

EE854 Theory of Optimal Control

HW#2

By April 25, Friday

Spring 2003

Problem 1.

For the first-order system

$$\dot{x} = u, \quad x(0) = x_0$$

with the criterion

$$J = \int_0^1 u^2 dt + 0.5[x(1)]^2$$

and free $x(1)$

find the optimal control using

- Hamiltonian.
- Riccati equation

Show that the solutions coincide.

Problem 2.

Find the optimal control for the linear system

$$\dot{x}_1 = x_2,$$

$$\dot{x}_2 = u$$

with the criterion

$$J = \int_0^{\infty} (x_1^2 + x_2^2 + ru^2) dt, \quad r > 0.$$

Analyze asymptotic behavior of the eigenvalues of the feedback system with the penalty gain r tending to zero.

Problem 3.

For the linear system

$$\dot{x}_1 = x_2,$$

$$\dot{x}_2 = a^2 x_1 + u$$

with the criterion

$$J = \int_0^{\infty} [(qx_1 + x_2)^2 + u^2] dt$$

find the set on the plane (q, a) , such that the optimal system is unstable.