LAB 3:
VARIABLE STRUCTURE POSITION CONTROL

Overview

The objective of this experiment is to apply the concept of variable structure control to the position control of a DC motor. Basically, the controller is to switch between two constant gain controllers in the feedforward path of the closed-loop system. The basic reference is: Bengiamin and Kaufman, "Variable Structure Position Control," *Control Systems Magazine*, Vol. 4, No. 3, pp. 3-8, August, 1984. Another reference to be checked is: Lin and Tsai, "A Microprocessor-Based Incremental Servo System with Variable Structure," *IEEE Trans. Industrial Electronics*, Vol. IE-31, No. 4, November, 1984. The plant will be an VAR-586 Electro-Craft DC motor and the digital computer will be used for control.

Preparation

- Study the references on variable structure control given above.
- Derive a detailed mathematical model (transfer function and state-space) of the DC motor from voltage applied to the angular position in radians. Reduce the order of the model as necessary.
- Design variable structure controller and prepare required program.
- Simulate your model and controller, including the various friction effects.

Task Description

In this task, you are to use variable structure control to control the position of an Electro-Craft VAR--586 Series permanent magnet DC servo motor. BE EXTREMELY CAREFUL when making your connections to the amplifier circuit and motor. **DO NOT APPLY POWER TO YOUR AMPLIFIER CIRCUIT OR MOTOR UNTIL YOUR TA HAS VERIFIED YOUR CIRCUIT!**

You are to investigate the effects of varying the sampling rate, gains, slope of the sliding surface in the phase plane, and the initial error.

Modeling the Motor

Often, motor dynamics are described by the following transfer function:

$$\frac{\omega}{\upsilon} = \frac{K}{(\tau_e s + 1)(\tau_m s + 1)}$$

where

ω : the shaft angular velocity
υ : the input voltage
K : the DC gain of the motor
τ_e : the electrical time constant
τ_m : the mechanical time constant

Assuming the mechanical time constant is much greater than the electrical time constant, $\tau_e << \tau_m$, then the model may be reduced and the motor dynamics can be approximated by

$$\frac{\omega}{\upsilon} = \frac{K}{(\tau_m s + 1)}$$

Since we are interested in controlling the shaft position, θ,

$$\frac{\theta}{\upsilon} = \frac{K}{s(\tau_m s + 1)}$$

In this lab, however, we will be using a transconductance amplifier between the computer and the motor. Therefore, the voltage applied from the D/A channel to the amplifier will result in a particular current applied from the amplifier to the motor. Therefore, we need a transfer function from the armature *current* to shaft position. To obtain this model, the following equations should be used:

$$T_{dev} = K_t i_a$$

$$\begin{aligned} T_L &= T_{dev} - T_{friction} \\ &= J\ddot{\theta} \end{aligned}$$

When obtaining a linear model of the motor, it is often assumed that the friction torque is zero. However, you will observe in the laboratory that the combined motor, gearing, tachometer and potentiometer can exhibit a substantial nonlinear stiction friction as well as ordinary rolling friction.

Ignoring friction, we obtain the linear motor transfer function

$$\frac{\theta}{i_a} = \frac{K_t / J}{s^2}$$

The state equations of the motor are easily found by letting

$$x_1 = a(\theta - \theta_{ref})$$
$$x_2 = b\omega$$

where a and b are the potentiometer and tachometer gains, respectively, and θ is the desired angle to be achieved.

Hardware Specifications

Load

As Figure 3.2. indicates, the plant assembly consists of a DC motor with an integral tachometer (a generator) to be used as a velocity transducer. Rigidly fixed to the shaft is a metal disk with a hub, introduced as a simulated inertial load. Modeling both the disk and the hub as right circular cylinders of uniform density, their contribution to the total inertia can be calculated using the following specifications:

- Disk radius, r_D = 2.0 in = 50.8 mm
- Disk thickness, t_D = 0.25 in = 6.35 mm
- Disk weight, ω_D = 0.85 lb = 3.7825 N
- Disk density, ρ_D = 7489.6 kg/m^3
- Hub radius, r_H = 0.5 in = 12.7 mm
- Hub thickness, t_H = 0.5 in = 12.7 mm
- Hub weight, ω_H = 0.05 lb = 0.2225 N
- Hub density, ρ_H = 3524.5 kg/m^3

The motor shaft is further coupled via a bellows coupling to a potentiometer to serve as the position transducer. Ignoring the inertia of the potentiometer and coupling, the total inertia is simply the sum of the inertias of the disk, the hub, and the armature given below.

$$J = J_a + J_D + J_H$$

DC Motor

Since we are using a current amplifier, only two motor specifications become critical. The following data are from the motor specification sheets for the Electro-Craft VAR--586 Series Model 0576-01-018:

- Rated voltage: $V_{max} = 30$ V
- Rated speed (no load): $\omega_{max} = 6900$ rpm
- Back emf constant: 3.86 V/krpm < K_e < 4.72 V/krpm
- Friction torque: $T_{friction}$ *(Motor should start and continue to run with 0.144 A to 0.43 A.)*
- Torque constant: $K_t = 5.8$ oz-in/A = 0.041 N-m/A (±10 %)
- Electrical time constant: $\tau_e = 2.1$ ms (±10%)
- Armature inertia: $J_a = 0.0055$ oz.in.s^2 = 4.0×10^{-6} kg.m^2

Note: the friction torque given here is the rolling friction for the motor only and does not take other factors such as the potentiometer into consideration.

Tachometer

The tachometer is basically modeled as a generator. The generator voltage gradient is given as

Voltage gradient: $b = K_{tach} = 3.0$ V/krpm = 0.029 V/rad/s

Potentiometer

The associated transducer gain, a, is readily obtained from the knowledge that the potentiometer is powered with 10 V and is mechanically limited to a 10-turn swing.

Amplifier

The nominal design for the current amplifier maintains a current limit set at I = 2 A, and its gain has been adjusted to 0.2 A/V so that full-scale input voltage (10 V) yields full-scale output current.

At the time of this writing, the components used in each amplifier vary from unit to unit. Thus, the following table is provided:

Unit number	Gain (A/V)	Current Limit (A)
1	0.78	6.94
2	0.21	2.67
3	0.78	6.94
5	0.40	N/A

Hardware Connections

Before connecting the hardware to the computer and running the experiment, you should test your software using the variable voltage supply. Manually change the voltages (as the motor would) and watch the output of the D/A on the oscilloscope. Also, test the hardware to make sure of the polarity of the potentiometer and tachometer to ensure that a ``positive" position and velocity correspond to positive voltages.

The necessary hardware connections are depicted in Figure 3.2. 24 VDC power supplies are connected in series to provide the ±24 VDC power for the amplifier. The variable power supply A and B outputs are to be adjusted to +10 VDC and -10 VDC to power the yellow and violet leads of the position potentiometer, respectively. The red and black motor leads are the winding power and connect to the yellow plugs on the amplifier.

Two analog inputs and one output are required for the controller. The inputs obtain the position and velocity feedback from the potentiometer (orange lead) and the tachometer (white lead), respectively. For proper ground referencing, the tachometer green lead and the variable supply common must be connected to the analog ground of the interface card.

Sliding Mode/Variable Structure Controller Design

As described in the references, the variable structure controller will switch the feedback gains (either $\psi = \alpha$ or $\psi = \beta$) based on the location of the current state in the phase plane. It is desired that the state move to the switching line and then slide along the switching line towards the origin as shown in Figure 3.1.

The switching line is described by

$$\sigma = Cx_1 + x_2 = 0$$

where C is some constant which determines the slope of the switching line in the phase plane. Note that this equation for σ is a first order differential equation and the line $\sigma = 0$ describes the system average behavior with the dynamics of our choice, provided the system can be driven onto this line.

To design the controller and find the feedback gains, choose as a Lyapunov function candidate

$$V = \sigma^2$$

Note that σ^2 is positive and nonzero everywhere except where $\sigma = 0$ (on the switching line). To achieve the desired trajectory, it is required that

$$\dot{V} = 2\sigma\dot{\sigma} < 0$$

This implies that we want

$$\sigma\dot{\sigma} < 0$$

Note that

$$\dot{\sigma} = C\dot{x}_1 + \dot{x}_2$$

A variable structure control law should be designed such that $\sigma\dot{\sigma} < 0$ is satisfied. By substituting and massaging this inequality, one may obtain inequality relations for the control u which may be solved as a function of the plant and switching surface parameters. Note that the inequality may be satisfied by adding or subtracting a positive quantity δ as appropriate.

$$u = \begin{cases} \psi x - \delta & for \quad \sigma > 0 \\ \psi x + \delta & for \quad \sigma < 0 \end{cases}$$

where ψ is a function of the plant and switching surface parameters. δ may be constant or may vary with the states. For this lab, it is recommended that

$$\delta = K_1 \,|\, x_1 \,| + K_2$$

where K_2 represents an input voltage large enough to overcome the friction torque of the motor and potentiometer. By increasing K_2, a larger input will allow the system to hit sliding mode quicker when the error is large.

With the appropriate control law which satisfies the basic inequality, the stabilizing feedback control will guarantee the state trajectory will reach the sliding surface and follow it to the origin. If it were possible to switch the gains instantaneously, the system could follow the sliding surface perfectly to the origin. However, due to delay and inertia, the state trajectory often overshoots the sliding surface which could result in damage to equipment.

Report

Phase-plane trajectories are to be generated and plotted. A full mathematical analysis and a detailed model and simulation is to be included in the report. Be sure to report and interpret all results and discuss the effects of varying the prescribed parameters.