

EE854 Theory of Optimal Control

HW#1

By April 16, Monday

Spring 2003

1. Find the extremal for each of the following fixed-end point problems:

1.1 $J = \int_1^2 \dot{x}^2 t^3 dt, \quad x(1) = 0, \quad x(2) = 3.$

1.2 $J = \int_1^2 \dot{x}^2 t^{-3} dt, \quad x(1) = 2, \quad x(2) = 17.$

1.3 $J = \int_0^\pi (\dot{x}^2 + 2x \sin t) dt, \quad x(0) = x(\pi) = 0.$

2. Find the extremal of

$$J = \int_1^T \dot{x}^2 t^3 dt,$$

$x(1) = 0, \quad T > 0, \quad T$ is finite and $x(T)$ lies on the curve

$$x(t) = 2/t^2 - 3.$$

3. Find the extremal for

$$J = \int_0^\infty (x^T Q x + \dot{x}^T \dot{x}) dt, \quad x^T = (x_1, x_2, \dots, x_n),$$

$$Q^T = Q, \quad Q > 0, \quad x(0) = x_0, \quad x(\infty) \text{ is free.}$$

Note that a nonsingular matrix D exists such that

$$D^{-1} Q D = \Delta, \quad \Delta = \text{diag}(\lambda_i)$$

λ_i 's are eigenvalues of $Q, \quad \lambda_i > 0,$

λ_i 's and D are assumed to be known.