbus campus, has location $N\ 44^\circ\ 00'\ 08''$, $W\ 83^\circ\ 00'\ 58''$. Use Google Earth$^{\mathrm{TM}}$ to find the latitude and longitude for your location.

2.2.2 Questions

1. Derive the resonant frequency of a parallel LC circuit. Specifically, find the transfer function, $H(j\omega)$, from $x(t)$ to $y(t)$ for the circuit shown in Figure 2.7. (In the figure, the resistance R approximates an antenna given by a long insulated wire.) Question 2.3

2. For 100 picofarad capacitance, what inductance is needed for a resonant frequency of 1150 kHz? Question 2.4

3. If the capacitance is increased, does the resonant frequency increase or decrease? Question 2.5

4. If the inductance is increased, does the resonant frequency increase or decrease? Question 2.6

5. Sketch a wiring diagram for your crystal radio (simply show connections between your actual components). Question 2.7

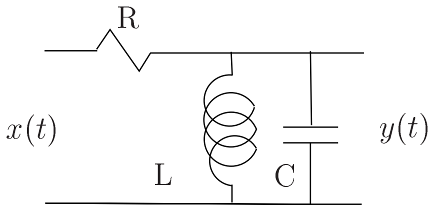

Figure 2.7: RLC circuit model.

2.3 Experiment: AM Transmission with Overmodulation

Use your IF transceiver to create an AM transmit signal and demodulate using your crystal radio components. First, copy the `Tx Streaming.vi` program from the course directory into your workspace.

1. Select an audio `.wav` file (limit size to approximately 150 kB, because the interpolation done in the MATLAB Script Node will greatly expand the file size.

2. Execute your `Tx Streaming.vi` program. Select `audio` for the input and choose, from the Front Panel directory listing, the path your `.wav` file. Configure your `Tx Streaming.vi` transmitter to broadcast in the AM radio band (530 – 1700 kHz), and set the `IQrate` low, so that the interpolated `.wav` file does not grow too large.

3. Connect your `AO-0` output to your envelope detector. This end, terminate a SMA cable with a PCB mount straight SMA jack. Connect to the center pin of the SMA jack using an alligator clip; use one of the four square legs as a ground. Receive the transmission with your crystal radio receiver. Tune the LC circuit for best volume as heard through the earphone.

4. Edit the MATLAB Script Node within the Block Diagram of your transmit program, `Tx Streaming.vi`, to adjust the modulation index, μ. Specifically, the program creates a signal using an offset, `beta`. The user selected input, `gamma` is inversely proportional to μ and determines the size of the offset:

   ```
   beta_fac = max(abs(Tx))*(gamma);
   Tx=(beta_fac+Tx); %add offset to message
   Tx= Tx/max(abs(Tx))*0.95; %scale final size
   ```

   Note that `gamma=1` corresponds to critical modulation, `gamma<1` corresponds to overmodulation, and `gamma>1` corresponds to undermodulation. To change your MATLAB code, stop execution of the vi, type any changes, save (`control s` or click on the checkmark in the menu bar), then resume execution.

Record your observed results at the receiver as a function of the input value `gamma`. For each `gamma`, list the corresponding value of the modulation index, μ. Offer an explanation for the observed behavior. Among your choices of modulation index, include at least three values: critical modulation; a large value of `gamma` (e.g., 5); and, a small value of `gamma` (e.g., 0.2).

Question 2.8

2.4 Experiment: Programmable waveform

Modify the `Tx Streaming.vi` template to transmit a single sinusoid.

2.4.1 Procedures

1. Modify the MATLAB Node Script in the `Tx Streaming.vi` Block Diagram to create a perceptually pleasant tone of 400 Hz at baseband

```
>> f=400;
>> Tx = cos(2*pi*f*[1:buffer]/TxRate);
```

In using the VI, set `TxRate` equal to the `ActualIQrate` as input variables to the MATLAB Script Node.

2. Use `Tx Streaming.vi` to modulate the baseband signal, $m(t)$, to an AM radio frequency, such as 1600 kHz. Receive the signal with your crystal radio receiver; use the PCB mount straight SMA jack to connect `AO-0` to the crystal radio, as in Experiement 2.2.

   (a) Use `gamma =1` for critical modulation and listen to the envelope detector.

   (b) Use `gamma = 0` for zero energy in the carrier (pilot) tone and listen to the envelope detector.

   Describe and explain your observations. (Hint: sketch the signal at the diode output.)

2.4.2 Questions

1. From the exercise in Section 2.4, list your MATLAB code (one line).
2. Explain any limitations on the baseband signal, $m(t)$, that are imposed by the `IQ rate` parameter set in the `Tx Streaming.vi` Front Panel. Question 2.9
3. Describe and explain your observations from the exercise in Section 2.4. Question 2.10

2.5 Experiment: Square-law detector

Modify the `Rx StreamingIntro.vi` template to implement a square-law detector for noncoherent reception of the AM transmit signal. Let the transmit message signal be a tone, as in Exercise 2.3. Observe the output of the receiver on the time and frequency plots avaiable at the Front Panel of `Rx StreamingIntro.vi`.

2.5.1 Procedures

1. Use `gamma=1` for critical modulation.

2. Use `gamma=0` for zero energy in the carrier (pilot) tone.

Describe and explain your observations.

To implement the square-law detector, you can use the graphical programming tools available in LabVIEW by following these six steps:

1. In `RxStreamingIntro.vi`, connect the received data (the wire connected to the `time signal` plot) to a `Square` function; the `Square function` can be generated via right click on the Block Diagram and menu selection `Numeric >> Square`). Press `Ctrl+h` to get help.

2. Click on the output from the `Square` function and connect it to the `Butterworth filter` function, which is available in `Signal Processing >> Filters >> Butterworth`. The help window (`Ctrl+h`) you have opened shows the connections to be made in the function over which the mouse hovers.

3. Select `LowPass` in filter type. To do so, first hover the mouse over the filter and look for the input `filter type` in the help window. Second, right lick at the input of `filter type` and choose `Create >> Constant`). Third, select the order of the filter to be 5 or higher by again using `Create Constant`. Note that a higher order yields a steeper transition from passband to stopband. Fourth, select the `Actual IQ rate` (from outside the while loop) for the sampling frequency. The while loop is observable as a grey colored box. Fifth, select the `fl` (low cutoff frequency) as the desired cutoff frequency by using right click on the input and using `Create >> Constant`).

4. Observe the signal by connecting the output from the filter to the `time signal` plot. If you find that the signal goes negative, then connect the output from the filter to an `Array Max and Min` function (right click on block diagram and select search to find it) and subtract the minimum value of the filtered data vector from the original vector by using `Numeric >> Subtract`.

5. Now take the `square root` of the vector (`Numeric >> Square root`).

6. To remove the DC offset due to the large-carrier AM, first find the mean (`Mathematics >> Prob & Stat >> Mean` and then subtract `Numeric >> Subtract`. Then, take the output and connect it to both the `time signal` plot and to the niput from which the frequency spectrum is calculated and plotted; that is, connect the final processed signal to the `re` input of the `real/imag to complex` function.

2.5.2 Questions

1. What is an appropriate choice of passband and stopband for your low-pass filter? Question 2.11
2. What is an appropriate choice for `IQrate` at the receiver? To avoid any aliasing of the squared signal? to allow aliasing, but prevent aliasing from corrupting the desired baseband signal? Question 2.12
3. Describe and explain the observed demodulate waveforms for `gamma=0` and `gamma=1`. Question 2.13